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EVIDENCE-BASED PRACTICES
LEARNING COLLABORATIVE

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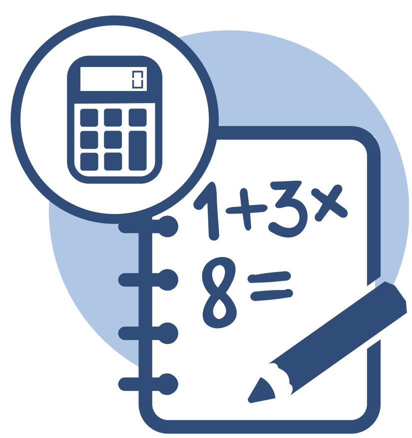
Five Evidence-Based Mathematics Practices That Teachers Can Employ to Support Accelerated Learning

Accelerated learning, learning loss, unfinished learning, and learning gaps are all terms used to describe the challenges educators will be facing for the next few years as a result of the COVID-19 pandemic. Data show that students are 4 months behind in mathematics compared to pre-pandemic benchmarks.[1] Moreover, recent data show variability in student need resulting in classes that are more heterogeneous than they were pre-pandemic; yet this is not the first time that students have regressed in skills or academically struggled in core classrooms. In addition, students with disabilities and students who struggle are showing even greater unfinished learning and a need for additional instruction or accelerated learning to diminish the gap. Thus, the state education agency needs to support local education agencies in providing evidence-based instructional practices in core academic classes and within intervention programs to support students through accelerated learning as a means to fill in unfinished learning so that students can begin to achieve grade-level standards. Using structures within a Multi-tiered System of Supports will help educators understand various student needs related to mathematics instruction and deliver instructional support that decreases the achievement gap. For more information see Five Considerations That Educators Can Employ to Accelerated Learning.

What Are Evidence-Based Mathematics Instructional Practices?

Data show that students, especially students with disabilities and students who struggle, may exhibit more academic regression in the area of mathematics because of learning challenges faced during the COVID pandemic. Educators need to maintain high-quality, accelerated mathematics instruction that includes evidence-based practices (EBPs). An EBP is a practice or strategy that has been proven effective through rigorous research.

This document summarizes five EBPs[2] that have been shown to increase student conceptual understanding and procedural fluency for all students but specifically for students with disabilities, students who have difficulties with mathematics, and English language learners. Research has shown that, when educators and interventionists embed EBPs, students' conceptual understanding and procedural fluency increase.



Five EBPs for Increasing Conceptual Understanding and Procedural Fluency

1. Explicit, Systematic Instruction

Explicit, systematic instruction involves teaching a specific concept or procedure in a highly structured and carefully sequenced manner. During this type of instruction, the teacher

- presents lessons that build on one another, moving from simple skills and concepts to more complex skills and concepts or from high-frequency skills to low-frequency skills;
- breaks complex skills into smaller, more manageable chunks, a method also known as *task analysis*;
- prioritizes and sequences tasks from easy to more difficult; and
- scaffolds instruction by providing temporary supports (e.g., manipulatives, written prompts, cues).

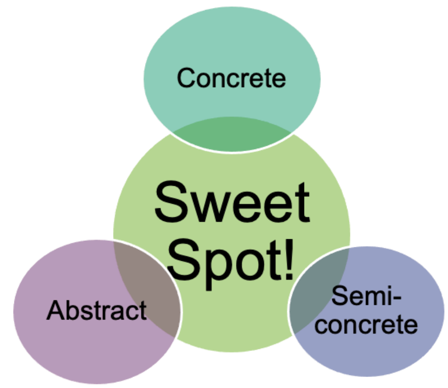
2. Multiple Representations

Multiple representations help to build student conceptual understanding of mathematics skills by presenting the skills in a way that is different from simply abstract only. Multiple representations include (1) concrete; (2) visual, pictorial, or semi-concrete; and (3) abstract representations. The concept of multiple representations is to find the "sweet spot" within instruction. Instruction should include all three types of representations rather than present each representation in isolation or in a linear progression. The intertwining of representations helps students to understand the skills and the relationships between the representations.[3]

Concrete: Objects or models that students can manipulate (e.g., base-10 materials, counters, integer chips, balance, algebra tiles)

Visual, pictorial, or semi-concrete: Pictorial representations of concrete objects or a picture or graphic organizer that represents a problem (e.g., diagrams, tables, pictures—drawn or created on the computer, number lines, virtual manipulatives)

Abstract: Mathematical symbols such as numbers, operation signs, expressions, or equations (e.g., variables, operation symbols, ordered pairs)



Students need explicit instruction in how to use each representation, when to use the representation, and when they should pair the representation with precise mathematical language. For example, if teachers are using base-10 materials, teachers need to show that 10 units are the same as a group of 10 or a rod in the same way that 10 groups of 10 is equal to a group of 100. Teachers also need to explicitly teach connections to help students understand that the concrete materials are the same as a picture, for example, teachers can draw a small circle to represent a unit, a line to represent a group of 10, and a square to represent a group of 100.

3. Schema and Attack Strategy Instruction[4]

Schema instruction and attack strategies, often referred to as metacognitive or cognitive strategies, have large effect sizes and have been proven to be an EBP for word problem solving for elementary grades through Grade 8. Most students, especially students who have difficulties with mathematics and students with disabilities, have trouble solving word problems. *Schema instruction*, rather than the use of keywords, helps students become more proficient at solving word problems by helping them recognize the underlying structure, or *schema*, of the word problem. Many teachers, especially teachers in elementary grades, tend to provide instruction by using keywords. Keywords lead to students understanding the problem, often cause misconceptions in solving, and prevent practice in comprehending mathematical situations.

An attack or metacognitive strategy is an easy-to-remember set of steps to help students solve word problems. Attack strategies are helpful across schemas and grade levels. Examples of attack strategies include **CUBES** (**C**ircle numbers, **U**nderline the question, **B**ox the key words, **E**valuate and draw, **S**olve and check) and **Q-TIPS** (**Q**uestion—underline, **T**hink about the problem, circle important **I**nformation, choose a **P**lan, find the **S**olution). Attack or metacognitive strategies help students learn to do the following:

- **Plan**. Students decide how to approach the mathematical problem, first determining what the problem is asking and then selecting and implementing an appropriate strategy to solve the problem.
- **Monitor**. As students solve a mathematical problem, they check to see whether their problem-solving approach is working. After working the problem, they consider whether the answer makes sense.
- **Modify**. If, as students work to solve a mathematical problem, they determine that their problem-solving approach is not working or that their answer is incorrect, they can adjust their approach.

Attack strategies are not enough. Regardless of the attack strategy, students need support in developing a plan to solve. For the plan and solve component, students use the schema-based strategy which highlights the importance of pairing the instructional attack strategies with the EBP of schema-based instruction.

Examples of schemas include additive schemas and multiplicative schemas.[5, 6]

Teachers can use the following three common *additive schemas* for addition and subtraction problems:

- **Combine**. Total; part-part-whole: *Mrs. Taylor bought 4 apples and 9 bananas. How many pieces of fruit did Mrs. Taylor buy?*
- **Compare**. Difference: *Ava has 9 fewer points than Giovani. Ava has 2 points. How many points does Giovani have?*
- **Change**. Join; separate: *Lou earned some money raking leaves. Then, he spent $12 at the mall and has $25 left. How much money did Lou have to start with?*

Teachers can use the following three common multiplicative schemas for multiplication and division:

- **Equal groups**. *Tara has 6 bags of oranges. There are 4 oranges in each bag. How many oranges does Tara have?*
- **Comparison.** *Pedro has 7 video games. Bronwynn has 21 video games. How many times as many video games does Bronwynn have than Pedro?*
- **Ratios and proportions.** Percentages; unit rates: *On Saturday, Naoki worked in the hot sun for 10 hours, helping to clean up and revitalize a neighborhood park. To prevent dehydration, she took a 5-minute water break every hour. What proportion of time did Naoki spend working compared to taking breaks?*

Each schema contains a specific equation and graphic organizer to help students set up the problem and plan to solve. It is imperative that teachers utilize explicit instruction to help students identify the type of schema within a word problem, build mastery of how to solve, and use precise mathematical language to describe the process.

4. Precise Mathematical Language

Vocabulary understanding has a huge effect on comprehension across many content areas, including mathematics. Instruction in mathematics needs to include skill instruction, as well as vocabulary. Students with disabilities and students who are multilingual are often more challenged to show mastery of skills as a result of vocabulary. The language of mathematics is complex and often includes words with multiple meanings within and outside of mathematics class. For example, teachers may use the word *table* to describe a *t*-table, which looks very different from a multiplication table and is vastly different from a table in the cafeteria. In addition, instruction in mathematics often includes words that are inaccurate or inconsistent, such as *zero as a placeholder, small or bigger number* rather than greater or less, *canceling* a fraction, or *plugging in* a variable. Misconceptions and delayed conceptual development of student skills can occur when teachers do not explicitly teach, model, and correct students to use precise mathematical language. Students with disabilities, students who are multilingual, and students who struggle need explicit instruction, pictures, and support in using precise mathematical language.

5. Student Discourse and Engagement

For students to be successful academically, they need to be engaged and participate. In the area of mathematics, many studies have begun to highlight the importance of discussions or discourse about mathematics. Student discourse is a practice that encourages students to express their mathematical reasoning. While refining their conceptual understanding, this practice allows students to become aware of their own problem-solving processes as well as the processes of others. Student discourse leads to increased student engagement, which allows the teacher to better assess student understanding.

Discussing or verbalizing mathematics is often challenging for students. Many students may lack the vocabulary or prerequisite skills or have skill deficits that hinder discussion. Teachers need to explicitly teach how to verbalize mathematics by modeling and using sentence starters, and they need to provide additional classroom practice for students to learn to talk about mathematics. Teachers pairing tasks in mathematics with explicit instruction and support while implementing instructional EBPs will lead to greater conceptual understanding and long-term success for students. For instruction to result in increased student discourse, the following practices should be part of instruction in mathematics:

- **Work solved problems.**[7] To encourage discourse, ask students to review a problem that has already been solved. Spending time reviewing or comparing two problems already worked or solved removes the challenge of computation, thus increasing student confidence and ability to discuss mathematics. For example, ask questions such as the following: What skills or methods did the sample student use to solve the problem? Do you disagree or agree with how they solved it? How would you solve it differently?
- **Present and compare multiple solutions.** Teaching multiple ways to solve a problem helps students develop flexibility (i.e., understand that a problem may be solved accurately using different procedures, be able to use efficient procedures) and might support conceptual understanding of the procedure.
- **Assess student understanding.** Assessing student understanding allows teachers to determine whether students have learned the mathematical procedures or concepts covered in class. Teachers can use different types of assessment data, including *formative assessment* and *error analysis*, to make instructional decisions (e.g., identify what they need to revisit or reteach, exit tickets).

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Resources

Achieve the Core

- Lesson Planning Tool
- Mathematics: Focus by Grade Level
- Coherence Map: Math Trajectories and Lesson Samples

Institute of Education Sciences: Practice Guides

- Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades
- Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students
- Improving Mathematics Problem Solving in Grades 4 Through 8
- Developing Effective Fractions Instruction for Kindergarten Through 8th Grade

IRIS Center Math Modules (self-paced, include links to research and additional resources):

- High-Quality Mathematics Instruction: What Teachers Should Know
- Evidence-Based Practices (Part 1): Identifying and Selecting a Practice or Program
- Evidence-Based Practices (Part 2): Implementing a Practice or Program with Fidelity
- Progress Monitoring: Mathematics

National Center on Intensive Intervention

- Intensive Intervention in Mathematics Course Content
- Mathematics Sample Lessons to Support Intensifying Intervention
- Mathematics Curriculum Crosswalk Grades 1–5

Endnotes

1 Kuhfeld, M., & Lewis, K. (2021). *Student achievement in 2021-22: Cause for home and continued urgency.* NWEA Collaborative for Student Growth Brief. https://www.nwea.org/content/uploads/2022/07/Student-Achievement-in-2021-22-Cause-for-hope-and-concern.researchbrief-1.pdf

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3 Bush, S. B., Karp, K. S., & Dougherty, B. J. (2021). *The math pact: Achieving instructional coherence within and across grades*. Middle School. Corwin Press, Inc. p. 49.

4 Woodward, J., Beckmann, S., Driscoll, M., Franke, M., Herzig, P., Jitendra, A., Koedinger, K. R., & Ogbuehi, P. (2012). *Improving mathematical problem solving in grades 4 through 8: A practice guide (NCEE 2012-4055).* U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. http:// ies.ed.gov/ncee/wwc/publications_reviews.aspx#pubsearch/

5 IRIS Module. *High-quality mathematics instruction: what teachers should know*. https://iris.peabody.vanderbilt.edu/module/math/challenge/#content

6 Powell, S. R., & Berry, K. A. (n.d.). *Pirate math equation quest*. https://piratemathequationquest.com/docs/PMEQ-1A-Teacher-Materials.pdf

7 Star, J. R., Caronongan, P., Foegen, A., Furgeson, J., Keating, B., Larson, M. R., Lyskawa, J., McCallum, W. G., Porath, J., & Zbiek, R. M. (2015). *Teaching strategies for improving algebra knowledge in middle and high school students (NCEE 2014-4333*). Washington, DC: National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Education Sciences, U.S. Department of Education. Retrieved from the NCEE website: http://whatworks.ed.gov

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