Pre-service and in-service Elementary School Teacher's Procedural and Representational Knowledge of Fractions

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Abstract: The main purpose of this study is to assess pre-service and public elementary mathematics school teachers' conceptual understanding and computational abilities of fractions. 20 pre-service mathematics teachers and 24 in-service mathematics teachers participated in this study. In-service teachers were divided into two categories; one for teachers having a degree in mathematics and the other for teachers having a degree in any other discipline. Results showed that both pre-service and in-service teachers' computational knowledge is greater than their representational knowledge. However, in-service teachers had difficulties in multiplication of mixed numbers (41.7% correct answers). The study revealed that regarding the computational knowledge no significant difference was found between in-service and pre-service teachers. When considering representational abilities, pre-service teachers were able to perform better than in-service teachers. The difference was significant ($p<0.005$). however, when we compared preservice teachers' performance to in service teachers who graduated from the faculty of pedagogy, there was no significant difference ($p=0.717$). Moreover, faculty of pedagogy graduate in-service teachers performed better than preservice teachers which shed a light on the importance of teachers' specialization even in elementary classes.

Keywords: Fractions, Representational abilities, Computational abilities, Inservice teachers, Preservice teachers

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Introduction

The NCTM' Principles and Standards for School Mathematics (NCTM, 2000) states that fractions are fundamental because of their significant application in daily-life situations. It is a concept that extends over most of the elementary grades (from grade 3 till grade 7 in the Lebanese curriculum and often revisited in all mathematics subject areas) and in which students have difficulties in almost all levels. Sowder & Wearne found that even in the middle grades, fractions represented a challenge for students. Results of their research showed that students have a weak understanding of fraction concepts (Sowder & Wearne, 2006). This lack of understanding is then transposed into difficulties with 'fraction computation, decimal and percent concepts, and the use of fractions in other content areas, particularly algebra' (Brown & Quinn, 2007; National Mathematics Advisory Panel, 2008).

In parallel, teachers that do not understand fraction and are not able to interpret them conceptually will find difficulties in helping their students make sense of that concept (Ball, 1990; Graeber, et al., 1989; Redmond & Utley, 2007). Representations and conceptual understanding are very important to help students understand operations on fraction (Mewborn, 2001).

In teacher education program in the Lebanese university, elementary mathematics education students experience three years of learning how to teach mathematics to students ranging from grade one to grade six. They learn methods of teaching mathematics through three required method courses in year one and year two. Yet, their certification is not a requirement for the Lebanese ministry of education to enable them to enter elementary classes in public schools. Any university degree is acceptable to teach elementary mathematics according to the ministry of education.

The purpose of this study is to assess Lebanese University mathematics pre-service teachers' and public elementary schools' teachers', whether having a degree in teaching mathematics or not, knowledge in fractions, including their conceptual understanding and computational abilities.

Review of literature

When students start learning about fractions, they start constructing their understanding of a different type of number system with its own representations and symbols (Wright, 2008). Students should be taught operations with fractions in concrete terms (Martin & Sebesta, 2004; Patterson, Capraro, Kemp, Standish & Sun, 2003) once the concept is introduced to them (Naiser, Wright & Capraro, 2004). That is why it is very important for teachers to construct activities for student to develop conceptual comprehension of fractions and operations involving fractions. The achievement of such conceptual comprehension can be accelerated through the use of mathematical models and representations like physical and mental actions; a picture, drawing, symbol, or a concrete means entailing the relationship conveyed by a mathematical concept. The representations can be employed to enable students develop new concepts and relationships in their minds, to help students establish the relationships between concepts and symbols, and to assess the level of comprehension in students' mind (Olkun and Toluk Uçar, 2012).

Literature has shown that pre-service teachers' understanding of fraction content knowledge is very weak (Simon, 1993; Cramer, Post, & del Mas, 2002). Ball (1990) found that pre-service teachers have difficulties with the concept of fractions and the meaning of division of fractions. Other researchers showed that pre-service teachers have difficulty in explaining fractions to children and why algorithms work (Chinnappan, 2000). Pre-service teachers will be teaching mathematics in elementary schools and their weak performance may cause serious problems. Therefore, this issue is extremely important and should be addressed.

In-service elementary teachers also showed difficulties in explaining the concept behind operations of fractions. In their research that aimed at looking for in service teachers' knowledge on multiplication and division of fractions at primary schools, Veloo & Puteh (2017) found that teachers prefer algorithmically approach in their work rather conceptual approach. Their practices showed that they lacked understanding of conceptual explanations. Although they came up with correct solutions, they were unable to give explanations for their work through drawings.

Teachers need to have the necessary specialized knowledge, such as knowledge of a variety of representations including concrete models and real-world problems, to help students understand mathematical concepts (Taber, 2000). Yetkiner & Capraro (2009) recommended that middle school teachers should be "equipped with the necessary knowledge to help students develop conceptual understanding of fractional concepts such as accurate and appropriate representations."

Theoretical background

The visual takes a wide space in our lives. Everything around us is filtered through our eyes before analyzing and understanding it. Also in the learning process, visual representations play a very important role. Many psychological studies confirm that using visual representations in teaching help a deeper understanding of concepts. “For a mathematical thinking and communication, we need to represent in some way the elements of mathematical structures. Communication requires external representation in the form of language resources, written symbols, figures and objects” (Lesh, Post& Behr, 1987).

Cognitive psychologists defined two types of representations:
-External representations that can be: enactive, iconic and symbolic (written and spoken language, symbols).
- Internal representations: they are the mental representations that cannot be directly observed but needed in order to conceptualize about a mathematical concept.
Cognitive psychologists have formulated two hypotheses on representations:
1) There is a connection between internal and external representation of a concept. We can make logical deduction about internal representation, about their quality with the help of manipulating external representations.
2) Internal representations are interconnected, they form a network, that of mathematical concepts and principals. These connections can be simulated by constructing the right connections between external representations (Ambrus, 2001, cited in Debrenti, 2013). External representations, such as figures and text definitions influence the nature of internal representation. This also holds the other way round. The way a student reperesents his/her knowledge externally shows the way he/she represents the information internaly (János, 1998 cited in Debrenti, 2013)
Visual representations facilitate sense making and understanding because people remember images better than words (Ambrus, 2001). “Using concrete and iconic representations is necessary not only for the so called slow students or elementary students. These representations are important for all students and are useful throughout the entire learning process” (Wittmann, 1998)
Didactics states that iconic representations are important in the early stages of learning, and as students’ age and

mental development increase symbolic representations take over. However, there are other views that suggest that iconic representation should be implemented at all stages (Ambrus, 2001).

According to NCTM (2000) representations are useful in all areas of mathematics because they help us develop, share, and preserve our mathematical thoughts. "[They] help to portray, clarify, or extend a mathematical idea by focusing on its essential features" (p 206)

Cognitive theories prefer the term representation while researchers prefer model and modeling. The two terms are to a large extent interchangeable especially when researches in science and mathematics are involved (Gravemeijer, Lehrer, Van Oers, & Verschaffel, 2003; Greer, 1997).

Research Questions

Three questions are posted:

1) To what extent could pre-service and in-service mathematics teachers solve problems involving operations on fractions and how do they compare?

2) To what extent could pre-service and in-service mathematics teachers demonstrate their solving of comparison, addition, subtraction, and multiplication of fractions with representations and how do they compare?

3) How do in-service teachers with mathematics education degree and in-service teachers with other degrees compare in solving and representing fraction problems?

Method

Participants

A total of 44 teachers participated in the study for data collection. Participants consisted of 20 pre-service mathematics teachers and 24 in-service mathematics teachers. Pre-service teachers are enrolled in a 3-year mathematics teacher education program at the Lebanese University, faculty of Pedagogy, branch1. These University- Faculty have completed the required 9 credit hours in mathematics education. In service teachers are actually teaching elementary mathematics in several public schools in Beirut. Ten of them have a degree in teaching mathematics from the Lebanese university and 14 of them are other faculties' graduates. The table below shows the profiles of the participants.

Table1. Teachers' profiles

Pre-service teachers	**In-service teachers**			
	Faculty of Education		Other Faculties	
	Years of experience			
20	**≤ 5**	**> 5**	**≤ 5**	**> 5**
	4	6	8	6

Instrument

One instrument was used in this study. A fraction knowledge test, adapted and adopted from several previous studies on teachers' knowledge of fractions, was constructed to provide emphasis on both procedural and conceptual knowledge. The test consisted of five tasks that measure areas related to: (1) comparison, (2) addition, (3) subtraction, (4) multiplication, and (5) division. An item exemplified as "How much is 3/4 of 2/3?" is considered as a procedural knowledge item, and an item exemplified as "Explain how you determined your answer by giving an illustration or representation for 3/4 of 2/3?" is considered as representational knowledge. The test contains contextual and non-contextual tasks.

Data collection and analysis method

Data was collected by using a fraction assessment test. Test was submitted by hand to each participant who answered the test items in presence of the researcher. All items were corrected according to a rubric that categorized the answers as true, partially true or false. An answer was considered false if no work was shown at all or the answer is inappropriate. Partially true if the answer is not complete and true if it is correct and complete. Descriptive statistics was used in order to answer the research questions. Percentages were found for each item of the test. In addition, Pearson correlation was used to explore whether there is a significant relationship between two sets of points.

Results

The data collected in this research was intended to portray pre-service and in-service teachers' knowledge of computing and representing fractions. Educational background of in-service teachers was also taken into consideration. The results are presented in the following section in a way to answer the research questions.

Comparison of fractions

When teachers were asked to compare two fractions the pre-service teachers were 85% able to give true answers while the in-service teachers had 100% true answers. When they were asked to use a representation (model) to clearly demonstrate the comparison only 30% of pre-service teachers and 16.7% of in-service teachers gave a full true answer. The below table illustrates the answers.

Question1: comparison of fractions					
Pre-service Teachers			In-service teachers		
Computation					
True	False	Partially true	True	False	Partially true
17	1	2	24	0	0
85%	5%	10%	100%	0%	0%

Representation					
True	False	Partially true	True	False	Partially true
6	12	2	4	16	4
30%	60%	10%	16.7%	66.7%	16.7%

Table2. Pre-service and in-service teachers' results of Q1

40% of in-service teachers who were graduated from the faculty of education were able to represent correctly the comparison of two fractions while 0% who graduated from other faculties were able to do so.

Table3. In-service teachers' results of Q1

Question1: comparison of fractions					
In-service- teachers Q1					
Faculty of Education teachers			Other faculties teachers		
Computation					
True	False	Partially true	True	False	Partially true
10	0	0	14	0	0
100%	0%	0%	100%	0%	0%
Representation					
True	False	Partially true	True	False	Partially true
4	4	2	0	12	2
40%	40%	20%	0%	86%	14%

Multiplication of fractions

When teachers were asked to multiply two fractions less than one, 85% of pre-service teachers had a completely true answer compared to 75% of in-service teachers. When they were asked to explain by giving an illustration or representation of how they multiplied the fractions, 45% of PST and 25% of IST gave complete true answers

Table 4. Pre-service and in-service teachers' results of Q2

Question2: Multiplication of Fractions					
Pre-service Teachers			In-service teachers		
Computation					
True	False	Partially true	True	False	Partially true
17	3	0	18	6	0
85%	15%	0%	75%	25%	0%
Representation					
True	False	Partially true	True	False	Partially true

9	4	7	6	12	6
45%	20%	35%	25%	50%	25%

Of all IST who had correct representation of multiplication of two fractions 40% were Faculty of Education graduates and 14% were from other faculties.

Table 5. in-service Teachers results of Q2

Q2: Multiplication of fractions. In-service teachers					
Faculty of Education teachers			Other faculties teachers		
Computation					
True	False	Partially true	True	False	Partially true
6	4	0	12	2	0
60%	40%	0%	86%	14%	0%
Representation					
True	False	Partially true	True	False	Partially true
4	2	4	2	10	2
40%	20%	40%	14%	72%	14%

Multiplication of mixed numbers

In question3 where teachers were supposed to calculate and represent multiplication of two mixed numbers, 70 % of pre-service teachers and 41.7 % of in-service teachers answered correctly while only 35 % of pre-service teachers and 8.3 % of in-service teachers were able to represent the multiplication correctly.

Table6. pre-service and in-service teachers results of Q3

Question3: Multiplication of Mixed Numbers					
Pre-service Teachers			In-service teachers		
Computation					
True	False	Partially true	True	False	Partially true
14	3	3	10	14	0
70%	15%	13%	41.7%	58.3%	0%
Representation					
True	False	Partially true	True	False	Partially true
7	9	4	2	22	0
35%	45%	20%	8.3%	91.7%	0%

60% of in-service teachers who are faculty of education graduates computed the multiplication of mixed numbers correctly compared to 86% of graduates of other faculties. On the other hand, 40 % of those who are FOE graduates represented the multiplication of mixed numbers correctly compared to 14% of other faculties graduates.

Table 7. In-service teachers result of Q3

Q3: Multiplication of mixed numbers. İn-service teachers					
Faculty of Education teachers			Other faculties teachers		
Computation					
True	False	Partially true	True	False	Partially true
6	4	0	12	2	0
60%	40%	0%	86%	14%	0%
Representation					
True	False	Partially true	True	False	Partially true
4	2	4	2	10	2
40%	20%	40%	14%	72%	14%

Addition of fractions

In question4 teachers were supposed to add two fractions and then to represent the addition by a model. 70% of pre-service teachers and 83.3% of in-service teachers did the computation correctly, while 60% of pre-service teachers and 25% of in-service teachers were able to give a model for representation

Table 8. pre-service and Inservice results for Q4

Question4: Addition of fractions					
Pre-service Teachers			In-service teachers		
Computation					
True	False	Partially true	True	False	Partially true
14	5	1	20	4	0
70%	25%	5%	83.3%	16.7%	0%
Representation					
True	False	Partially true	True	False	Partially true
12	4	4	6	12	6
60%	20%	20%	25%	50%	25%

80% of in-service teachers from the faculty of education added correctly two fractions and 40% of them were able to represent this addition, while 85.7% of teachers from other faculties added the two fractions correctly and only 14.3% were able to represent the addition.

Table 9. in-service teachers result for Q4

In-service teachers Q4	
Faculty of Education teachers	Other faculties teachers

Computation					
True	False	Partially true	True	False	Partially true
8	2	0	12	2	0
80%	20%	0%	85.7%	14.3%	0%
Representation					
True	False	Partially true	True	False	Partially true
4	6	0	2	6	6
40%	60%	0%	14.3%	42.9%	42.9%

Division of fractions

In question 5 teachers were asked to divide then model a division between two fractions. 95% of pre-service teachers and 91.7% of in-service teachers did the division correctly, but only 40% of pre-service teachers and 16.7 % of in-service teachers gave a correct model.

Table 10. pre-service and in-service results for Q5

Question5: Division of fractions					
Pre-service Teachers			In-service teachers		
Computation					
True	False	Partially true	True	False	Partially true
19	1	0	22	2	0
95%	5%	0%	91.7%	8.3%	0%
Representation					
True	False	Partially true	True	False	Partially true
8	6	6	4	20	0
40%	30%	30%	16.7%	83.3%	0%

100% of in-service teachers from the faculty of education divided the two fractions correctly and 40% of them represented the division, while 85.7 % of in-service teachers from other faculties divided correctly and no one of them was able to represent the division by a model

Table 11. in-service teachers result for Q5

In-service teachers Q5					
Faculty of Education teachers			Other faculties teachers		
Computation					
True	False	Partially true	True	False	Partially true
10	0	0	12	2	0
100%	0%	0%	85.7%	14.3%	0%
Representation					
True	False	Partially true	True	False	Partially

					true
4	6	0	0	14	0
40%	60%	0%	0%	100%	0%

A t test comparison of percentages of correct answers for representations of fractions between in-service and pre-service teachers showed a significant difference ($p=0.006221$, $p<0.05$), while no significant difference was noted when comparing their computational skills.

A t test comparison of percentages of correct representations of different types of fraction operations between in-service teachers having a degree in mathematics education and those having other university degrees revealed a significant difference ($p<0.05$). no significant difference was revealed between the two types of in-service teachers regarding the computational skills.

Conclusion

In conclusion, the results of this study showed that there is a difference between computational abilities and representational abilities of both in-service and pre-service teachers in all studied domains: comparison of fractions, multiplication of fractions and of mixed numbers, addition of fractions and division of fractions. The results showed that pre-service and in-service teachers' computational knowledge is greater than their representational knowledge. However, in-service teachers had difficulties in multiplication of mixed numbers (41.7% correct answers). The study revealed that regarding the computational knowledge no significant difference was found between in-service and pre-service teachers.

When considering representational abilities, pre-service teachers were able to perform better than in-service teachers in all the studied domains. The difference was significant ($p<0.005$). however, when we compared preservice teachers' performance to in service teachers who graduated from the faculty of pedagogy, there was no significant difference ($p=0.717$). Moreover, faculty of pedagogy graduate in-service teachers performed better than preservice teachers in all domains which shed a light on the importance of teachers' specialization even in elementary classes.

There still are some limitations to this study. First, the sample was small in size. Second, only students enrolled in the education program of the Lebanese public university participated in the study. Also, in-service teachers were only public-school teachers. Another study with a larger representative sample could validate the results of this study if its results were similar.

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