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We will be pleased to receive feedback in the form of comments, questions, or corrections. Please direct such feedback to Robert C. Schoen (rschoen@lsi.fsu.edu).

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Introduction

The first teacher professional development (PD) program based on Cognitively Guided Instruction (CGI) was created more than three decades ago. That original CGI PD program was created to bridge between research and practice by giving teachers access to robust, research-based frameworks for understanding children's mathematical thinking (Carpenter et al., 1989). Over the past 35 years, the definitive CGI book—*Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 1999; 2015)—has sold more than 200,000 copies, and tens of thousands of teachers have participated in CGI-based PD programs.

Schools in the United States spend billions of dollars every year on teacher professional development (PD) programs. While rigorous evaluation of the impact of PD programs through randomized controlled trials (RCTs) is rare, the latest meta-analysis of RCTs of teacher PD programs focused on content and pedagogy suggests that these types of programs typically have little detectable effect on student learning in mathematics (Pellegrini et al., 2021).

Cognitively Guided Instruction (CGI) is one of very few teacher PD programs that has been found to have a potentially positive impact on student learning in mathematics. Several experimental and quasi-experimental studies have found that CGI PD can have positive effects on student learning (Carpenter et al., 1989; Jacobs et al., 2007; Schoen et al., 2018; 2020; 2021b; Secada & Brendefur, 2000; Villaseñor & Kepner, 1993)[1].

As part of a program of research focused on CGI in Florida, the first long-term (i.e., five-year), experimental study to estimate the impact of a CGI PD program on student achievement found that the CGI program had a positive impact on school mathematics achievement (Schoen et al., 2021b). The results of that study are particularly noteworthy, because it was the first to measure the effects of a CGI PD program on student achievement using the state-mandated, high-stakes mathematics assessment (i.e., Florida Standards Assessment) as the outcome of interest. We anticipate that this result will contribute to a broader interest in teacher professional learning opportunities that are based on CGI.

While some core features of CGI and CGI PD tend to be consistent across different CGI-based programs, the particulars of the design and implementation of CGI PD programs—such as structure and duration, mathematics content, role and qualifications of the learning leader(s), target participants, and focal activities—vary substantially. Understanding the core features of a program and the context in which it was studied is important for interpretation of results and replication of the positive effects.

The purpose of this paper is to describe the core features of the CGI-based PD programs that were implemented in Florida through a series of projects that occurred during the past 8 years—from Summer 2013 through Spring 2020[2]. These core characteristics have been upheld across more than 69 cohorts of teachers supported by over 15 different workshop leaders. In this paper, we describe some of

[1] We note that not every experimental study of CGI has found positive effects on students, and the positive effects that do occur are not necessarily found on all measures or for all students. There is still much more research to do before researchers and practitioners fully understand whether and how the CGI program has a positive impact on teachers, teaching, and students.

[2] We intentionally limited the scope of this paper to describing the model of CGI-based professional development prior to the onset of the COVID-19 pandemic. Conditions of the pandemic forced the CGI PD program implemented in Florida to adapt from in-person PD to an online model. This necessitated significant adjustments to the PD program that will be the topic of a future report.

the core characteristics of the CGI PD model implemented in Florida. We provide this information as a way to help researchers and practitioners to understand the context in which these studies occurred and interpret the available and forthcoming research findings related to the impacts of the interventions. Additionally, we aim to enable informed comparison of this model with other mathematics PD programs. This paper also offers information for school leaders, policy makers, and teachers who are potential consumers of CGI-based PD, including those who may be considering participation in the model of CGI PD described.

Background

Before describing the CGI-based PD programs implemented in Florida, we briefly describe CGI and the history and general characteristics of CGI PD.

Cognitively Guided Instruction

The premise of Cognitively Guided Instruction (CGI) is that children have experiential knowledge and informal knowledge of mathematics that can serve as the basis for learning much of the formal elementary mathematics curriculum (Carpenter et al., 1996; 1999; 2015). For example, research has demonstrated that children can solve word problems without explicit instruction on how to do so, and they do it by modeling the action and relations in the problems (Baroody, 1987; Baroody & Ginsburg 1986). Children can then build on this intuitive knowledge to develop progressively abstract understanding of and formalized strategies for addition, subtraction, multiplication, and division with single-digit whole numbers, multi-digit whole numbers and fractions, as well as the base-ten number system and fractions concepts (Carpenter et al., 1996). A CGI approach to teaching is mindful that children often view mathematics differently than adults do and that striving to understand the child's perspective is an important part of teaching. The teacher's job is to design instruction that leverages and elevates children's ways of knowing and understanding such that they are used as a foundation for building new knowledge. It is to be expected that deep and meaningful learning requires an extended period of time—a phenomenon that holds true for students and teachers alike.

CGI in Practice

CGI involves teachers in designing instruction that is attentive and responsive to students' thinking, leading to the following principles (Carpenter et al., 1989; Carpenter & Franke, 2004):

1. Instruction should develop understanding by stressing relationships between skills and problem solving, with problem-solving serving as the organizing focus of instruction.
2. Instruction should be organized to facilitate students' active construction of their own knowledge *with understanding* (Hiebert & Carpenter, 1992), and each student should be able to relate problems, concepts, or skills being learned to the knowledge that he or she already possesses.
3. Instruction provides opportunities for students to engage with each other's mathematical ideas through analysis and discussion of their peers' problem-solving strategies.
4. Teachers should continually assess their students' thinking processes—often using informal methods of assessment—and use information gathered to guide their moment-by-moment instructional decisions as well as their day-to-day and longer-term instructional plans.

5. Teachers learn about student thinking by observing, questioning, and listening to students and working to understand their observations.

When a teacher's mathematics instructional practice aligns with CGI principles, classroom instruction involves: posing problems to students that they have not shown students how to solve, paying close attention to students' thinking as they solve problems, and adjusting the instructional plan based on what they learn about their students. Mathematics teaching and learning involves extensive interaction among students and teachers, wherein the members of the class strive to communicate their thoughts and understand each other's perspective and ideas.

CGI-aligned mathematics instructional practice centers children's ways of thinking and reasoning about mathematics. CGI emphasizes mathematics as a sensemaking activity and advocates for a corresponding bottom-up (Hiebert & Carpenter, 1992) approach to teaching and learning. Rather than focusing attention on those things students do not know or things they cannot do—an orientation that privileges a deficit perspective on teaching and learning—CGI focuses on knowledge and skills that students do have and builds toward more sophisticated knowledge or abilities. In contrast with the deficit perspective, this latter perspective views all understandings—from the least sophisticated to the most sophisticated—as partial understandings and can be described as an asset-oriented approach to teaching.

Implementation of these principles can be embodied in many different forms. While research-based frameworks for types of problems (e.g., word problems, equations) and associated student strategies can be salient features of many CGI-based PD programs, the manner in which the frameworks are incorporated will vary within and between different programs.

CGI Professional Development for Teachers

Parallel to CGI's foundational assumptions about children's abilities to construct knowledge of mathematics, CGI PD aims to support teacher learning by activating and building on teachers' existing knowledge of children's thinking (Carpenter, Fennema, & Franke, 1996). This aspect of the design of CGI teacher PD is consistent with the CGI principles as they relate to student learning as well. Table 1 outlines aspects of this parallel structure. The following paragraphs and sections provide further elaboration.

Table 1. Seven Parallel Principles in the Design and Implementation of Learning Opportunities for Students and Teachers Through CGI.

For students learning mathematics through CGI…	For teachers learning to teach mathematics through CGI PD…
1. Problem solving serves as an organizing focus of mathematics instruction.	1. Teaching is conceptualized as a problem-solving endeavor.
2. Assumes students have experiential mathematical knowledge that can serve as a foundation for learning much of the formal elementary mathematics curriculum	2. Assumes teachers have experiential knowledge of children's mathematical thinking that, when organized, can serve as a foundation for making instructional decisions
3. Instruction leverages and elevates children's ways of knowing by facilitating students' active construction of their own mathematical knowledge	3. PD facilitates teachers' construction and testing of models of students' mathematical thinking and pedagogy
4. Students are regularly assigned problems that they have not been taught how to solve. They learn to approach mathematics as a sense-making activity and decide how they might use the knowledge and skills that they do have to devise strategies to solve novel problems.	4. PD does not provide curriculum and is not prescriptive with respect to how teachers use what they learn in workshops. Teachers learn to attend to students' mathematical thinking and consider how instruction can be tailored to advance students' mathematical understanding.
5. Instruction cultivates students' abilities to explain, elaborate, and reflect on their own strategies and mathematical thinking	5. PD cultivates teachers' abilities to recount and interpret what they notice when observing students' mathematical activity
6. Instruction advances students' mathematical understanding through social interaction with teacher and peers focused on analysis and discussion of problem-solving strategies generated by peers.	6. PD advances teachers' understanding of students' mathematical thinking and related pedagogy through social interaction focused on analysis and discussion of specific examples of students' thinking and instruction
7. Teachers learn about student thinking by observing, questioning, and listening to students and working to understand their observations.	7. Workshop leaders learn about the thinking of teacher participants by observing, questioning, and listening to participants and working to understand their observations.

Research predating CGI PD indicates that experienced teachers have a considerable amount of knowledge about children's mathematical thinking, but that knowledge is often fragmented and disorganized, thereby limiting its use in teachers' decision making (Carpenter et al., 1988). Carpenter and Franke (2004) assert that fundamental changes in teacher practice can result from understanding and building upon students' mathematical thinking. Consequently, CGI PD aims to support teachers with organizing and building on their informal knowledge of children's thinking in specific content domains to construct and test models of student thinking such that they can use them to make instructional decisions.

Also consistent with the principles of CGI, CGI PD conceptualizes teaching as a problem-solving endeavor (Carpenter, 1989), in which teachers can use information gained by attending to the mathematical thinking of their students to further refine their knowledge of children's thinking and how instruction can be designed to support its further development (Franke et al., 1998, 2001). The original CGI programs set out to support teachers with building and organizing their understanding of children's mathematical thinking, in part by introducing them to research-based taxonomies for types of word problems and research-based frameworks for students' strategies for solving problems (Carpenter et al., 1989; Fennema et al., 1996). Research conducted over many decades suggests that children use these strategies as they make sense of mathematics, regardless of how they are instructed to solve problems (Baroody, 1987; Berglund-Gray & Young, 1940; Carpenter et al., 1996, 2015; Christou & Philippou 1998; De Corte & Verschaffel, 1987; Fuson, 1992; Gibb, 1956; Hiebert, 1982; Nesher et al., 1982; Riley et al., 1983; Schoen et al., 2016; Verschaffel et al., 2007). However, even in these first CGI PD offerings, the acquisition of knowledge of the formalized, research-based frameworks was not viewed as an end in and of itself. Rather, these frameworks serve as a lens to support teachers' interpretation and cultivation of instructional practices that build on their own students' thinking. The goal was (and is) to stimulate teachers' engagement in practical inquiry that leads to better outcomes for students and generative learning for teachers (Franke et al., 1998).

CGI PD does not provide teachers with a curriculum to follow or promote specific prescriptions on how to implement CGI in the classroom. The researchers involved in the original CGI studies emphasize the importance of not having received explicit guidelines for classroom instruction (Fennema et al., 1996):

> Teachers had to decide how to consider students as they selected problems, how to question children, and how to organize their classrooms. In order to do this, they had to reflect on what the research-based model of children's thinking meant for their classroom with their own students. This was not easy. The teacher had to deal with the complexity of children's problem solving as well as the myriad of other factors that are always present. But by doing so, the teachers transformed the model and it became part of their knowledge. (p. 432)

As a result, part of the work of a CGI teacher is to determine how to organize classroom instruction in a way that makes sense in their own context. CGI teachers learn to select or create mathematical tasks that expose student thinking in relation to various learning goals, and they become increasingly adept at using students' ideas to support the learning of individuals and that of the class.

CGI Professional Development Models

The five authors of the definitive CGI book also wrote and published a guide for CGI workshop leaders (Fennema et al., 1999). It is important to recognize that the guide for CGI workshop leaders does not attempt to define the one and only way to support teacher learning through CGI. Subsequent CGI PD programs—including those created by the authors of the guide for workshop leaders—vary significantly, employing structures such as workshops (e.g., Carpenter et al., 1989; Fennema et al., 1996; Moscardini, 2014; Schoen et al., 2018; 2020), teacher work groups (e.g., Franke & Kazemi, 2001), and university classes for preservice teachers (e.g., Philipp et al., 2007; Vacc & Bright, 1999). CGI PD models have also blended these approaches. For instance, the CGI PD reported by Jacobs et al. (2007) utilized a combination of workshops, teacher work groups, and on-site support visits to teacher's classrooms. The target audience and mathematical focus of CGI PD has also varied across the different offerings. The duration of PD support has also varied in the extant studies. Some programs offered just a few days of teacher workshops, whereas others have provided up to one year of support. None of the experimental studies that have been completed and published thus far have followed teachers or their students across multiple years, but Fennema et al (1996) did conduct an observational study of support for teachers over a period of three years, and a follow-up study was conducted in the fourth year (Franke et al., 2001).

Focal Topics in Mathematics

The original CGI PD program only involved first-grade teachers and focused on addition and subtraction on whole numbers (Carpenter et al., 1989). Subsequent CGI PD efforts have supported teachers at all elementary grade levels, and the focus of the subject matter has expanded to include a wider range of topics, including: multi-digit addition and subtraction, single digit multiplication and division, and base-ten concepts (Carpenter et al., 1999; 2015; Fennema et al., 1996), algebraic reasoning (Carpenter et al., 2003), and multi-digit multiplication and division, and fractions and decimals concepts and operations (Empson & Levi, 2011). The experimental and quasi-experimental studies have focused on number, operations, and algebraic reasoning—the mainstay of the elementary mathematics curriculum (Carpenter et al., 1989; Villaseñor & Kepner, 1993; Jacobs et al., 2007; Schoen et al., 2017; 2018; 2020; 2021b). Other important topics, such as geometry, measurement, and data analysis have not been the primary focus of many published studies of the impact of CGI-based interventions, but there are CGI-based programs that do explore how to use the CGI principles to guide efforts to support teaching and learning in those other domains of mathematics.

CGI PD Programs Implemented in Florida

A team led by Robert Schoen at Florida State University has implemented three grant-funded projects that have provided CGI PD to Florida teachers (see Table 2). All three projects offered CGI PD for teachers of early elementary grades (i.e., K–2), and the latter two projects included CGI PD for teachers of intermediate grades (i.e., 3–5). Many research studies have been and continue to be conducted through these projects. Four RCTs have been conducted through these efforts, with the largest one currently in progress.

Table 2. Overview of Three Grant-Funded CGI PD Projects in Florida

Characteristic	Grant-funded CGI professional-development projects in Florida		
	Project 1	Project 2	Project 3
Project title	Replicating the CGI Experiment in Diverse Environments	Foundations for Success in STEM	Foundations for Success: Developing Effective Mathematics Educators Through Cognitively Guided Instruction
Funding agency	Institute of Education Sciences (IES)	Florida Department of Education	U.S. Department of Education
Funding program	Education Research Grants: Efficacy and Replication program	Math-Science Partnership (MSP)	Supporting Effective Educator Development (SEED)
Awardee	Florida State University	Florida State University	Florida State University
Grant award number(s)	R305A120781	371-2355B-5C001; 371-2356B-6C001; 371-2357B-7C004	U423A180115
Principal investigator	Robert C. Schoen	Robert C. Schoen	Robert C. Schoen
PD provider	Teacher Development Group (TDG)	Teacher Development Group (TDG)	CGI Math Teacher Learning Center (CGI Math-TLC)
CGI PD director	Linda Levi	Linda Levi	Linda Levi
PD program(s) offered and target audience	CGI for Grades 1–2 teachers	CGI[1] for Grades K–2 teachers ECM for Grades 3–5 teachers	CGI K–2 for Grades K–2 teachers CGI 3–5 for Grades 3–5 teachers
Duration and timeline	2 years, starting Summer 2013	3 years, starting Summer 2015	3 years[2], starting January 2019
PD Structure	4 days of workshops in summer; 2 days of workshops in fall; 2 days of workshops in winter	5 days[3] workshops in summer; 2 days of workshops in fall; 2 days of workshops in winter;	4 days of workshops in summer[4]; 2 days of workshops in fall; 2 days of workshops in winter

Notes. ECM = Extending Children's Mathematics.

[1] Approximately half of the participating K–2 teachers also participated in weekly FACT meetings (Bauduin et al, 2016; Bray et al., 2019) during the 2015-2016 and 2016-2017 school years.

[2] Three years have been offered at the time publication. Additional years are anticipated.

[3] In addition to the TDG workshop leader providing CGI PD, a university mathematics faculty member taught mathematics for 90 minutes per day on five summer workshop days.

[4] The first group of 163 grades K–2 teachers participated in 8 days of workshops on school days between January 2019–April 2019.

This series of projects and associated studies have involved increasingly large numbers of teachers[3] participating in CGI programs during each school year from 2013 to the present (see Table 3). During this 8-year period, over 1,900 teachers have participated in at least the first year of the program, and more than 800 of those teachers have participated in two or more years. We also note that more than 700 teachers are serving in the waitlist comparison condition for an ongoing study at the time of writing, which suggests that the demand for the program is high among eligible teachers.

Table 3. Number of Florida Teachers Participating in each Year of the CGI PD Program, Split by Track and Year

		CGI program						
		K–2			3–5			
School year	Project[a]	Yr 1	Yr 2	Yr 3	Yr 1	Yr 2	Yr 3	*Annual total*
2013-2014	1	115	0	0	0	0	0	*115*
2014-2015	1	27	80	0	0	0	0	*107*
2015-2016	2	221	0	27	55	0	0	*303*
2016-2017	2	84	133	0	54	23	0	*294*
2017-2018	2	17	0	23	14	14	0	*68*
2018-2019	2 & 3	166	0	0	19	0	0	*185*
2019-2020	3	221	48	19	268	6	8	*570*
2020-2021	3	174	174	47	241	180	0	*816*
2021-2022	3	113	87	75	126	78	71	*550*
Cumulative total		*1138*	*522*	*191*	*777*	*301*	*79*	

Notes.
[a]Table 2 provides more information about Projects 1, 2, and 3.

In all of these projects, the CGI PD program designer/provider has been external to Florida State University and the research team. All of these CGI PD programs were led by a team of CGI workshop leaders under the direction of Linda Levi. The CGI PD programs in Projects 1 and 2 were created and taught by Teachers Development Group (TDG) under the direction of Linda Levi, the Director of CGI Initiatives for TDG and a co-author of three of the definitive CGI books (Carpenter et al., 1999; Carpenter et al., 2003; Empson & Levi, 2011), a manual for CGI workshop leaders (Fennema et al., 1999), the 2nd edition of the first CGI book (Carpenter et al., 2015), and many CGI-related research articles. At the time of these projects, TDG was the world’s largest provider of CGI professional development for teachers. In 2018, Levi discontinued her affiliation with TDG and founded the CGI Math Teacher Learning Center (CGI Math-TLC). CGI Math-TLC is the CGI PD provider for the third Florida project.

We note that teacher participation in the CGI PD program was always voluntary in these projects. Teachers were remunerated for their participation in PD during time when they were not paid by their districts and for participation in data collection that occurred outside of their contracted time with their school districts, and they did receive credit for hours of in-service training for the purposes of renewing their teaching credentials, but their participation was neither mandated nor paid by their schools or school districts.

[3] We use the term *teachers* inclusively to describe classroom teachers as well as instructional coaches, special education teachers, ESOL teachers, interventionists, and paraprofessionals.

Design and Structure

Figure 1 depicts the two tracks: one focused on grades K–2, the other focused on grades 3–5. The K–2 and 3–5 programs of CGI PD provided by TDG and CGI Math-TLC in Florida each consist of three years of professional development. Each of the three-year CGI PD programs that have been implemented in Florida thus far have involved a teacher participant in four or five days of workshops during the summer[4], a two-day follow-up session in the fall, and a two-day follow-up session in the winter or spring. The summer workshops consisted of approximately 7 hours per day of direct contact among participating teachers and the workshop leaders. The four days of follow-up sessions during the school year add an additional 24 hours of contact. All of the K–2 and 3–5 programs in Florida between the years of 2013–2020 maintained this structure. Teachers spend additional time outside of workshops completing reading activities, posing problems to students, analyzing their students' work, and participating in CGI team meetings with their colleagues in their schools. This long-term approach to encountering ideas in workshops, interacting with students, and revisiting those ideas in subsequent workshop days is a key feature of the CGI PD programs that have been implemented in Florida.

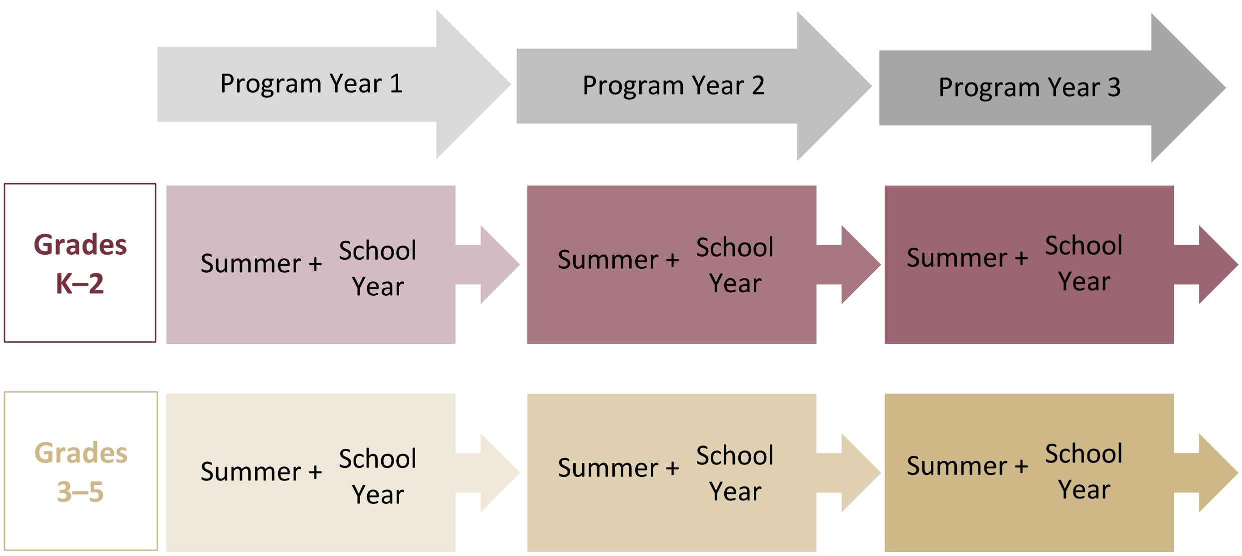

Figure 1. Two tracks in the three-year CGI PD programs for mathematics educators.

The spacing of workshops across the summer and academic school year enables teachers to attend the workshops and return to their respective schools to interact with students and colleagues. After interacting with students in their schools, teachers then return to the next round of workshops. The summer workshops provide teachers with a concentrated period in which to be immersed in CGI PD content and to prepare for the school year. In those workshops, they discuss their experiences in their classrooms and are exposed to additional ideas (or to the same ideas but from a new perspective).

[4] Project 2 included a fifth workshop day in the summer; but the amount of time spent with a CGI facilitator on CGI-specific PD was similar to the other projects, because approximately 7.5 hours of summer PD was designed and delivered by university mathematics faculty who were not members of TDG.

During the school year following the summer PD, teacher participants experiment with implementing CGI in structured and unstructured ways before returning to fall PD. This cycle repeats again before the follow-up workshops held in the winter. The follow-up PD sessions are designed to leverage their placement during the school year by including experiences with students and time in a host teacher's classroom and by incorporating activities that involve analysis of student work completed by the students of the teacher participants.

The teacher change process is thought to occur in an iterative manner over an extended duration of formal and informal experiences. The iterative back-and-forth between workshop and school-based experiences provides a supportive structure for receiving feedback, direction, and new ideas that promotes sustained engagement and greater take-up of ideas. It also creates opportunities for teachers to situate their learning into their own practice. As a result, teacher-learners play an active role in creating coherence between their daily work and the ideas they encounter in PD. This dynamic allows for the changes in knowledge and beliefs that may occur in the workshops to transfer into long-term, significant changes in instructional practice.

The multi-year program extends the opportunities for teachers to continue to develop their understanding, beliefs, and instructional practice, as these are thought to be affected incrementally across the three years of the program. The content and substance in each year of the PD experiences becoming increasingly sophisticated over time. Revisiting topics is meant to facilitate a deeper understanding as teachers' knowledge and experience grows. As teachers revisit a given topic in subsequent sessions and years, they focus on the topic from a more advanced perspective that is meant to facilitate a deeper understanding.

The placement of topics on PD agendas in Florida has varied over time, with teachers encountering a given topic earlier or later based on the iteration of the program in which they are enrolled. See Appendix A for an overview of the distribution of topics in different iterations of the program. In Project 1, for example, the first year focused more on grade 1 material than on grade 2 material. Findings from the first randomized controlled trial (RCT) in Florida—which involved first- and second-grade teachers and their students—suggested that the program might be working better for grade 1 students (Schoen et al., 2020). Subsequent versions of the CGI PD program in Florida were redesigned to be sure to address mathematics standards at each grade level in the program for that grade band during each summer and follow-up sessions in each year[5].

While individual teacher participants can complete the three-year program in consecutive years, the program is designed to allow for single-year commitments. Completion of the program in three consecutive years is recommended, but it is not required.

The program intent is for a single CGI PD workshop leader to facilitate workshops with an assigned cohort—composed of the same group of up to 30 teachers—throughout a given year of the program; some allowances have been made for teachers unable to attend a given workshop with their assigned cohort to attend sessions with a different cohort. There is a preference for cohorts to remain intact within a single program year, but they are not expected to remain in the same cohort of ~30 teachers for more than one year of the program.

[5] We also note that the topics and timeline presented in Appendix A represent the planned program. Implementation can (and does) differ from the planned agenda. Reporting on implementation for each program year is beyond the scope of the present paper.

To enable teachers participating in CGI PD to hold ongoing discussions and inquiry into student thinking with CGI-knowledge colleagues in their schools, the program designer(s) recommend having at least three teachers from the same school participate in the PD together. More than three is preferred.

The iterative nature of the interactions among teachers, workshop leaders, students, and other factors related to the context in which teachers are interacting and learning is depicted in the program theory of change in Figure 2. The theory of change depicts an indirect impact of the CGI PD program on teaching and learning, because the CGI PD program does not provide curriculum materials and is not prescriptive with regard to how teachers should teach. Rather, the program has a direct impact on teachers' knowledge, attitudes, and beliefs, and they use their new perspective to decide how they will approach mathematics instruction. This theory of change also acknowledges a myriad of contextual factors that may enhance or impede teachers' learning, changes to instructional practice, and student learning; we will not explore or examine those factors in detail in the current paper.

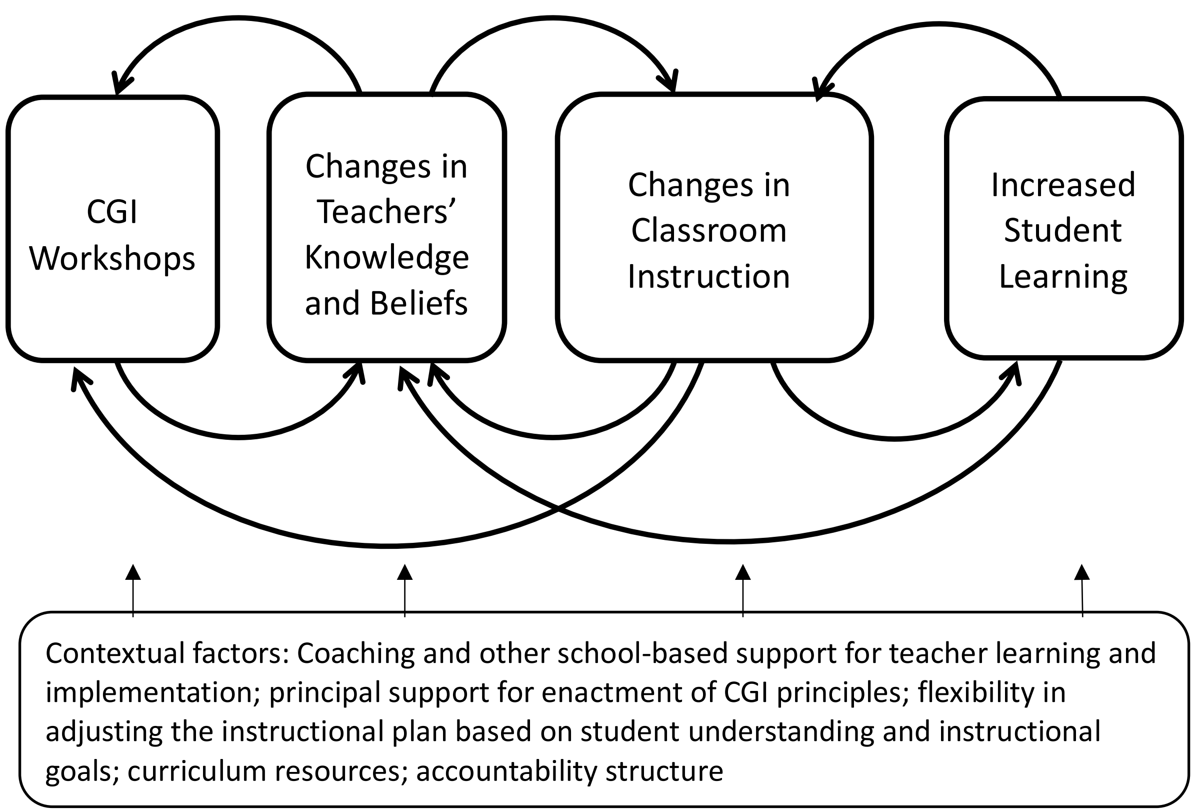

Figure 2. CGI program theory of change.

Another important design element of the CGI PD is the stringent set of qualifications of the workshop leaders. A challenge for every scaled-up PD effort is maintaining a high-quality intervention across program sites. As director of CGI Initiatives for TDG (for Projects 1 and 2) and Director of TLC (for Project 3), Linda Levi has served as the point person for vetting and training workshop leaders as well as determining readiness to facilitate Years 2 and 3 of the programs. Through all three projects, the workshop leaders for CGI PD in Florida have been experienced CGI teachers and/or university mathematics educators who met specific requirements to facilitate workshops. Figure 3 details the minimum requirements established for teacher educators to lead workshops in Year 1 of the three-year program. As workshop leaders become more experienced and complete further training, they become eligible to facilitate PD for Years 2 and 3 of the CGI programs.

Minimum Facilitator Requirements to Lead CGI Year 1 Workshops in CGI PD in Florida:

- Have a strong understanding of the CGI frameworks (e.g., problem types, solution strategies, relationship between problem types and solution strategies).
- Have at least 5 years of experience with CGI in one or more of the following ways:
  - actively implementing CGI as a classroom teacher.
  - actively supporting/implementing CGI as a math coach working with expert CGI teachers.
  - actively supporting/implementing CGI as a CGI researcher working closely with expert CGI teachers.
- Have at least 3 years of experience leading CGI PD for teachers in their own communities.
- Be able to recognize the formal mathematical concepts embedded in children's intuitive strategies.
- Be able to design a problem in real time that would engage children with a particular property of operation within a particular number domain.
- Have strong pedagogical skills when working with adult learners.

Figure 3. Requirements to serve as a Year 1 facilitator in the CGI PD in Florida.

Other Key Features of the CGI PD Programs Implemented in Florida: 2013–2020

We have already described some of the key design features of the CGI programs that have been implemented in Florida, such as the eight (or nine) days per year of real-time, in-person workshops that are spaced throughout the summer and academic year during the period from 2013–2020.

Next, we will describe additional key features of the CGI PD in Florida. (See Figure 4.) We organize these key features into key resources and key learning experiences. We refrain from repeating some other key features listed previously in the following sections, such as the presence of a qualified workshop leader. We note that these key features are selected for their salience to these authors, but they are not comprehensive of every feature in this PD program. We also note that these are salient features of all the programs that have been implemented thus far in Florida, while other programs may or may not emphasize these same features.

Key Resources

- Research-based frameworks for problem types and strategies
- CGI books and related publications
- Children to interview during workshops and teach during classroom-embedded lessons

Key Learning Experiences

- Noticing and interpreting details of students' solution strategies and connecting strategies to research-based frameworks
- Conducting mathematical interviews with individual children
- Acquiring knowledge of mathematical content and language through analysis and discussion of student thinking
- Engaging in collaboration and inquiry during classroom-embedded workshop days
- Developing instructional strategies to support students' fluency with number facts and computation
- Bridging between workshops and classrooms
- Understanding and using professional vernacular for teaching elementary mathematics

Figure 4. Some of the key features of the CGI PD program(s) implemented in Florida (2013–2020).

Key Resources

Research-based frameworks for problem types and strategies. The CGI PD programs in Florida are centered around supporting teachers to develop their understanding and use of two complementary types of research-based frameworks:

1. Problem-types frameworks, which describe how the structure of a problem influences how children think about the mathematical concepts embedded in the problem, and
2. Strategies frameworks, which describe the developmental progressions of children's mathematical thinking as illustrated by their strategies for solving problems within the problem types frameworks.

These frameworks, summarized in the CGI books, are grounded in decades of research and consensus established in the research community (Carpenter & Moser, 1983; Fuson, 1992; Riley et al., 1983; Schoen et al., 2021a; Verschaffel et al., 2007). The CGI PD program(s) implemented in Florida involve teacher participants in constructing their own understanding of the frameworks through in-depth analysis and discussion of carefully sequenced sets of mathematics problems and videos of students solving mathematics problems.

In this CGI PD program, the teachers are introduced to the research-based, CGI frameworks for problem types and student strategies during the first days in the first year of the program. As the workshops progress, teacher participants frequently revisit the frameworks and refine their understanding of them as they work to make sense of the mathematical thinking of students observed during the workshops and in their own classrooms.

The frameworks of focus in the CGI K–2 PD program implemented in Florida describe children's thinking about: (a) addition and subtraction of single-digit and multi-digit numbers, (b) multiplication and division of single-digit numbers, (c) base-ten number concepts, and (d) early algebraic ideas (Carpenter et al., 1999; 2003; 2015).

The frameworks addressed in the CGI 3–5 PD program implemented in Florida focus on children's thinking about: (a) multiplication and division on whole numbers, (b) fraction concepts, (c) operations on fractions, (d) base-ten number concepts involving whole numbers and decimals, (e) addition and subtraction on whole numbers, and (f) early algebraic ideas (Carpenter et al., 1999; 2003; 2015; Empson & Levi, 2011; Schoen et al., 2018).

See Appendix A for an overview of how aspects of the frameworks for problem types and students' strategies have been distributed in a spiraling manner throughout Years 1, 2, and 3 of the CGI K–2 PD and the CGI 3–5 PD in different iterations of the program.

CGI books and related publications. As part of the CGI programs in Florida, teacher participants receive and regularly use the CGI books associated with the content of the program in which they are enrolled. The books offer an authoritative reference for teachers to review and reflect between (and after) workshops on the topics explored during workshops. Teachers enrolled in CGI K–2 use the Children's Mathematics: Cognitively Guided Instruction (Carpenter et al., 1999; 2015) as a core text and reference. In the CGI 3–5 PD, the core text is Extending Children's Mathematics: Fractions and Decimals (Empson & Levi, 2011). Book chapters are typically assigned for review and reflection after the ideas of focus in the chapter are introduced in a given workshop. The books serve as a common tool to reference the problem-type frameworks and strategies frameworks within the workshop setting. Between and after workshops, the CGI books serve as a resource that includes examples of different types of problems, descriptive examples of children's thinking, and links to videos illustrating children's thinking.

Children to interview during workshops and teach during classroom-embedded lessons. Direct interactions in real time with students who represent the target age groups for each program (i.e., K–2, 3–5) have always been provided for teachers in the Florida CGI PD. This is particularly important during the summer workshops in year 1 of the respective programs, but it has also been part of the summer workshops in year 2 of the program (where the focus of the interview tends to be on fluency) and sometimes in the third year. The real-time interviews provide a different kind of learning opportunity for teachers—one that differs from studying student work or videos of students. During the classroom-embedded workshop days, students are also needed. Teachers interview these students, plan a lesson for the students, and then teach the lesson (or observe the lesson being taught) in a classroom setting. Subsequent sections describe these experiences in more detail.

Key Learning Experiences

Noticing and interpreting details of students' solution strategies and connecting strategies to research-based frameworks. A central goal in CGI PD is to support teachers' ability to notice the details of students' strategies with increased specificity so they can connect the strategies used by their own students with research-based frameworks for students' thinking. This is based on a theory that argues the following: When teachers can structure their knowledge of their students' mathematical thinking around principled frameworks, they can more readily use this knowledge to guide their instruction. This process begins in the CGI PD sessions, wherein teachers observe and analyze videos of individual children solving mathematics problems and explaining their thinking processes.

Throughout the CGI PD in Florida, workshop leaders use video and examples of written student work extensively to introduce the frameworks of problem types and solution strategies and to promote analysis of student thinking. After viewing video of students solving problems, workshop leaders routinely press participants to describe in detail what they see children doing, often probing for additional detail at a finer grain size than initially provided. Workshop leaders and participants also discuss conjectures about how children's strategies are related to their mathematical understanding.

The medium of video creates a context in which a given student's strategy can be reviewed multiple times, allowing teachers to notice aspects of a student's solution that they may have missed if they did not have the capability to study the details multiple times. Early experiences with viewing, discussing, and reviewing examples of students solving problems afford opportunities to slow down the work of teaching and focus participants' attention on noticing a finer level of detail. Through discussion, workshop leaders and participants draw distinctions between what is observed and what is inferred based on observation. Through these early experiences, the workshop leader establishes norms for the grain size of detail that is desirable to observe and how the cohort will discuss what they observe. PD leaders press the participating teachers to link the observed students' strategies to the frameworks for problem types and solutions strategies.

As the CGI PD progresses, teachers continue to cultivate their skills of noticing and interpreting the details of children's thinking as they analyze students' written work, conduct math interviews with individual children, as they interact with students in their own classrooms, and as they observe students during classroom-embedded workshop days.

Conducting mathematical interviews with individual children. A cornerstone activity in which teachers engage within each year of workshops is conducting semi-structured mathematics interviews with children. The workshop leader typically provides a set of mathematics problems to pose to a child in an interview, manipulatives that student interviewees can choose to use (or not), and guidance on how to conduct the interview. Each child is typically interviewed in the presence of two or more workshop participants, with one teacher taking the lead with conducting the interview and the other observing. During a given session of interviews with children, each teacher will typically alternate between roles of interviewer and observer, but that specific arrangement does not always occur.

Participants are encouraged to use the interview to gather as much information as they can about the child's mathematical thinking by asking questions about a child's strategies. Teachers often find this process challenging, because they are accustomed to providing explicit instruction and asking questions to support a child to follow a specific procedure. The interview creates an opportunity to practice the art of questioning and observing, attending to student thinking processes, and interpreting student behaviors. The interview is not intended to be a tutoring or teaching session for the student. On some occasions, the children to be interviewed are all from the same class at a single grade level. On other occasions, they are drawn from a youth summer or afterschool program and represent a wider range of students in different grade levels.

The interviews with children during workshops serve multiple purposes. During the summer workshops in the first year of the program, participating teachers interview children soon after they have been introduced to a framework of solution strategies children typically use to solve particular types of problems. The interviews at this point offer the participating teachers an opportunity to consider how their experiences with the children they interview compare to what they observe in the videos of children solving problems. Participating teachers are often surprised to find that the children they interview are solving problems in similar ways to those of the children in the videos and that the children they interview can solve problems that they haven't been taught to solve. In this way, these initial experiences interviewing children reinforce the validity of the research-based frameworks, and in doing so, also often spark curiosity and interest.

Interviewing children always provides opportunity for participants to practice observing and eliciting the details of students' strategies. Allocating workshop time to interviewing children ensures that participants can focus on developing their skills of attending to a child's thinking and asking questions to elicit details about a child's thinking without the complexity of daily teaching responsibilities. Interviewing children within the workshops also creates a shared experience in which participating teachers can compare and contrast what they noticed in the strategies of individual children and across the group of children interviewed in relation to the relevant solution strategies framework, allowing opportunity for deepening understanding of the framework through reflection.

Acquiring knowledge of mathematical content and language through analysis and discussion of student thinking. As part of discussions focused on noticing the details of children's strategies and linking those observed strategies to the research-based frameworks, workshop leaders also begin guiding participants to draw connection to the mathematical relationships and concepts embedded in students' strategies. For example, after observing a young child solve a simple addition problem with concrete objects, they might discuss what this child understands about early number concepts and/or addition. As another example, after watching an older child use an abstract invented strategy to multiply a fraction by a whole number, they might discuss what properties of multiplication were embedded in this child's strategy and/or what fraction concepts this child understands. Discussions linking students' strategies to mathematical concepts start on the first day of the first year of the workshops and grow in sophistication throughout the three years of the program. Through these discussions about children's thinking, workshop participants gain a deeper understanding of the mathematics they teach.

Additionally, the workshop leaders are careful and deliberate in their approach to developing participants' mathematical language abilities. They take care to use formal terms and notation in a manner that conforms to conventions, but they also introduce and accept informal notation and vocabulary, especially when it supports problem solving in the moment or provides an alternative method to common misuses of mathematical terms or symbols.

Just as participants learn mathematics content in the CGI PD through the study and analysis of children's thinking, they also learn mathematics language—verbal and written—through that same study and analysis. The workshop leaders emphasize the use of mathematics notation to represent mathematical thinking. For example, suppose a child is evaluating the product of 8×7 and says, "eight fives are 40, and eight twos are 16, so 56." Through discussion and analysis, the participants and workshop leader might translate the verbal expression into a written expression by writing: $8 \times 7 = (8 \times 5) + (8 \times 2)$. They may discuss whether that statement reflects what was expressed verbally, whether the statement is true, whether it would always be true, and how that idea might be used to evaluate 8×12. Discussion about how the strategy and notation relate to laws of operations—and the formalized names of those laws—are also common in these moments.

Engaging in collaboration and inquiry during classroom-embedded workshop days. Each of the two-day follow-up sessions held during the school year typically includes one classroom-embedded workshop day (Levi, 2017; Nielsen et al., 2016). A classroom-embedded day engages participants in collectively planning, implementing, and reflecting on mathematics instruction for students in an actual class of students in their community—usually the class of one of the participants in the workshop. In Florida, these lessons have almost always been taught by the workshop leader. The design and implementation of the classroom-embedded days are informed by the purposeful pedagogy model (Jaslow & Evans, 2012) and Smith and Stein's five practices for orchestrating productive mathematics discussions (2011; 2018). The purposeful pedagogy model is a tool intended to help participating teachers use information about student thinking to guide instruction. It consists of the following steps:

1. Assess the level of understanding held by each student in the class (through interviews or samples of student work) related to a specific mathematical concept. The selected concept is directly related to benchmarks in specific the state curriculum standards for mathematics.
2. Set a learning goal for each student based on his or her understanding of this concept and the grade level standards.
3. Design instruction to engage children in the established learning goal(s), utilizing the elements in Smith and Stein's discourse model (2011; 2018).

The classroom-embedded day typically begins with each workshop participant interviewing and/or observing another participant interview one or more students from a volunteer host classroom—usually the class of one of the workshop teacher participants. As described previously, these mathematical interviews with children utilize a set of problems provided by the workshop leader. Participating teachers are directed to pose each problem in a way that supports the student to understand the context of the problem, provide students with tools that they can use (or not) to solve the problems, and use observation and questioning to gather information on the student's solution strategy and their understanding of the math concepts. Participants are reminded to avoid using the interview as a time to show students how to solve problems.

After the interviews, and with support from the workshop leader, the cohort of participants in the workshop analyze students' strategies with the goal of categorizing students into groups of students who hold similar understanding of the math concept. The focus is on what each group of students understands rather than whether or not they correctly solved the problems. Strategies used by each student are linked to the relevant CGI frameworks, and learning goals are established for each student and then aggregated up to establish a learning goal for the class and/or for subgroups of students in the class. The goal is to design a single lesson that will advance the understanding of each individual student in the class[6]. Although the learning goals may be different for different groups, all learning goals are linked to the same mathematical concepts. Under the direction of the CGI PD leader, the participants collectively design a lesson that will engage the host class of students with the content of the learning goals.

In the afternoon (typically after lunch), the cohort of workshop participants goes to the host classroom to observe the workshop leader[7] implement the lesson that was collectively developed by the cohort.

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[6] The *Teachers Analyze Student Thinking and Write Detailed Learning Goals* story in the *What's Next?* story collection at www.teachingisproblemsolving.org/whats-next-stories provides a more detailed description of how this can occur.

[7] In other locations, the classroom teacher or some other person in the group may teach these lessons. In these Florida projects thus far, it has been the workshop leader.

Participants in the workshop are asked to observe the overall lesson while paying close attention to students they interviewed earlier in the day to see whether any changes appear in the way the students are thinking and solving the problem(s) posed in the lesson. The lessons developed by participant cohorts, with guidance from the workshop leader, typically include a component in the beginning of the lesson in which the workshop leader requests that select students explain their approaches to solving one of the interview problems. The workshop leader[8] facilitates class discussion of these shared solutions in accordance with the plan established by the workshop participant cohort. Based on the strategies used by students in the interviews, the strategies of focus in this initial class discussion are typically determined in advance by the cohort of participants, with each solution purposefully selected to support surfacing an important idea relevant to the established learning goals. Once the class has finished discussing the purposefully selected strategies/solutions from the interview problems, the class is presented with a newly developed problem to solve while the workshop participants observe. If time allows, strategies for this new problem are discussed and/or participants return to the students they interviewed to collect information on how these students solved the new problem. Students' work for the new problem are collected and brought back to the workshop meeting room.

At the conclusion of the classroom lesson, the participants return to their workshop meeting room to discuss their observations of students and instructional practice during the classroom lesson. Then to close these follow-up days, participants often repeat portions of the protocol using the student work from their own classrooms. After analyzing their own students' strategies, the participants set learning goals for their own students and design one or more problems that will engage students with the learning goals. The participants anticipate the variety of ways their students might solve the designed problem(s) and develop a plan for carrying out a class discussion of strategies that their own students may use to solve the problem(s).

The classroom-embedded day offers a collective experience in which participants weave together the myriad things they are learning through the workshops to contribute toward the creation of a lesson that operationalizes and approach to practicing CGI.

For a more detailed view of the classroom-embedded day experience applied to different grade levels and mathematics concepts, readers are encouraged to sample the collection of "What's Next?" stories available at https://teachingisproblemsolving.org/whats-next-stories (Schoen & Champagne, 2017). Each story provides a detailed, narrative account of an actual classroom-embedded day that occurred during Project 2 (see Table 2).

Developing instructional strategies to support students' fluency with number facts and computation. Teachers starting the first year of CGI PD typically think of fluency with number facts and computation exclusively as the ability to quickly obtain the correct answer or accurately perform a given procedure. Through CGI PD, participating teachers participate in a variety of experiences that foster a more comprehensive view of fluency that reflects the position of the National Council of Teachers of Mathematics (NCTM):

> Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently. The computational methods that a student uses should be based on mathematical ideas that the student understands" (2000 p. 152).

[8] This is one aspect that is true of the Florida CGI workshops thus far, but it is not ubiquitous. In other places, the classroom teacher—and not the workshop leader—may teach the lesson.

Initial work on supporting teachers to develop a more comprehensive understanding of fluency focuses on what it means for a student to understand a computational method. This approach is based on a premise that students who generate their own strategies to solve problems—without the teacher telling or implying the strategy that they should use—understand the strategies they employ.

In rejecting a pedagogy of *telling students how to solve problems*, CGI PD guides participating teachers to honor and build on students' existing competence with number, operations, and equality in ways that will lead to greater fluency. The focus often turns to helping students who are using concrete—and sometimes time-consuming—strategies draw meaningful connections between their own strategies and more efficient strategies with understanding. Participants also explore how to select or design sequences of problems that stimulate student consideration of particular ways of reasoning about numbers. For example, students who solve a problem such as $77 - 20 = n$ after they solved the problem $70 - 20 = h$ might use the fact that $70 - 20 = 50$ in solving $77 - 20 = n$. An intermediate-grades example of sequencing problems might involve solving $12 \times \frac{3}{4} = n$ after solving $12 \times \frac{1}{4} = h$.

In the year 2 and year 3 CGI PD programs, participants learn to support the development of students' computational flexibility through discussions of student-generated strategies. For example, suppose students in a class were solving the problem $401 - 392 = n$, and one student solved the problem using this strategy: 401– 300 → 101 – 90 → 11 – 2 → 9, and another student used this strategy: 392 + 8 → 400 + 1 → 401; 8 + 1 = 9. After the teacher ensured that students understood both strategies, they might lead a class discussion about which strategy was better suited to this problem as a way to help students increase their fluency. Sometimes students need to be asked questions such as, "even though both strategies are good strategies, which strategy would be better if you needed to solve this problem quickly?" Participants learn to design problems where one type of strategy is better suited to the numbers in the problem than another so that they can have these discussions with their students.

Bridging between workshops and classrooms. As noted earlier, CGI PD is different than many other teacher PD experiences, because it does not provide participants with a curriculum or prescriptions regarding what or how to teach. Instead, CGI PD aims to support participants with cultivating a practice of on-going practical inquiry with the goal of developing and refining teaching that is optimally responsive to students' thinking. To support this aim, the CGI PD in Florida is intentional about incorporating multiple mechanisms to support participants with bridging between workshop and classroom. The classroom-embedded experiences (described above) provide one such mechanism. The following paragraphs discuss several others.

One bridging mechanism is a structured, between-workshop assignment given at the end of summer and fall workshops. Workshop leaders provide the cohort of participants with a small set of problems to pose to their own students prior to the next workshop. Participants are directed to bring their students' work on these problems to the next workshop for further examination. In the follow-up workshop, extensive time is dedicated to analyzing the student work collected by the group and using this student work to plan a discussion of students' strategies and choose a follow-up problem. The common experience of this assignment allows participants to discuss what they are noticing in their own students and questions provoked by the experience. The next between-workshop assignment involves posing this participant-developed problem as well as problems provided by the workshop leader to their students. These between-workshop assignments are instrumental, particularly in Year 1 PD, at scaffolding teachers' initial efforts to experiment with using CGI in the classroom. Like the student interviews, these between-workshop assignments serve as a mechanism for teachers to test the validity of the solution strategies frameworks with their own students. As with the interviews, teachers are often surprised that their own students are able to solve problems without being told how to solve them and that their strategies reflect those studied in the workshops. We believe that participants' surprise heightens their curiosity and motivation to continue to practice CGI.

A second bridging mechanism is the allocation of time within workshops for participants to meet with other teachers from their school and/or grade level to design and prepare for implementing mathematics instruction using CGI. In Project 1, participants from the same school collaborated during workshops to plan their own CGI-based lessons and were encouraged to schedule a time to meet between workshops to discuss their efforts and findings. In Project 2, participants also engaged in planning for their own classrooms within workshops, but in this project the collaboration was designed to be with participants from their school grade-level team. In Project 2, participants were required to enroll in PD with at least three participants from the same grade-level team at a given school. During the first year of this project, some grade-level teams enrolled in the CGI K–2 PD were randomly assigned to participate in weekly CGI team meetings with an external facilitator. In these meetings, the grade-level team worked through a cyclical two-week meeting protocol in which they designed a problem to gather information about student thinking in relation to a learning goal, implemented the problem with their students, and worked as a team to analyze the students' strategies and consider implications. For more information on what occurred in the weekly team meetings with an external facilitator, see Bauduin et al. (2016) and Bray et al. (2019). In Project 3, workshop time was allocated for participants to work in grade-level specific groups—including participants from their own school and other schools—to design problems and instruction to implement with their students. In all cases, the intent of these activities has been to foster a support network of colleagues to help bridge between the CGI workshops and the particular context in which individual participants are working in their schools. In all of these projects, the current Florida standards documents were made available to participants, and time was spent making connections between the ideas studied through CGI PD and specific standards in the Florida mathematics curriculum.

Understanding and using professional vernacular for teaching elementary mathematics. Connecting students' strategies to research-based frameworks and identifying the mathematical concepts embedded in students' strategies supports participants in the CGI program to become more knowledgeable about professional vernacular for mathematics and teaching mathematics and mathematical representations. The language supports professional conversations about mathematics, mathematics teaching, mathematics learning, mathematics curriculum, and more. The language abilities include written notation that is consistent with formal conventions for mathematical communication as well as informal notation that is useful for expressing and recording elementary school students' thinking processes. The language participants learn supports the development of their understanding and insight into student thinking, curriculum resources, assessment strategies, and student learning progressions. This vernacular can support efficient and sophisticated conversations among teachers on a grade-level team who may be planning, discussing, and reflecting on student thinking and learning as well as curriculum, instruction, and assessment.

The resulting increases in professional vernacular has the potential to support collaboration among teachers in a school. At the same time, it can create new challenges in a school when some teachers have acquired this knowledge and insight and other teachers have not, especially when these two groups are expected to collaborate and discuss mathematics teaching and learning. This is one reason that the program encourages teachers to participate in the program with their peers. It may also be one reason why we have seen higher rates of recruitment and retention to the CGI PD program in schools where teachers have participated in CGI than in schools that do not have a history of teachers being involved in CGI (Schoen et al., 2021b).

Closing

This paper has described the scope, structure, and key features of the series of CGI PD programs that have been implemented in Florida, primarily through support from grant-funded projects, between 2013 and 2020. These core characteristics have been upheld in the series of Florida projects across more than 69 cohorts of participants supported by over 15 different workshop leaders. In efforts to scale CGI PD, these Florida projects have been intentional about embracing the constant tension between facilitating a set of common experiences across cohorts and designing each PD to build on the particular contexts, needs, and concerns of each cohort of participants. This paper aims for full disclosure and transparency about the core characteristics of the model of CGI-based PD implemented in these Florida projects.

We hope this paper will enable comparison of this CGI PD program with others in pursuit of answers to the questions about how to design learning opportunities for teachers with the ultimate goal of improving mathematics teaching and learning. We also view this paper as an essential foundation for interpreting available and forthcoming research findings related to the impacts of these projects. While discussion of research findings on these projects is beyond the scope of this paper, interested readers are encouraged to visit our research lab website—www.schoenresearch.com—for updated information about existing and future reports, publications, presentations, and additional forms of scholarship that have emanated from these projects. Finally, we hope this paper will be a helpful source of information for school leaders, policy makers, and teachers who are potential consumers of CGI-based PD and are considering participation in the model of CGI PD described.

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Appendix

Comparison Tables of Distribution of Major Topics on Overview Agendas for Years 1, 2, and 3 of CGI K–2 and CGI 3–5 Programs Across School Years

Table A1. CGI K–2 Year 1 Overviews by School Year

School Year	Summer					Fall		Winter	
	Day 1	**Day 2**	**Day 3**	**Day 4**	**Day 5**	**Day 1**	**Day 2**	**Day 1**	**Day 2**
2013-2014	• Introduction to CGI • Direct Modeling • CGI Problem Types	• Strategies for Addition and Subtraction Problems	• Strategies for Multiplication and Division Problems • Interview Children* • (*depends on scheduling)	• Introduction to Multi-digit Strategies • Using CGI in your own Classroom		• Classroom Embedded Work	• Developing Base Ten Understanding	• Classroom Embedded Work	• Strategies for solving addition and subtraction problems with large numbers
2014-2015	• Introduction to CGI • Direct Modeling • CGI Problem Types	• Strategies for Addition and Subtraction Problems	• Strategies for Multiplication and Division Problems • Interview Children* • (*depends on scheduling)	• Introduction to Multi-digit Strategies • Using CGI in your own Classroom		• Classroom Embedded Work	• Developing Base Ten Understanding	• Classroom Embedded Work	• Strategies for solving addition and subtraction problems with large numbers
2015-2016	• Introduction to CGI • Direct Modeling • CGI Problem Types	• Strategies for Addition and Subtraction Problems	• *Strategies for Multiplication and Division Problem • Interview Children	• Introduction to Multi-digit Strategies	• Using CGI in your own classroom	• Classroom Embedded Work	• Developing Base Ten Understanding	• Classroom Embedded Work	• Solving addition and subtraction problems
2016-2017	• Introduction to CGI • Direct Modeling • Addition and Subtraction Problem Types	• Children's Strategies for Addition and Subtraction Problems	• Children's Strategies for Multiplication and Division Problem • Interview Children	• Multi-digit Strategies Addition and Subtraction Strategies • Learning to Count	• Developing an Understanding of how operations work • Using CGI with your students	• Classroom Embedded Work – Base Ten in Second Grade	• Developing Base Ten Understanding	• Classroom Embedded Work – Multi-Digit Addition and Subtraction in First Grade	• Developing efficiency for multi-digit addition and subtraction

(Continued)

School Year	Summer					Fall		Winter	
	Day 1	**Day 2**	**Day 3**	**Day 4**	**Day 5**	**Day 1**	**Day 2**	**Day 1**	**Day 2**
2019-2020	• Direct Modeling • Single Digit Addition and Subtraction	• Learning to Count • Single Digit Multiplication and Division • Grade Level Work Session	• Math Interviews with Children • Components of a CGI Classroom • Using CGI with your students	• Multi-digit Addition and Subtraction • Grade Level Work Session		• Base Ten Number Concepts	• Interview students, plan and observe a second-grade lesson on developing base ten understanding	• Interview students, plan and observe a first-grade lesson on strategies for multi-digit addition and subtraction	• Multi-digit addition and subtraction Using CGI with your students
2020-2021	• Introductions • Children's intuitive problem-solving strategies • Addition and subtraction word problem types	• Children's strategies for *single digit addition and subtraction* – a developmental progression • Using children's thinking to guide your instruction – part 1	• Children's strategies for *multi-digit addition and subtraction* – a developmental progression – part 1 • Learning to count	• Children's strategies for *multi-digit addition and subtraction* – a developmental progression – part 2 • Common components of a CGI Classroom		• Base Ten Number Concepts	• Interview students, plan and observe a second-grade lesson on developing base ten understanding	• Interview students, plan and observe a first-grade lesson on strategies for multi-digit addition and subtraction	• Multi-digit addition and subtraction • Using CGI with your students
2021-2022	• Children's intuitive problem-solving strategies • Addition and subtraction word problem types	• Single digit addition and subtraction – a developmental progression • Using children's thinking to guide your instruction – part 1	• Multi-digit addition and subtraction – a developmental progression – part 1 • Counting as a problem-solving endeavor	• Multi-digit addition and subtraction – a developmental progression – part 2 • Common components of a CGI Classroom		• Base Ten Number Concepts	• Classroom embedded Day – focus on base ten number concepts	• Classroom embedded day – focus on addition and subtraction	• Multi-digit addition and subtraction

(Continued)

School Year	Summer					Fall			
	Day 1	Day 2	Day 3	Day 4	Day 5	Day 1	Day 2	Day 3	Day 4
2017-2018[a]	• Introduction to CGI • Addition and Subtraction Problem Types and Strategies	• Multiplication and Division Problem Types and Strategies	• Problem Types and Solution Strategies • Interview Children	• Using CGI in your classroom • Multi-digit Addition and Subtraction Strategies	• Multi-digit Addition and Subtraction Strategies • Planning for instruction	• Developing Base Ten Understanding	• Classroom Embedded Work	• Strategies for solving addition and subtraction problems with large numbers	
School Year	**Winter/Spring**								
	Day 1	**Day2**	**Day 3**	**Day 4**	**Day 5**	**Day 6**	**Day 7**	**Day 8**	
2018-2019[b] (Group A)[3]	• Direct Modeling • Addition and Subtraction Problem Types and Strategies	• Multiplication and Division Problem Types and Strategies • Observation of Math Interview	• Interview Students • Common Components of a CGI Classroom • CGI Kindergarten small group lesson	• Base Ten Problem Types and Strategies	• Interview students, plan and observe a second-grade lesson on developing base ten understanding	• Addition and Subtraction with multi-digit numbers	• Interview students, plan and observe a first-grade lesson on early invented strategies for multi-digit addition and subtraction	• Multi-digit addition and subtraction (continued) • Closing thoughts	
2018-2019[b] (Group B)[c]	• Direct Modeling • Addition and Subtraction Problem Types and Strategies	• Multiplication and Division Problem Types and Strategies • Observation of Math Interview	• Addition and Subtraction with multi-digit numbers • Interview Children	• Common Components of a CGI Classroom • CGI Kindergarten small group lesson	• Base Ten Problem Types and Strategies	• Interview students, plan and observe a second-grade lesson on developing base ten understanding	• Interview students, plan and observe a first-grade lesson on early invented strategies for multi-digit addition and subtraction	• Multi-digit addition and subtraction • Closing thoughts	

Note.

[a]This school year is out of chronological order because the agenda did not follow the same format. All follow-up days occurred during fall. .

[b]This school year is out of chronological order because the agenda did not follow the same format. All PD days occurred between mid February and mid May.

[c]This school year is out of chronological order because there were two different groups and a different agenda for each group. Although the content was the same across the two groups, the content was delivered in a different sequence.

Table A2. CGI K–2 Year 2 Overviews by School Year

School Year	Summer					Fall		Winter/Spring	
	Day 1	**Day 2**	**Day 3**	**Day 4**	**Day 5**	**Day 1**	**Day 2**	**Day 1**	**Day 2**
2014-2015	• Teacher Decision Making • Understandin g the relationships between problem types	• Solution Strategies – Assessing understandin g of addition and subtraction	• Integrating Base Ten with Multiplication and Division • Purposeful Pedagogy Model	• Developme nt of Base Ten Understand ings • Introductio n to developing strategies with number facts		• Classroom Embedded Work – focus on developing strategies for number facts	• Relational Thinking • Properties of Operations • Properties of Equations	• Classroom Embedded Work – focus on developing students' strategies for multi-digit addition	• Multi-digit Addition and Subtraction • Becoming Self-Sustaining Generative Professional
2016-2017	• Teacher Decision Making • Addition and Subtraction Problem Types • Direct Modeling	• Addition and Subtraction Strategies – Counting and Derived Facts • Problem Difficulty	• Developing Addition and Subtraction Relationships • Multiplication and Division Problem Types • Developing Base Ten Number Concepts	• Multi-digit addition and subtraction • Purposeful Pedagogy and Discourse Instructiona l Model	• Developing Fluency with Number Facts • Using CGI in your classroom	• Classroom Embedded Day – focus on developing fluency with number facts	• Developing Addition and Subtraction Relationships • Base Ten Number Concepts	• Classroom Embedded Day – focus on developing students' strategies	• Developing Fluency with Multi-digit addition and subtraction
2019-2020	• Teaching with Understandin g • Addition and Subtraction Problem Types • Direct Modeling	• Addition and Subtraction Strategies – Counting and Derived Facts • Problem Difficulty	• Multiplication and Division Problem Types • Developing Base Ten Number Concepts	• Multi-digit addition and subtraction • Introductio n to Developing Fluency with Number Facts		• Developing Addition and Subtraction Relationships • Base Ten Number Concepts	• Classroom Embedded Day – focus on developing fluency with number facts	• Classroom Embedded Day – focus on developing students' strategies	• Developing Fluency with Multi-digit addition and subtraction

(Continued)

School Year	Summer					Fall		Winter/Spring	
	Day 1	**Day 2**	**Day 3**	**Day 4**	**Day 5**	**Day 1**	**Day 2**	**Day 1**	**Day 2**
2020-2021	• Addition and Subtraction Problem Types • Direct Modeling	• Addition and Subtraction Strategies – Counting and Derived Facts	• Developing Base Ten Number Concepts • Planning for your students – part 1	• Multi-digit addition and subtraction • Planning for your students – part 2		• Developing Addition and Subtraction Relationships	• Classroom Embedded Day – focus on developing fluency with number facts	• Classroom Embedded Day – focus on developing students' strategies	• Developing Fluency with Multi-digit addition and subtraction
2021-2022	• Addition and Subtraction Problem Types • Direct Modeling	• Addition and Subtraction Strategies – Counting and Derived Facts	• Developing Base Ten Number Concepts • Planning for your students – part 1	• Multi-digit addition and subtraction • Planning for your students – part 2		• Developing Addition and Subtraction Relationships	• Classroom Embedded Day – focus on developing fluency with number facts	• Classroom Embedded Day – focus on developing students' strategies	• Developing Fluency with Multi-digit addition and subtraction

Table A3. CGI K–2 Year 3 Overviews by School Year

<table>
<tr><th rowspan="2">School Year</th><th colspan="5">Summer</th><th colspan="2">Fall</th><th colspan="2">Winter/Spring</th></tr>
<tr><th colspan="5">Days 1–5</th><th>Day 1</th><th>Day 2</th><th>Day 1</th><th>Day 2</th></tr>
<tr><td>2015-2016</td><td colspan="5">Strategy progression for addition and subtraction problems with multi-digit numbers

The influence of number choices and problem difficulty

Notating children’s strategies for solving problem

Properties of Operations
• The associative property of addition

Purposeful Pedagogy and Instruction Discourse Model with special attention to Professional Noticing

Developing Base Ten Number Concepts

Developing an understanding of the Equal sign

Developing a CGI Professional Learning Community

Properties of Operations
• the distributive property Florida Standards

Using the Purposeful Pedagogy and Instructional Discourse Model with a textbook

Number facts
• Teaching through sequence of equations
• Assessing with Number Fact Assessment</td><td>Classroom Embedded work – teaching number facts with understanding</td><td>Choosing efficient derived facts

Fact Fluency

Continued work with CGI PLC</td><td>Classroom Embedded Work – Computational Fluency</td><td>Computation al fluency

Continued work with CGI PLC</td></tr>
<tr><td rowspan="3">2019-2020</td><th colspan="5">Summer</th><th colspan="2">Fall</th><th colspan="2">Winter/Spring</th></tr>
<tr><th>Day 1</th><th>Day 2</th><th>Day 3</th><th>Day 4</th><th>Day 5</th><th>Day 1</th><th>Day 2</th><th>Day 1</th><th>Day 2</th></tr>
<tr><td>• Purposeful Pedagogy
• Common Characteristi cs of CGI Classrooms
• Equal Sign</td><td>• Addition and Subtraction with single digits
• Equation situations to represent relationships</td><td>• Multi-digit Addition and Subtraction
• Notation to represent students’ strategies</td><td>• Base Ten Number Concepts
• Number choice and Problem Difficulty</td><td></td><td>• Developing Fluency with Addition and Subtraction Facts</td><td>• Classroom Embedded Day – Learning Subtraction Facts</td><td>• Classroom Embedded Day – Developing Computational Fluency</td><td>• Anticipatory Thinking</td></tr>
</table>

(Continued)

School Year	Summer					Fall		Winter/Spring	
	Day 1	**Day 2**	**Day 3**	**Day 4**	**Day 5**	**Day 1**	**Day 2**	**Day 1**	**Day 2**
2020-2021	• Introductions and Welcome • CGI Math Lessons • Single Digit Addition and Subtraction	• 3 types of Mathematical Notation • Teaching Number Facts	• Multi-digit addition and subtraction. • Developing fluency	• Base Ten Number Concepts • Planning for your students		• Kindergarten Teaching Experience	• Learning Subtraction facts – Classroom embedded work	• Developing Computational Fluency – Classroom Embedded work	• Anticipatory thinking – putting it all together
2021-2022	• Introductions and Welcome • CGI Math Lessons • Single Digit Addition and Subtraction	• 3 types of Mathematical Notation • Teaching Number Facts	• Multi-digit addition and subtraction. • Developing fluency	• Base Ten Number Concepts • Planning for your students		• Kindergarten Teaching Experience	• Learning Subtraction facts – Classroom embedded work	• Developing Computational Fluency – Classroom Embedded work	• Anticipatory thinking – putting it all together

School Year	Summer					Fall			
	Day 1	**Day 2**	**Day 3**	**Day 4**	**Day 5**	**Day 1**	**Day 2**	**Day 3**	
2017-2018[1]	Characteristics of CGI Classrooms Purposeful Pedagogy Developing an understanding of the equal sign Our goals for you in CGI year 3	Building the solution strategy chart. Notation Review Examining Relationships in Derived Facts Developing Fact Fluency	Multi-digit strategies progression for JRU and SRU problems Relationships embedded in multi-digit strategies Engaging students with each other's strategies	Multiplication and Division Problems and Strategies Children's understanding of 10 as a unit Influence of Number choice on Problem Difficulty		Classroom Embedded Work –Fact Fluency CE Day – Subtraction	Classroom Embedded Work – Computation Fluency CE Day	Pulling it all together. Developing an understanding of how operations work	

Table A4. CGI 3–5 Year 1 Overviews by School Year

School Year	Summer					Fall		Winter/Spring	
	Day 1	Day 2	Day 3	Day 4	Day 5	Day 1	Day 2	Day 1	Day 2
2015-2016	• Teaching Math for Understanding • Multiplication and Division Problems	• Relational Thinking – Engaging students with properties of operations • Start Problem Solving notebook	• Student Interviews • Understanding Fractions as Quantities – Equal Sharing Problems	• Multiplication and Division with Fractions • Students' Strategies for Multiple Groups Problems.	• Understanding Equations • Using ECM with your students.	• Classroom Embedded work – Equal Sharing Problems	• Developing an Understanding of Fractional Quantities	• Classroom Embedded Work – Multiplication and Division	• Fraction Computation – Multiplication and Division with a whole number of groups
2016-2017	• Teaching Math for Understanding • Multiplication and Division Word Problems • Students' Strategies for solving Multiplication and Division Problems	• Understanding Fractions as Quantities - Equal Sharing Problems • Students' Strategies for Equal Sharing Problems	• Multiple Groups • Fraction Multiplication and Division • Student Interviews	• Relational Thinking – Engaging students with properties of operations – Fraction Multiplication and Division	• Partial Groups – Fraction Multiplication and Division • Understanding the equal sign • Using ECM with your students.	• Classroom Embedded Work – Equal Sharing in 3rd of 4th grades	• Base Ten Number Concepts	• Classroom Embedded Day – Base Ten Number Concepts	• Multi-digit Multiplication and Division
School Year	**Summer**					**Fall**		**Winter/Spring**	
	Day 1	**Day 2**	**Day 3**	**Day 4**	**Day 5**	**Day 1**	**Day 2**	**Day 1**	**Day 2**
2019-2020	• Multiplication and division of Whole Numbers • A view into a CGI classroom	• Using Equal Sharing Problems to develop an understanding of fractional quantities • Fraction concepts embedded in students' solutions	• Fraction multiplication and division – part 1 • Student Interviews	• Fraction multiplication and division – part 2 • Getting ready to use CGI in your classroom		• Classroom Embedded Work – developing fraction concepts	• Base ten number concepts – decimals and Whole Numbers	• Classroom Embedded Work – fraction multiplication	• Division of whole numbers

(Continued)

School Year	Summer					Fall		Winter/Spring	
	Day 1	Day 2	Day 3	Day 4	Day 5	Day 1	Day 2	Day 3	Day 4
2020-2021	• Introductions • Children's intuitive problem-solving strategies • Whole number multiplication and division of Whole Numbers	• Using Equal Sharing Problems to introduce fraction concepts • Linking students' strategies to state math standards	• Students' strategies for fraction multiplication and division – part 1 • Using CGI with your students – writing and posing problems	• Fraction multiplication and division – part 2 • Using CGI with your students – posing problems and discussing students' strategies		• Classroom Embedded Work – developing fraction concepts	• Base ten number concepts – decimals and Whole Numbers	• Classroom Embedded Work – fraction multiplication	• Division of whole numbers
2021-2022	• Introductions • Children's intuitive problem-solving strategies • Whole number multiplication and division of Whole Numbers	• Using Equal Sharing Problems to introduce fraction concepts • Linking students' strategies to state math standards	• Students' strategies for fraction multiplication and division – part 1 • Using CGI with your students – writing and posing problems	• Fraction multiplication and division – part 2 • Using CGI with your students – posing problems and discussing students' strategies		• Classroom Embedded Work – developing fraction concepts	• Base ten number concepts – decimals and Whole Numbers	• Classroom Embedded Work – fraction multiplication	• Division of whole numbers
School Year	**Summer**					**Winter/Spring**			
	Day 1	**Day 2**	**Day 3**	**Day 4**	**Day 5**	**Day 1**	**Day 2**	**Day 3**	**Day 4**
2017-2018	• Multiplication and Division Word Problems – Whole Numbers	• Understanding Fractional Quantities - Equal Sharing Problems	• Fraction Multiplication and Division – Part 1 • Student Interviews	• Using CGI with your students.	• Fraction Multiplication and Division – Part 2	• Classroom Embedded work – Equal Sharing Problems	• Base Ten Number Concepts – Decimals and Whole Numbers.	• Division – Whole Numbers and Fractions	

Table A5. CGI 3–5 Year 2 Overviews by School Year

School Year	Summer					Fall		Winter/Spring	
	Day 1	Day 2	Day 3	Day 4	Day 5	Day 1	Day 2	Day 1	Day 2
2016-2017	• Multiplication with Whole Numbers • Equations to represent properties of operations.	• Division with Whole Numbers • Purposeful Pedagogy and Discourse Instructional Model	• Equal Sharing Problems. • Developing an understanding of Equivalent Fractions	• Multiple Groups Problems. • Developing Fraction Concepts and Properties of Operations.	• Partial Groups Problems • Intro to developing fluency with number facts	• Fact Fluency Classroom Embedded work	• Developing an Understanding of Base Ten – Decimal Numbers • Addition and Subtraction with • Whole Numbers	• Classroom Embedded Work Decimal Numbers	• Developing Fluency With Computation
2017-2018	• Multiplication with Whole Numbers • Equations to represent properties of operations.	• Division with Whole Numbers • Purposeful Pedagogy and Discourse Instructional Model	• Equal Sharing Problems. • Developing an understanding of Equivalent Fractions	• Multiple Groups Problems. • Developing Fraction Concepts and Properties of Operations. • Partial Groups Problems		• Developing an Understanding of Base Ten – Decimal Numbers • Addition and Subtraction with Whole Numbers		Classroom Embedded Work	Partial Groups Problems
2019-2020	• Multiplication with Whole Numbers • Notating Students' Strategies. • Equations to represent properties of operations. • Equal Sign	• Multi-digit Division. Division relationships. • Purposeful Pedagogy and Discourse Instructional Model	• Equal Sharing Problems. Important Fraction Concepts. • Developing an understanding of Equivalent Fractions	• Multiple Groups Problems. • Developing Fraction Concepts and Properties of Operations. • Partial Groups Problems		• Developing an Understanding of Base Ten – Decimal Numbers • Addition and Subtraction with Whole Numbers	• Classroom Embedded Work – Base Ten - Decimal Numbers	• Teaching Multiplication Facts	• Division – whole numbers and fractions

(Continued)

School Year	Summer					Fall		Winter/Spring	
	Day 1	Day 2	Day 3	Day 4	Day 5	Day 1	Day 2	Day 1	Day 2
2020-2021	• Introductions and Orientation • Multiplication whole numbers • Analyzing students' written work	• Multidigit division • Planning instruction from students' written work	• Equal Sharing problems and • equivalence • Grade level meetings with expert CGI teachers	• Fraction multiplication and • division • Using CGI with your students – posing problems and discussing students' strategies		• Whole number addition and subtraction	• Learning and teaching multiplication facts - Classroom embedded work	• Fraction multiplication and division	• Multiplication and division of fractions – Classroom embedded work
2021-2022	• Introductions and Orientation • Multiplication whole numbers • Analyzing students' written work	• Multidigit division • Planning instruction from students' written work	• Equal Sharing problems and equivalence • Grade level meetings with expert CGI teachers	• Fraction multiplication and division • Using CGI with your students – posing problems and discussing students' strategies		• Whole number addition and subtraction	• Learning and teaching multiplication facts - Classroom embedded work	• Fraction multiplication and division	• Multiplication and division of fractions – Classroom embedded work

Table A6. CGI 3–5 Year 3 Overviews by School Year

School Year	Summer				Fall		Winter/Spring	
	Day 1	Day 2	Day 3	Day 4	Day 1	Day 2	Day 1	Day 2
2019-2020	• Welcome back • Developing an Understanding of Equivalent Fractions • Addition and Subtraction – Fractions and Whole Number	• Multiplication with Whole Numbers • Multiple Groups Multiplication • Partial Groups Multiplication	• Division with Whole Numbers • Multiple Groups Division – part 1 • Partial Groups Division	• Multiple Groups Division – part 2 • Partial Groups Division	• Partial Groups Problems	• Classroom Embedded work – Fraction Division	• Classroom Embedded Work – Addition and subtraction of fractions	• Tying things together – Anticipatory Thinking
2021-2022	• Developing an Understanding of Equivalent Fractions • Addition and Subtraction – Fractions and Whole Number	• Multiplication with Whole Numbers • Multiple Groups Multiplication • Partial Groups Multiplication	• Division with Whole Numbers • Multiple Groups Division – part 1 • Partial Groups Division	• Multiple Groups Division – part 2 • Partial Groups Division	• Partial Groups Problems	• Classroom Embedded work – Fraction Division	• Classroom Embedded Work – Addition and subtraction of fractions	• Tying things together – Anticipatory Thinking