Bridging between Real World & Mathematics Ideas through Modelling Task

Abolfazl Rafiepour

Faculty of Education and Arts (FLU), Nord University, Levanger, Trondelag, Norway

Abstract: In this paper duality between real world phenomenon and mathematics will be discussed. This duality exists for many years which considerably remarked in the history of mathematics curriculum. One of the good potential for filling the gap between real world phenomenon and mathematical ideas would be modelling tasks which require performing the modelling cycle. In this paper after discussion about modelling in more details, two modelling cycles will be elaborated in the context of mathematical tasks that are related to everyday life. Main ideas behind different modelling cycles is starting point that would be from real world situation. Then gradually mathematical elements of real world phenomenon identified and mathematics problem will have shaped in a form that could be solved through mathematical problem solving techniques. Furthermore, it would be important phase that call interpretation of mathematics answer and check it in front of real world situation. This paper will be followed by discussion about learning theories that support the idea of modelling. Finally, two educational challenges (Design good modelling tasks and assessing of students performance) in mathematical modelling activity will be discussed.

Keywords: Real world, Mathematics, Modelling Task, Modelling Cycle, Word Problems.

Citation: Rafiepour, A. (2023). Bridging between Real World & Mathematics Ideas through Modelling Task. In M. Shelley, O. T. Ozturk, & M. L. Ciddi, *Proceedings of ICEMST 2023-- International Conference on Education in Mathematics, Science and Technology* (pp. 148-156), Cappadocia, Turkiye. ISTES Organization.

Introduction

> Ultimately, only life educates, and the deeper that life, the real world, burrows into the school, the more dynamic and the more robust will be the educational process. That the school has been locked away and walled in as if by a tall fence from life itself has been its greatest failing. Education is just as meaningless outside the real world as is a fire without oxygen, or as is breathing in a vacuum (Vygotsky, 1997).

Mathematics exist everywhere around us. We can see mathematics in everyday phenomenon, in high-tech instruments like mobile phone and satellite. It is true that mathematical ideas start from human being real life and real world phenomenon but mathematics can quickly lose its connection with reality because of its nature and essence. But, sometimes students feel that the mathematical concepts which they learnt at school are useless and meaningless. They couldn't understand the situation of mathematics problems and they couldn't solve it, so they start to complain about that and some of them experience math as boring and stressful subject. What we can do in this regards as teachers and educators of mathematics at school and university level? Is there any excellent way for making connection between mathematics ideas and students real life? How we can organize mathematical lessons around that excellent way?

Upon my 20 years of experiences in teaching mathematics at school level and based on my teaching and research experiences in different universities around the world, I could recommend to use modelling approach as an excellent way for overcoming this challenge. In many mathematics curriculum documents at many countries we can find some sort of modelling approach as their learning outcomes. At the university level, also we can find or define connection between real life experience and mathematics (which explicitly refer to modelling approach) as learning outcome.

It is seems to be an easy task for implementing modeling approach as learning outcome at actual mathematics classroom setting, but there are many details which reveals the complexity and difficulties of implementing this approach at the mathematics classroom. In the next section, brief history related to using application of mathematics in curriculum will be discussed and then modelling process will be define through modelling cycle and modelling approach will explain with more details.

Literature Review

> How we can maximize learning of students in mathematics? This is an old question! But, is there a new answers for that old question?

Mathematics curriculum change during last centuries for many time to prepare good response for above question. Niss, Blum and Galbraith (2007) discuss about the swing of the curriculum pendulum between pure and applied mathematics during the last two centuries. For example in early 1800, mathematics curriculum focused on applied mathematics while in late 1900 mathematics curriculum focused on pure mathematics. Upon Niss, Bloom and Galbraith (2007), in the past, mathematics as a field of human knowledge included its adjacent fields such as physics, astronomy and engineering and as a result, until the beginning of the 19th century, mathematics was a part of natural sciences and was included other practical activities. From the beginning of the 19th century, much attention was paid to purely mathematical education in order to strengthen intellectual powers (Nice, 1996). Since the late 19th century, most post-primary curricula have emphasized both pure and applied mathematics components. But during the 20th century, the school mathematics curriculum has always fluctuated between these two divisions - applied mathematics and pure mathematics.

Over time, the balance has sometimes been in favor of teaching pure mathematics and sometimes in favor of teaching applied mathematics. These successive changes have been based on social tendencies and various changes in the educational and learning needs of students; For example, in England, industrialists claimed that school graduates are unable to use their mathematical knowledge in real world situations (Pollack, 1979). The protest of the industries was actually because the students were only able to use their knowledge to solve familiar problems and they weren't able to use their mathematical knowledge for solving unfamiliar and non-routine problems. This protest of the industrial owners led to the introduction of some applications in the school mathematics curriculum. As a result, to enable students to solve real world problems. Therefore, mathematical modeling and application appear in school mathematics curriculum. Although according to Nice, Bloom and Galbraith (2007), attention to the modeling and application in mathematics education has a long history and rooted in cultural activities of Islamic, Indian, Chinese, and Egyptian group. For example finding the direction of Qibla or calendar setting in Islamic cities during Islamic golden age were a modelling problems.

Mathematical modelling is a round-trip process that connects the real world and the mathematical world (Steelman, 2010). There have been several interpretations for mathematical modelling cycles. The four steps modelling cycle (see picture 1) is one of the well-known (Vershafel, 2002, Kaiser and Schwartz, 2006, Rafiepour, Stacey and Gooya, 2012) one which the modeling process starts with a problem from the real world and then it is formulated and becomes a mathematical problem; then the mathematical problem is solved by mathematical problem solving techniques and the mathematical answer is interpreted in the real world to measure its compatibility with the real world. Finally the answer should be checked in front of real situation to control sensemaking of the final answer.

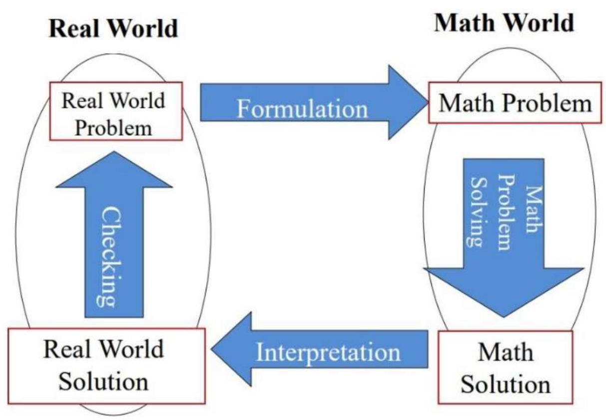

Figure 1. Four steps modelling cycle

As an example for modelling task, you can consider travelling from one city to another one for the purpose of participating in a workshop which there are different possible ways for traveling contain using personal car, train and flight. If you asked to find best way in terms of time consuming, or in terms of economical point of view or regard to environmental concerns and air pollution, what would be your choose? You have to choose one of these ways upon your situation, so this problem start from real world and you have to make mathematical problem based on given information through process of formulation (see Figure 1). Then in mathematical world, you have to use your mathematical knowledge and mathematical problem solving techniques. Then after finding the answer in math world, you have to do interpretation and checking phase of modelling cycle (see Figure 1). There is another well-known and well defined modelling cycle (see Picture 2) which has seven steps and it will be more suitable for explaining formulation part of modelling cycle which mathematical elements of real situation realized with more detailed steps. Specifically the terms of situation model refer to mental representation of the situation.

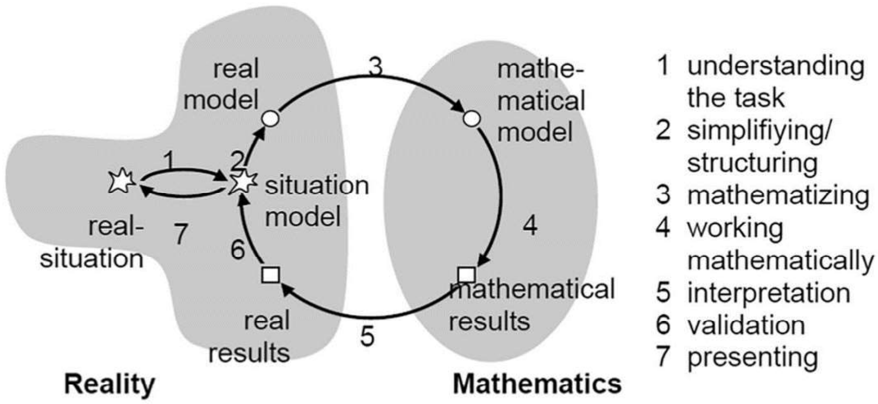

Figure 2**.** Seven steps modelling cycle (Blum and LeiB, 2007)

Learning theories which support the idea of modelling and application

No single logic is strong enough to support the total construction of human knowledge. (Jean Piaget, 1970)

Like as Piaget, Rienecker, Jørgensen, Dolin, and Ingerslev, (2015) believe that there is no "correct" learning theory that explains the learning of everything in all situations. Even in teaching and learning of a single subject like mathematics, it should be more effective to use different theories of learning. In this paper, modelling and application introduced as a mean for filling the gap between real life experiments and mathematical ideas. Several learning theories support the idea of using mathematical modelling and application in the process of teaching and learning mathematics. For example, in the modelling cycle learners start from real world situation which is familiar for them and this component alien with constructivism point of view that discuss for starting teaching activities from students experiences and try to build knowledge up on these experiences. Rienecker, Jørgensen, Dolin, & Ingerslev, (2015) believe that the key to constructivism as a learning theory is that each individual person constructs his or her knowledge through interaction with his or her surroundings.

Essence and identity of modelling tasks put based on learners group working and discussion about every component of modeling cycle. So, this element of mathematical modelling alien with social cultural theory of Vygotsky. In the line with Rienecker, Jørgensen, Dolin, & Ingerslev, (2015) which discuss about consequences of applied social constructivism theory of learning for teaching practice, it would be consider that in modelling tasks students would be able to work into the social situations where they can apply and reflect on the application of the culture's artefacts. In this situation, they have many dialogues to exchange views and understandings.

Another theory of learning that support modelling point of view will be critical reflection that introduced by Paulo Freire (1987, 2005). Students who start to work in the modelling tasks, have to formulate real world problem to math world problem and after solving this problem, they have to interpret and check the solution in front of real world situation. In this phase of modeling cycle, they have to use their critical reflection skills to make final answer.

Designing a good modelling activity

One of the most important and challenging part of using modelling activities in the mathematics classroom would be finding or designing a good modelling activity that is suitable, believable and authentic. Mathematical modelling task should be suitable for the students and that means the task must related to level of students mathematical knowledge. Mathematical modelling task should be believable by students and that means the task must choose or designed based on students real life experiences. Finally, the mathematical modelling task must be authentic for the students and that means whether modelling task make sense for students or not?

Mathematical modelling tasks could be taken from literature or designed by mathematics teachers or instructor. Mathematical modelling tasks could be designed by teachers or instructors in advanced, or develop by teacher and with corporation of students based on their common experiences, or developed based on students experiences and support with teachers feedback. Noticed that the last method for designing modelling task will be suitable just for experienced classroom that complete several modelling activities. One of the useful researches which help beginner and give them some ideas for designing mathematical modeling task is Bonotto paper (2007) which discuss how we can replace ordinary word problems in the textbooks by realistic mathematical modelling activities.

As an example for mathematical modelling activities consider below modeling activity which related to students everyday life who live in society based on agriculture and horticulture. Teacher can bring different type of apples (size, shape and weight and color) in the mathematics classroom and ask students to help apple farmers in order to classified these apples for different market. Mathematical modelling task related to classification of apple presented in Figure 3.

You are apple farmers and must classify the apples in your basket for several buyers:

- high quality gift basket distributors (the "best" apples);
- supermarkets ("apples which can be sold");
- school districts ("smaller apples for lunch");
- and the least attractive apples which can be used for making purees and juices.

Each group of students has tools for measuring, weighing…and a blank piece of paper for writing the results. You must classify your apples for the different buyers and then explain how you have decided to classify the apples and why. In order to do this, each group has to prepare a presentation for the other apple farmers in order to help them understand how to classify their apples for potential buyers. In the end, we will bring together all of your explanations.

Figure 3. apple classification modelling task (Alsina and Salgado, 2022)

As we can see working on mathematical modelling activities like apple classification modelling task (see figure 3) can activated the high level (Relational level and Extended abstract level) of SOLO taxonomy as well as medium levels (Uni-structural level and Multi-structural level). Action like calculate, identify variable, collect data (related to level 2 of SOLO taxonomy), describe, formulate, solve, express, display, report, simulate (related level 3 of SOLO taxonomy), analyze, explain, compare, summarize, design, optimize, construct (related to level 4 of SOLO taxonomy), and discuss, estimate, evaluate, interpret, predict, criticize, reflect (related to level 5 of SOLO taxonomy), to be done during apples classification modelling activities.

Mathematics teachers or educators also can choose mathematical modeling task from literature and modified it based on students cultural and social context. For example consider below task from Jensen (2009).

- How does the tax you pay depend on the income tax percentage and the VAT percentage?

Different societies and countries have different rules and regulation for tax, and this problem can be modified based on contextual information and used in different societies and countries.

Assessment and supervision in modelling activities

One of the important part of implementing mathematical modelling activities in mathematics classrooms is related to supervision of students and give them continuous feedback. It is not easy and straightforward task for teachers to provide constructive feedback to students, but it is challenging and enjoyable task for them. Usually students specially novice one in mathematical modelling have difficulties in building situation model (see figure 2), to formulate (mathematizing) mathematics model from reality (see step 3 in Figure 2) and in completion of mathematical modelling cycle. For troubleshooting, teachers have to provide different feedback for the students. For example, when the students didn't complete covering of mathematical modelling cycle, teacher can notify them about that. In many cases, students forget to do final steps of modelling cycle (step 6 and 7 in figure 2). In this cases teacher just remind them for do these steps. Sometime students couldn't formulate (mathematizing) mathematics model from reality. In this case teacher can help them to define and consider variable that is important in the situation without reveal direct reference to solution of the problem.

In supervision of students when they are working on mathematical modelling activities, it is very important to provide them useful feedback. Hattie and Timperley (2007) believe that an ideal learning environment occurs when both teachers and students seek answers to three below questions related to feedback.

- Where am I going? (feed up dimension)
- How am I going? (feed back dimension)
- and Where to next? (feed forward dimension)

In feed up dimension, teacher have to ask students to read carefully mathematical modelling activities and understand the context and focus on demand of question. In this way, students can understand all components of mathematical modelling activities and will be able to formulate problem. This understanding help students to develop shared commitment which is necessary for feed up dimension. In feed back dimension, teachers provide information relative to the mathematical modelling activity or students' performance and their success or failure on solving that problem. In feed forward dimension, teachers encourage students to generalized the problem that they solved during their engagement in mathematical modelling activity through changing the assumption.

In last section one of the most important and challenging part of using mathematical modelling activities in the mathematics classroom was discussed. Another challenge of implementation of mathematical modelling activities in the mathematics classroom is related to assessment of students activities when they engage into the modelling activities. Although there are several rubrics for assessing students' performance in modelling activity (e.g. Ludwig and Xu, 2010), but, as Galbraith (2007) mentioned that mathematical modelling community of research need to do more research in this area to find most effective ways for assessing students work when they engaged in solving mathematical modeling tasks. Ludwig and Xu (2010, p. 80) framework for assessing mathematical modelling have six different consecutively levels (see Figure 4).

- Level 0: The student has not understood the situation and is not able to sketch or write anything concrete about the problem.
- Level 1: The student only understands the given real situation, but is not able to structure and simplify the situation or cannot find connections to any mathematical ideas.
- Level 2: After investigating the given real situation, the student finds a real model through structuring and simplifying, but does not know how to transfer this into a mathematical problem (the student creates a kind of word problem about the real situation).
- Level 3: The student is able to find not only a real model, but also translates it into a proper mathematical problem, but cannot work with it clearly in the mathematical world.
- Level 4: The student is able to pick up a mathematical problem from the real situation, work with this mathematical problem in the mathematical world, and have mathematical results.
- Level 5: The student is able to experience the mathematical modelling process and validate the solution of a mathematical problem in relation to the given situation.

Figure 4. six level for assessing mathematical modelling competency (Ludwig and Xu, 2010, p. 80)

Concluding Comment

Many students and teachers who participated in one of my previous mathematical modelling workshops acknowledged that they enjoyed from participation on the workshop and they learnt mathematics in meaningful way. But, there are still two challenges for implementing mathematical modelling activities in mathematics classrooms. One of them related to design suitable and meaningful mathematical modelling activities based on students everyday life. Another one related to assessing students during engagement and solving mathematical modelling activities.

I believe that, one of my challenges as mathematics educator who interested to expand mathematical modelling at the mathematics classroom is that supporting prospective mathematics teachers (teachers students at pre-service university education) through designing many mathematical modelling activities in several social-cultural context related to the students and teachers everyday life. Another challenges of mine is that to encourage prospective mathematics teachers to design mathematical modelling activities based on real life phenomenon around them through careful observation of surrounding phenomenon and eliciting mathematical ideas from that phenomenon. In this way they need more and more mentoring through expert (mathematics educators who work in this domain of research) supervision to find their own motivation, courage and self-confidence.

References

Alsina, A. and Salgado, M. (2022). Understanding Early Mathematical Modelling: First Steps in the Process of Translation Between Real-world Contexts and Mathematics. International Journal of Science and Mathematics Education. 20:1719– 1742.

Biggs, J. B., & Tang, C. (2011). *Teaching for quality learning at university* (4th ed.). Open University Press.

Blum, W., & Leiβ, D. (2007). How do students and teachers deal with modelling problems? In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling: Education, engineering and economics* (pp. 222–231). Woodhead Publishing Limited.

Bonotto, C. (2007). How to Replace Word Problems with Activities of Realistic Mathematical Modelling. In: Blum, W., Galbraith, P.L., Henn, HW., Niss, M. (eds.) *Modelling and Applications in Mathematics Education*. New ICMI Study Series, vol 10. Springer, Boston, MA. https://doi.org/10.1007/978-0-387-29822-1_18

Freire, P. (1987). *A Pedagogy for Liberation: Dialogues on Transforming Education*. London: Macmillan.

Freire, P. (2005). *Education for Critical Consciousness*. York New: A Continuum Book the Seaburry Press.

Galbraith, P. (2007). Beyond the Low Hanging Fruit. In: Blum, W., Galbraith, P.L., Henn, HW., Niss, M. (eds.) *Modelling and Applications in Mathematics Education*. New ICMI Study Series, vol 10. Springer, Boston, MA. https://doi.org/10.1007/978-0-387-29822-1_6

Hattie, J. & Timperley, H. (2007). The Power of Feedback. *Review of Educational research*, *77*(1), 81-112. https://doi.org/10.3102/003465430298487

Jensen, T. J. (2009). Modellering versus problemløsning – om kompetencebeskrivelser som kommunikationsværktøj. *MONA*, 2: 37-54.

Kaiser, G., & Schwarz, B. (2006). Mathematical modelling as bridge between school and university. *Zentralblatt für Didaktik der Mathematik*, 36(2), 196–208.

Niss, M. (1996). Goals of mathematics teaching. In A. Bishop et al. (Eds.), *International handbook of mathematical education* (pp. 11–47). Dordrecht: Kluwer.

Niss, M., Blum, W., Galbraith, P. (2007). Introduction. In: Blum, W., Galbraith, P.L., Henn, HW., Niss, M. (eds) Modelling and Applications in Mathematics Education. New ICMI Study Series, vol 10. Springer, Boston, MA. https://doi.org/10.1007/978-0-387-29822-1_1

Piaget, J. (1970). Genetic Epistemology. Translated by Eleanor Duckworth. New York: Columbia university press.

Pollak, H. O. (1979). The interaction between mathematics and other school subjects. In UNESCO (Ed.), *New trends in mathematics teaching* (Vol. IV pp. 232–248). UNESCO: Paris.

Rafiepour, A.; Stacey, K. & Gooya, Z. (2012). Investigating Grade Nine Textbook Problems for Characteristics Related to Mathematical Literacy. *Mathematics Education Research Journal*, 24 (4), 403-421.

Rienecker, L., Jørgensen, P. S., Dolin, J., & Ingerslev, G. H. (Eds.) (2015). *University Teaching and Learning.* Samfundslitteratur.

Verschaffel, L. (2002). Taking the modelling perspective seriously at the elementary school level: promises and pitfalls. In A. Cockburn & E. Nardi (Eds.), *Proceedings of 26th annual meeting of the international group for the psychology of mathematics education* (pp, Vol. 1, pp. 64–80). Norwich: University of East Anglia.

Vygotsky, L. (1997). Educational psychology. Boca Raton, FL: CRC Press LLC.