Effects of Fourth- and Fifth-Grade *Super Solvers* Intervention on Fraction Magnitude Understanding and Calculation Skill

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Across the elementary-school grades, the curriculum is designed to gradually expand understanding of number by consolidating principles of whole numbers and rational numbers into a single numerical framework (Siegler, Thompson, & Schneider, 2011). This transition is a critical juncture in students' mathematics development, because competence with rational numbers, often indexed in the form of fraction magnitude understanding, is important for algebra and other forms of more advanced mathematics learning (Booth & Newton, 2012; Booth, Newton, & Twiss-Garrity, 2014; Geary Hoard, Nugent, & Bailey, 2012; Empson & Levi, 2011).

Yet, the shift from whole to rational numbers, which begins with fractions in the U.S. curriculum at third grade, represents a challenge for many students (Durkin & Rittle-Johnson, 2015; Kallai & Tzelgov, 2009; Obersteiner, Van Doorena, Van Hoof, & Verschaffel, 2013; Siegler et al., 2011; Vamvakoussi & Vosniadou, 2010). This developmental hurdle is especially challenging for students who have struggled in the primary grades with whole-number learning. According to Namkung, Fuchs, and Koziol (2018), students with below grade-level whole-number knowledge are 32 times more likely than students with adequate grade-level whole-number knowledge to struggle with fractions.

The source of student errors often resides with misapplication of whole-number principles to fractions. For example, when comparing two fractions, at-risk fourth graders commonly choose the fraction with greater numeral as the fraction greater magnitude, and a high proportion of errors in ordering three fractions also reflect whole-number thinking (Malone & Fuchs, 2017). A similar pattern of error types occurs among at-risk fourth graders when adding and subtracting fractions (Schumacher & Malone, 2017).

The sizable achievement gap in fractions knowledge between students with prior histories of whole-number learning and their not-at-risk classmates (Namkung et al., 2018) indicates the need for intervention to supplement the schools' classwide instructional program for at-risk learners. In a series of randomized controlled trials (RCTs), Fuchs and colleagues (Fuchs et al., 2016; Fuchs, Schumacher et al., 2013, 2014, 2016; Malone, Fuchs, Sterba, Fuchs, & Foreman-Murray, in press) tested the effects of *Fractions Face-Off!*, a fourth-grade intervention focused on fraction magnitude, which is designed to be used as Tier 2 intervention within a multi-tier system of supports. At the start of fourth grade, participants were identified as performing below the 35% percentile on a nationally-normed math achievement test. Approximately half of the sample was below the 15^{th} percentile; half between the 16^{th} and 34^{th} percentile.

In each study, findings indicated superior performance for the intervention condition over the control group on fraction magnitude (FM) understanding, as indexed on comparing fractions, ordering fractions, placing fractions on number lines, and a measure of overall fractions performance based on released NAEP items. Effect sizes (ESs) ranged from 0.37 to 2.50 depending on study year and outcome, with most in the moderate to large range.

Yet, as demonstrated in Fuchs et al. (2015), the focus in schools across the U.S. changed over the course of these studies, as a function of the challenging *Career- and College-Ready Standards* (CCRS) national reform movement. Contrary to the common perception that CCRS increase the challenge of earlier standards, Fuchs et al.'s analysis (2015) showed that earlier state standards on *fourth-grade fractions* were comparably difficult to CCRS. Instead, CCRS decreased content coverage by treating fewer topics at a given grade *in greater depth*. The new standards also strengthened the curricular focus on fractions in *third grade*, such that students entered fourth grade with stronger fractions performance than had previously been the case, even as CCRS re-focused fourth- and fifth-grade curricular attention on fraction magnitude understanding, while solidifying conceptual focus within fraction calculations.

The present study is part of a larger effort aimed at developing builds on *Fractions Face-Off!*'s instructional design to align more closely with the enriched emphases presently operating in U.S. schools. These extensions involved development and efficacy testing of a *third-grade* fractions intervention for improving multiplication, a foundational skill essential for fractions competence, as well as fraction magnitude understanding, fraction word problems, and fraction calculations (see Wang et al., in press for pilot study and Wang et al., 2019 for efficacy study).

It also included a next iteration of *Fractions Face-Off!* that reflects the enriched fraction context in U.S. schools at grades 4 and 5. This suite of fractions interventions (one designed at grade 3; the other at grades 4-5), which builds on and extends *Fractions Face-Off!*, is referred to as *Super Solvers*. (The previous name is retained because some users may wish to implement the earlier iteration, depending on the population for which the fraction intervention is intended.)

The *main purpose of the present study was to assess the efficacy of two of the three main instructional components of Super Solvers at grades 4-5*: the component focused on fraction magnitude (FM) and the component focused on fraction calculations (CA). The third component, not addressed in the present study, is fraction word problems (see Malone et al., 2019 for word-problem efficacy data). The present study's secondary purpose was to examine whether an instructional strategy focused on conceptual and strategic error analysis of fraction calculations (EA) provides added value on at-risk students' fraction calculation learning. The FM and CA components represent the standard form of the *Super Solvers* FM and CA components, and the study's two intervention conditions (FM+CA and FM+CA+EA) include both of these standard Super Solvers conditions. The FM+CA+EA also includes the EA condition.

Method

Participants

Participants were fourth- and fifth-grade students identified as at risk for mathematics difficulties in a U.S. Southeastern metropolitan city. Risk was operationalized as scoring at or below the 20th percentile on the Wide Range Achievement Test (4th ed.; WRAT-4; Wilkinson & Robertson, 2006). In this study's population, this screening measure, which reflects whole-number calculation skill, is predictive of end of fourth-grade fractions competence (Namkung et al., 2018).

From a randomly selected pool of 341 students who met this criterion during large-group screening, we excluded (a) students who scored below the 9th percentile on both subtests of the Wechsler Abbreviated Scales of Intelligence (2nd ed.; WASI; Wechsler, 1999) or were identified by teachers as having severe behavior difficulties (these exclusions were applied to target students specifically at-risk for learning disabilities); (b) students who had very low English proficiency or were identified by teachers with strong math achievement (these exclusion were applied to avoid false positive identifications of risk for learning disabilities); and (c) students who had a history of chronic absenteeism.

The 183 students who thus qualified for study entry were randomly assigned, at the individual student level, to three study conditions: Super Solvers (FM+CA); Super Solvers + EA (FM+CA+EA), and the control groups (standard school practice, which includes fraction classroom instruction with supplemental intervention for some students). During the study, 27 students moved with families to outside the study's reach; 8 developed scheduling conflicts; 3 withdrew before posttesting; and 2 were discontinued due to severe behavior challenges. To assess whether attrition occurred differentially, we applied What Works Clearinghouse's "liberal" standard (WWC; https://files.eric.ed.gov/fulltext/ED579501.pdf), because attrition was largely unrelated to intervention (with three severe behavior challenge exclusions related to intervention).

We considered differential attrition in accord with the study's two orthogonal contrasts (see data analysis section). For the contrast between combined intervention (25% attrition) versus control (15% attrition), overall attrition was 22%, with 10% differential attrition; this falls within the WWC's acceptable (green) zone. For the contrast between intervention conditions (26% FM+WP attrition; 25% FM+WP+EA attrition), overall attrition was 25%, with 1% differential attrition, again within the WWC's acceptable (green) zone. Also, ANOVA and chi-square analyses indicated comparability between students who left the study versus those who remained on screening, demographic, and pretest fraction variables.

In the final sample, screening scores for the three conditions, respectively, were as follows: on *WRAT-4*, 24.39 (*SD* = 1.98), 24.85 (*SD* = 1.84), and 24.42 (*SD* = 2.63); on *WASI Vocabulary*, WASI 23.34 (*SD* = 4.82), 22.59 (*SD* = 5.69), and 23.98 (*SD* = 5.39); and on *WASI Matrix Reasoning*, 12.50 (*SD* = 3.73), 12.07 (*SD* = 3.21), and 12.43 (*SD* = 3.97). In the three respective conditions, 52%, 50%, 47% were female; 43%, 46%, 43% were African-American, 23%, 20%, 29% Caucasian, 25%, 24%, 21% Hispanic, 9%, 10%, 7% other; 9%, 14%, 13% received special education services; 27%, 28%, 23% were English learners; and 52%, 52%, 58% qualified for subsided lunch. There were no significant differences among study conditions on these variables.

Screening Measures

With *WRAT-4* (Wilkinson & Robertson, 2006), students have 10 min to complete 40 computation problems of progressive difficulty. Reliability for this age group is .94. With *WASI Vocabulary*, students identify pictures (4 items) and define words (38 items). Students receive a score of 1 (correct) or 0 (incorrect) on picture items and receive a score of 0, 1, or 2 on word items. Testing discontinues after five consecutive scores of 0. Reliability for this age group is .88. With *WASI Matrix Reasoning* (Wechsler, 1999) students solve puzzles to complete patterns by selecting choices. Testing discontinues after four consecutive errors or four errors in any five items. Reliability for this age group is .93.

Outcome Measures

FM understanding. The *0-2 Fraction Number Line* task (Hamlett, Schumacher, & Fuchs, 2011, adapted from Siegler et al., 2011) requires students to place fractions on a number line marked with endpoints 0 and 2: $\frac{2}{3}$, $\frac{7}{9}$, $\frac{5}{6}$, $\frac{1}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{19}$, $\frac{3}{8}$, $\frac{7}{4}$, $\frac{3}{2}$, $\frac{4}{3}$, $\frac{7}{6}$, $\frac{15}{8}$, $1\frac{1}{8}$, $1\frac{1}{5}$, $1\frac{5}{6}$, $1\frac{2}{4}$, $1\frac{11}{12}$, $\frac{5}{5}$, 1), without relying on FM+CA's paper and pencil strategies. Each item's score is the absolute difference between where the student places the fraction and the actual value of the fraction. Scores are divided by 2 (for the 0-2 number line) and averaged across items to yield the average absolute error. Because scores reflect error, we multiplied scores by –1 so higher scores reflect greater accuracy. Test-retest reliability is .80.

Ordering Fractions (Malone & Fuchs, 2017) includes 10 items, each requiring students to order three fractions from least to greatest. Items includes a mix of fractions less than 1, equal to 1, and greater than 1, without three fractions with the same numerator or same denominator. Sample-based α = 81.

The last FM assessment comprises *17 released items from the National Assessment of Education Progress* (NAEP-revised; U.S. Department of Education, 1990-2009). In the series of prior randomized controlled trials testing *Fractions Face-Off!* (see summary in Fuchs et al., 2017), we had used a different problem set with 22 NAEP items. To test *Super Solvers*, we revised deleted 10 items from the earlier measure: 5 easy part-whole understanding (e.g., identifying the part of a pizza that was eaten) and 5 easy pre-algebraic knowledge (e.g., ___ - 8 = 21). The revised problem set includes 12 items from the earlier set plus five new word-problem items involving proportional reasoning (we first revised NAEP in this way during the previous year's study because they were relevant to the *Super Solvers* WP component, but this WP component was not included in the present trial). Of 17 items on NAEP-*revised*, 12 assess FM, 2 proportional reasoning; 3 identifying fractions and fraction equivalencies with pictures (part-whole understanding). Testers read each problem aloud (twice, if requested). Sample-based α = .82.

Fraction calculations. With *Fraction Calculations* (Malone & Fuchs, 2017), students have 5 min to complete two fraction addition items (one with like and the other with unlike denominators), one subtraction item (with unlike denominators), three multiplication items (one with like denominators, one with unlike denominators, one with multiplication of a whole number with a fraction), and four division items (one with a divisor and a dividend with the same denominator, one with both fractions with unlike denominators, one with a whole number divided by a fraction, one with a fraction divided by a whole number). Sample-based α = .89. (See more information on problem types in Supplemental File.)

Intervention

Both FM conditions were delivered in dyads, three times per week for 13 weeks (40 min per session), using *Super Solvers* – Grades 4 - 5 (Fuchs, Malone, Wang, Schumacher, Krowka, & Fuchs, 2019). Although *Super Solvers* is partially scripted, scripts are provided to guide implementers with a concrete representation of session content, instructional methods, and the nature of explanations. Tutors are not permitted to read or memorize scripts, and they must pass fidelity checks without relying on scripts before intervention begins. Once all problem types are taught, scripts are replaced with activity guides.

As already noted, in this efficacy trial, we tested two major program emphases of the *Super Solvers* program: *Fraction Action*, designed to build FM understanding, and *Calculations Quest*, designed to build fraction CA skill. (*Super Solvers* third major emphasis, *Problem Quest*, focuses on fraction word problems, including proportion, comparing, and splitting word problems.)

At the start of *Super Solvers*, the major emphasis shifts is *Fraction Action*; this shifts to *Calculation Quest* in Lesson 22. *Fraction Action* lasts 17 min in the first 21 lessons but drops to 10 min of review in Lesson 22. *Calculations* Quest lasts 5 min in Lessons 13-21 but increases to 15 min in Lesson 22. To support FM and CA, a small program focus on whole-number multiplication is addressed in Lessons 4-13 (5 min) and then again in Lesson 22 with a *Multi-Minute* (2 min) activity designed to build multiplication fluency. *Power Practice*, the final 7-min activity in Lessons 4-39, provides independent practice. Every lesson also includes a 3-min of *Brain Boost* discussion on self-regulated learning and a 3-min *Fraction Flash* magnitude fluency-building activity.

On some weeks, progress monitoring replaces *Power Practice*. Progress monitoring extends the *Brain Boost* activity with an additional 7 min dedicated to administration of and feedback on two curriculum-based measurement tasks. One focuses on FM (*Super Challenge*: 20 items, representing all *Fraction Action* problem types, appearing on each alternate test form in a new order; Lessons 9, 15, 21, 27, 33, and 39). The other is focused on CA (*Conquer*

Calculations: 10 items, representing all *Calculation Quest* problem types, appearing on each alternate forms in a new order; Lessons 18, 24, 30, and 36).

Instruction common to both FM conditions. To develop FM, *Fraction Action* (Lessons 1-39) incorporates three activities/problem types: comparing fractions, ordering fractions, and placing fractions on number lines (0-1 and 0-2). In initial lessons, students learn to compare fractions with same denominators or same numerators using conceptual activities and fraction tiles and circles. For fractions with same denominators, students learn to look at numerators and think about which fraction has more same size parts; for fractions with same numerators, to look at denominators and think about which fraction has bigger parts.

Then, benchmarking to $\frac{1}{2}$ and 1 is introduced. Students label each fraction according to its value relative to 1 (L1, =1, and G1, where L stands for less than and G stands for great than) or $\frac{1}{2}$ using similar labels. If both fractions are < or > $\frac{1}{2}$, they learn to find an equivalent fraction with the same numerator or denominator. Students gradually extend these strategies from comparing to ordering and number-line placement. A *Compare Card*, which is gradually faded over the 39 weeks, provides students easy reference to a consolidated set of strategies across three magnitude assessment problem types. Manipulatives are used to support key ideas. For ordering and number line placement, students convert each G1 fraction to a mixed number; then, compare whole numbers to decide which mixed number is bigger; and if needed, compare the L1 fraction of each mixed number to determine the greater value.

Fraction Flash (Lessons 4-39) is a timed magnitude fluency activity, with two main flashcard activities. In the first, students compare the magnitude of two fractions. Some comparisons have same numerators or denominators; the majority require benchmarking. In the second activity, students assess whether a single fraction is <. =, or > a benchmark, alternating between $\frac{1}{2}$ and 1 as the benchmark and sometimes also pointing to where the fraction belongs on a 0-2 number line. For incorrect responses, students provide the correct answer with an explanation before the tutor presents the next flashcard, as time elapses. They work cooperatively as a pair, taking turns responding. If the team beats the previous session's score, they earn fraction "money" for their "bank account."

The purpose of *Brain Boost* is to develop students' self-monitoring, goal-setting, and meaningful participation in intervention. Students begin each lesson by reading a story from a comic series called *Brain Boost Adventures*. Stories focus on increasing *brain power* (working hard to make your brain grow stronger) by setting and striving to achieve math goals. Stories address self-sufficiency (avoiding unnecessary reliance on strategy cards and other program supports), partner support (asking for/providing help), goal setting, taking responsibility for planning one's own learning activities, and tracking one's own progress. Students use *Super Challenge* progress-monitoring graphs to adjust goals and identify productive learning activities.

Multi Minute (Lessons 4-12 and 22-39) focuses on whole-number multiplication (factors 1-10). First, students learn rules for multiplying by 1 and by 10, then to skip count by 10s. Next, they practice skip counting with factors 2 and 5. For the 9-family, they learn a trick using fingers. For 3, 4, 6, 7, and 8 families, they learn decomposition strategies (using an easy or known fact to solve a hard fact), write out the relevant skip-counting sequence on their work, and memorize facts without reliable strategies. On Lesson 22, *Multi Minute* is introduced as a 1-min flashcard fluency activity. Students work cooperatively as a pair. For incorrect responses, the correction and money-earning procedures are the same as described above.

Power Practice involves independent practice problems on that day's content as well as cumulative review and corrective feedback. The program also includes a behavior-management system. Each lesson begins with a description of behavior expectations: listening, trying your best, being respectful. Behavior is tracked through the lesson with a timer set as random intervals. At each beep, tutors decide if students are on-task and following rules; if so, they deliver behavior-specific praise and deposit “money” into “bank accounts.” Tutors award additional “money” for correct *Power Practice* work and providing good explanations. With money, students purchase tangible rewards or opportunities to assist with *Super Solvers* tasks (setting the timer, passing out papers). Money can be saved to purchase higher-valued rewards.

Instruction differentiating the two FM conditions. Differences between FM+CA and FM+CA+EA focused on error analysis (EA) to check fraction calculations. EA instruction and practice occurred during *Calculations Quest* (Lessons 13-39). Other features of *Calculations Quest* are the same across FM+CA and FM+CA+EA.

In both conditions, students learn operational procedures to add, subtract, multiply, and divide fractions, with support from a *Fraction Calculations Card.* Students first identify if a problem is addition, subtraction, multiplication, or division; then they follow the solution steps taught for that operation. To add or subtract fractions, students identify whether the fractions have the same denominator. If so, they write the same denominator in the answer and add or subtract the problem stem numerators. If denominators differ, students use *Fraction Action* skills to find an equivalent fraction with the same denominator. For multiplication, students first multiply denominators, then numerators. For division, they multiply by the reciprocal: keep the first fraction, change the sign from division to multiplication, flip the second number, then multiply denominators and then numerators.

Students spend three lessons adding and subtracting fractions with like and unlike denominators (when one fraction is $\frac{1}{2}$); they add or subtract fractions with unlike denominators with non-$\frac{1}{2}$ equivalencies after *Fraction Action* covers that. They learn multiplication in Lesson 16 and division in Lesson 22. Calculations practice occurs in interleaved format (practice sheets present operations in mixed format, without blocking operation types), with an equal distribution of the four operations throughout the *Calculations Quest* lessons, even before all operations are introduced. Also, in Lessons 18, 24, 30, and 36, the *Brain Boost* progress-monitoring probe is *Conquer Calculations*.

For each operation, EA instruction first provides conceptual instruction on judging the reasonableness of answers. This relies on "go-to" problems, for each operation involving the same simple fractions ($\frac{1}{2}$ and $\frac{1}{4}$) and number lines to support thinking about how starting amounts change. Students are taught to invoke the go-to problem for a concrete reminder about what happens with each fraction operation. This is connected to the EA of each operation's common calculation error types. The *Brain Boost* characters depict use of these EAs, and students practice applying EAs on work produced by those characters. Students identify errors and error types (a careless mistake or a misconception error or a basic fact error or a strategy error) and provide and explain correct answers. To control for EA intervention time, FM+CA students solve and explain their strategy use for four whole-number multiplication problems with factors of 6, 7, or 8 during each *Calculations Quest* lesson.

For addition and subtraction, the EA comprises two steps: Students check that (1) they did not add or subtract denominators (the common addition/subtraction error type) and (2) the answer makes sense (for addition, the numerator in the answer is greater than the numerator in the starting amount; for subtraction, the numerator in the answer is less than the numerator in the starting amount. (EA occurs before students reduce answers.) For multiplication, conceptual instruction teaches students to think of the multiplication sign as *of*; to write *of* underneath the sign; and to re-read the problem (e.g., for $\frac{1}{2}$ times $\frac{1}{4}$: $take\ \frac{1}{2}\ of\ \frac{1}{4}$). With the same methods, students learn that multiplying a number by a fraction greater than 1 produces an answer greater than the starting amount. The EA is to check that (1) the answer makes sense and (2) denominators are multiplied, to avoid the common error of carrying the same denominator into answers. Division parallels multiplication (when a number is divided by a fraction < 1, the answer is greater than the starting amount; hen a number is divided by a number > 1, the answer is less than the starting amount).

Fidelity of Implementation

Every intervention session was audio-recorded. To provide corrective feedback on fidelity of implementation (FOI) on a weekly basis, we conducted live observations and listened to audio-recordings. To quantify FOI, we randomly sampled 20% of the 2,379 recordings, with tutor, condition, and group sampled comparably. Research staff listened to the 476 sampled recordings using an FOI checklist to code the extent to which tutors implemented intervention lessons as intended. Coding agreement was assessed at 97% by an independent coder on 20% (96) of the fidelity checks. When a discrepancy exceeded 3%, a third coder resolved differences.

For intervention components common across conditions, tutors addressed 90.42% (*SD* = 10.63%) of essential points: 90.13% (*SD* = 10.79%) in FM+CA and 90.73% (*SD* = 10.48%) in FM+CA+EA. For *Calculations Quest*, tutors addressed 94.32% (*SD* = 9.52%) of the CA condition's items and 81.79% (*SD* =20.04%) of the CA-EA condition's items. FOI for common components was comparable across conditions, but stronger in the CA than in the CA-EA condition, F (1, 335) = 54.20, p < .001. Problems in CA-EA reflected (a) slippage in early lessons occurred when student verbalizations of the checks were shaky for multiplication and division and (b) once students became more proficient in later lessons, some tutors became inconsistent in requiring them to verbalize the checks.

Fraction Instruction in the Control Condition

To describe the schools' fourth- and fifth-grade fraction instruction, we relied on two sources: an analysis of the fraction components of the district's fourth- and fifth-grade math standards and guidelines and a mathematics instruction survey completed by the 36 teachers who taught math in the 49 participating classrooms.

The district's program. The district advocates for supporting a growth mindset in mathematics to promote self-regulation and achievement; relies on *Go MATH!* (Houghton-Mifflin Harcourt, 2015-2016); and provides a scope and sequence for each content standard, with sample lessons, teaching materials, assessments, and resources.

With respect to *assessing fraction magnitude*, the sample units on comparing fractions include (a) using fraction bars, area models, and number lines to understand equivalence and visualize fraction models; (b) generating equivalent fractions by multiplying the numerator and denominator by the same number; and (c) comparing fractions by generating an equivalent fraction with same numerator or same denominator or by creating a visual model.

With respect to *fraction calculations*, the sample units on fraction addition and subtraction include (a) using understanding of addition and subtraction of whole numbers to add and subtract fractions with like denominators; (b) composing and decomposing fractions and mixed numbers into unit fractional quantities; (c) using concrete and pictorial representations and explanations to show fraction addition and subtraction; (d) rewriting mixed numbers as equivalent fractions to add or subtract with regrouping; and (e) finding equivalent fractions to add or subtract fractions with unlike denominators; and (f) checking the reasonableness of math answers. The sample units on fraction multiplication and division include (a) using visual models (such as rectangular arrays) to multiply a whole number by a fraction or divide a fraction by a whole number and (b) evaluating whether solutions make sense by using understanding of whole numbers to check for the reasonableness of math answers by knowing whether the solution to a fraction multiplication or division problem will be less than, equal to, or greater than each of its factors (or dividend/divisor).

Mathematics instruction survey. Teachers described how they taught fraction magnitude, whole-number multiplication, and fraction calculations. Thirty reported following Common Core State Standards (National Governors Association Center, 2010); six followed a combination of CCSS and other district guidelines. See Table 1 for an overview of how teachers reported teaching *fraction concepts* side by side with the FM intervention's approach.

In terms of *calculations*, 28 (78% of) teachers reported teaching fraction addition and subtraction; 1 of the 28 reported not teaching multiplication and 16 of the 28 reported not teaching division. See Table 2 for an overview of teachers' relative emphasis on calculation problem types. In terms of EA activities, the 28 teachers spent approximately one-fourth of their time promoting conceptual understanding by defining operational symbols (69%), using number lines (86%), and using representational circles or tiles (64%). (Teachers could select more than one option.) Fifteen reported spiral review (reviewing problem types previously taught); 17 required students to explain why answers make sense. These opportunities occurred in approximately half of math lessons.

Major distinctions between instruction in the study's intervention conditions versus control instruction. Based on these sources, we conclude the intervention study conditions spent a larger proportion of time on *fraction magnitude* using number lines, whereas the control group allocated greater emphasis on shaded pictures. Study intervention conditions also focused more on benchmarking fractions and understanding the meaning of numerator and denominator, while the control group placed greater emphasis on procedural methods (e.g., cross multiplication and finding common denominators) and pictorial representations. With respect to whole-number *multiplication*, intervention conditions spent the majority of time practicing skip counting, whereas the control group spent more time on rote memorization.

In terms of *fraction calculations*, 22% of math teachers reported not addressing calculations, despite that fraction calculations are a central part of the fourth- and fifth-grade curricula. Also, whereas both intervention conditions continuously reviewed previously-taught operations with interleaved presentation of practice items, half of teachers did not provide consistent review, and when provided, without interleaved practice. The FM+CA+EA condition had a strong emphasis on conceptual understanding of operations, explaining why answers make sense and identifying common errors. This was less true in the control group and in the FM+CA condition.

Mathematics instruction time for the study's intervention and control students. Math instruction time for the study's intervention versus control group students was comparable. The classroom mathematics block was 60-90 min five days per week, but to receive this study's intervention, half of students in the two intervention conditions missed 120 min per week of classroom math instruction; the other half missed math seat work or instruction unrelated to math. Approximately 16% of FM+CA students received school supplemental math intervention (M = 134.29 min per week, SD = 100.18), 11% of FM+CA+EA students (M = 118.00, SD = 104.92), and 15% of control group students (M = 149.38, SD = 91.20).

Procedure

In August and September, we administered the WRAT screening measures and NAEP items in one whole-class session. In mid-September, students who met the WRAT criterion were individually tested on WASI, and exclusions were applied. Remaining students completed one small-group testing session including Fraction Calculations, Fraction Ordering, and 0-2 Fraction Number Line Assessment. The 13-week intervention began in late October and ended in mid-February. In late February and March, we re-administered NAEP items in one whole-class session, 0-2 Fraction Number Line individually, and Fraction Ordering and Fraction Calculations in one small-group session. Teachers completed instructional surveys in March.

Trained tutors employed by the grant delivered intervention was delivered. Most were master's or doctoral-level students, each responsible for 2 - 5 groups. Each tutor delivered both intervention conditions. The first phase of training involved 20 hours of overview, demonstration, and tutor-paired practice. Tutors practiced until achieving 95% implementation accuracy before initiating tutoring with students. In the second phase, tutors met weekly with research staff to solve the preceding week's problems and train on the upcoming week's content.

Testers were graduate research assistants (RAs) who received training and passed fidelity checks on testing procedures prior to administering tests. Two independent RAs scored and entered data for each test. All scoring discrepancies were discussed and resolved. All testing sessions were audiotaped; 20% of tapes were randomly selected, stratifying by tester, for accuracy checks by an independent scorer. Agreement on test administration accuracy was 98%. Testers were blind to conditions when administering and scoring tests.

Data Analysis

In preliminary analyses, we confirmed that effects were not moderated by grade level (the pattern of effects held at each grade). We conducted other preliminary analyses to evaluate the cross-classified, partially nested data structure, in which nesting occurred at the intervention dyad level for conditions (not the control group) and at the classroom and school levels for all study conditions. To estimate variance due to classroom, dyad, and student, we obtained intraclass correlations (ICCs) by fitting a pair of models to each outcome first for observations nested in school and then for observations nested in a cross-classification of classroom and group, controlling for schools using dummy codes. ICCs (see Table 3) were large enough to justify retaining school, classroom, and dyad as sources of variance in further analyses. However, because there were only 13 schools, we used the strongly preferred fixed effects approach, replacing a level with *k* – 1 dummy codes for cluster membership (McNeish & Stapleton, 2017). At this stage, ICC analyses indicated a Bayes estimator be used, school membership be modeled using fixed effects, and student-level outcomes be modeled as nested in a cross-classification of classroom and dyad.

We next accounted for the partial nesting of the data, in which both intervention conditions have students nested in dyads but control students do not. We used the Roberts and Roberts (2005) method (in Bauer, Sterba, & Hallfors, 2008), in which ICC for dyad was defined for FM+CA and FM+CA+EA but undefined for the control group. We obtained ICC results separately for each of the three conditions, but they shared a common Level 1 residual variance. Then we conducted regression analyses to test the contrasts of interest, using the ICC code as a basis and adding pretest scores as covariates. The contrasts of interest were intervention (combined) versus control and FM+CA versus FM+CA+EA. The final full model equation was:

$$y_{ijk} = \gamma_{00} + \sum_{j=1}^{12} \gamma_{oj}\, d_j + u_{0j} + (\gamma_{10} + u_{1j} + u_{1k})c_{1i} + (\gamma_{20} + u_{2j} + u_{2k})c_{2i} + \gamma_{40} y_{0ijk} + e_{ijk}$$

where y is a generic outcome, y_0 is pretest, c is dummy code for condition (00 = control; 10 = FM+CA; 01 = FM+CA+EA), d is dummy code for school, i denotes individual student, j denotes classroom, and k denotes dyad. For FM+CA+EA versus FM+CA, the difference was $\gamma_{20} - \gamma_{10}$.

For average (combined) intervention versus control, the difference was $[(2\gamma_{00} + \gamma_{10} + \gamma_{20})/2] - \gamma_{00}$.

Results

Table 4 shows pre- and posttest means by study condition. Testing for equivalence revealed no significant differences among conditions on any pretest fraction measure. Results of the Bayes estimation are provided in Table 5, in which credible intervals (CrIs) excluding zero indicate significant effects. (With Bayesian estimation, a 95% CrI has a 95% probability of containing the parameter; this is preferred to *p*-values and frequentist confidence intervals.)

As shown, the intervention (combined) condition produced stronger outcomes than the control group on all outcomes except NAEP. As hypothesized, the intervention conditions performed comparably on Ordering, Number Line, and NAEP; contrary to expectations, the two intervention conditions performed comparably on Calculations. Effect sizes (ESs; Hedges *g*; Hedges & Citkowicz, 2014), using adjusted posttest means and posttest *SD*s, are provided in Table 6.

Discussion

The main purpose of the present study was to assess the efficacy of two of the three components of *Super Solvers* at grades 4-5: the component focused on fraction magnitude (FM) and the component focused on fraction calculations (CA). (For efficacy data on the third Super Solvers component, focused on word problem, see Malone et al., 2019.) The present study's secondary purpose was to examine whether an instructional strategy focused on conceptual and strategic EA of fraction CA problems provides added value for improving at-risk students' fractions CA performance. In this discussion, we refer to the FM+CA components as *Super*

Solvers because it represents the standard form of the *Super Solvers* FM and CA components. We refer to the FM+CA+EA condition as *Super Solvers* + EA.

We begin by discussing the pattern of *Super Solvers* effects on FM outcomes. This question involves the study's main intervention contrast, the combined *Super Solvers* conditions (both of which received the same FM and CA components) versus the control group, on the three FM outcomes. We next consider whether *Super Solvers* improves CA performance, a question also involving study's main contrast, but this time on the CA outcome. Finally, we consider the study's secondary question: whether conceptual and strategic EA provides added value for supporting fraction CA, beyond what occurs for *Super Solvers* (without EA).

Superior FM Outcomes for *Super Solvers* over Control

Across the two variants of the intervention, *Super Solvers* produced stronger outcomes than the control group on each FM measure except the NAEP items. On the ordering task, ESs were large: 1.47 for the combined condition (1.42 and 1.35 for each intervention condition vs. control). Finding that the *Super Solvers* FM component produces stronger ordering performance may seem unsurprising given that intervention students spent more time than control students on this activity. Yet, what probably contributes more strongly to the control group's relatively poor showing on the ordering task is its heavy instructional focus on the procedural cross-multiplying strategy, which not only lacks a conceptual basis but also is procedurally cumbersome for ordering more than two fractions.

The meaningfulness of the intervention conditions' superior performance on the ordering task is bolstered by the combined intervention conditions' convincingly stronger performance, compared to the control group, on the 0-2 number line task. Here, the ES was 1.22 (for each intervention vs. control, 1.27 and 1.31). This large effect is impressive for two reasons. First, although *Super Solvers* placed somewhat greater emphasis on the number line than did control group instruction, the difference was small (20% vs. 14%). Moreover, the computerized number line outcome task requires a purer form of FM estimation than the *Super Solvers* number line instructional benchmarking strategies, which require paper and pencil to executive and are thus not available during the computerized number line task. Second, the robustness of the computerized number line task in predicting advanced mathematics performance has been clearly demonstrated (e.g., Siegler et al., 2011).

So it was disappointing that effects on these two measures reflecting superior FM understanding were not revealed on the NAEP items, where *Super Solvers* effects were negligible to small (combined ES = .05; separate ESs -0.1 for *Super Solvers* and 0.11 for *Super Solvers* + EA). A similar pattern occurred in Malone et al. (2019), which first introduced this revised problem set, with a mean ES of 0.90 across other FM measures but a somewhat larger 0.27 on NAEP. So it is important to note that our revised NAEP problem set assessed multiple interpretations of fractions as well as items focused on proportional reasoning. By contrast, the line of randomized controlled trials focused on the earlier iteration of *Super Solvers* (i.e., *Fractions Face-Off!*) relied on a problem set with easier items and without a focus on proportional reasoning. In that earlier line of studies, effects were consistently significant, with ESs in the moderate to strong range.

It is tempting to think this diminution in NAEP effect may be due to CCRS's deepened focus on FM in control group instruction with CCRS. However, stronger number line performance suggests otherwise. What seems more likely is that positive effects in the more recent studies on the well-accepted number line task but not NAEP are more likely due the revised NAEP problem set. This suggests idiosyncrasy as a function of how NAEP problem sets are constituted and the need for caution when interpreting effects based on released NAEP items. This possibility is bolstered in Malone et al. (2019), where the NAEP ES for the condition with strongest word-problem instruction, which included proportional reasoning problems, was 0.41. Few if any well-designed distal measures of fractions knowledge are available, and development of a measure specifying a well-motivated domain of fraction knowledge appears needed.

Superior CA Outcomes for *Super Solvers* over Control

A novel feature of the present study, beyond testing the refined FM component just discussed and the testing of the EA strategy discussed later, is the testing of the newly added, innovative *Super Solvers* CA component. Results support this innovation, which incorporates three features designed to introduce fraction calculations in an instructionally coordinated manner across the four operations. The first feature is that students learn to start every CA problem by explicitly identifying which of the four operations is required. Second, they learn a problem-solving process that highlights differences and similarities among the operations and their problem subtypes. Third, practice is provided in interleaved format, with problem sets mixing the four operations, which are distributed equally across the operations throughout all CA lessons, even the earliest lessons ones.

This coordinated cross-operation approach runs contrary to methods traditionally used in schools for teaching fraction CA, with which the four fraction calculations are introduced sequentially, with few opportunities to compare and contrast solution procedures across operations and few if any opportunities to perform fraction CA in mixed problem sets. The *Super Solvers* approach to CA similarly violates conventions reflected in mathematics textbooks (Braithwaite, Pyke, & Siegler, 2017) and the manner in which CCRS standards are specified.

The scientific motivation for *Super Solvers'* unconventional approach resides partly in studies showing that traditional fraction CA instruction fails many children (Fuchs et al.,s 2014; Jordan et al., 2013; Lortie-Forgues, Tian, & Siegler, 2015; Newton, Willard, & Teufel, 2014). For example, Siegler and Pyke (2013) and Siegler, Thompson, and Schneider (2011) found low accuracy that across the four operations: 32% - 46% correct at sixth grade and 57% - 60% correct at eighth grade. Moreover, when Braithwaite, Pyke, and Siegler (2017) applied a computational model of fraction CA learning to the problem sets of a population textbook, they demonstrated that the statistical distribution of blocked problem sets conventionally used in schools is counter-productive, predictably mirroring the pattern of errors students commit.

So it is noteworthy that our large ES of 1.77 *SD*s favors our unconventional, coordinated instructional approach over the control group's traditional methods, in which few if any opportunities were provided to contrast operational procedures across operations and did not interleaved practice. Further, 8 of the 36 teachers who taught math reported not addressing CA, despite that fraction CA are a central part of the fourth- and fifth-grade curricula; 1 of the remaining 28 teachers did not teach multiplication and 16 did not teach division. This undoubtedly contributed to the very large ES. Research is therefore needed to contrast two *Super Solvers* conditions within the same study: one that combines the FM component with *Super Solvers* coordinated approach to CA versus one that combines the FM intervention with the more traditional operation-by-operation approach to CA with blocked practice.

Does Conceptual and Strategic Error Analysis Provide Added Value on CA Outcomes?

The present study's second extension to the literature was the question it posed concerning whether conceptual and strategic error provides added value on fraction CA outcomes. For each operation, EA instruction provided conceptual instruction on judging the reasonableness of answers by relying on “go-to” problems, with accompanying number line representations, to remind students about how each fraction operation changes the value of the operands. For each operation, the EA requires students to check whether answers make sense and check that they avoided the most common sources of error.

Yet, we found no significant advantage on CA performance for the FM+CA+EA condition over *Super Solvers* (FM+CA) without EA, with an ES favoring the EA condition of only .05 *SD*s. This may be due to lower fidelity of implementation of the EA strategy than was the case for the other parts of the CA condition: Tutors addressed 94.32% (*SD* = 9.52%) of CA instruction with EA component but 81.79% (*SD* = 20.04%) of the condition with EA. Slippage occurred in early lessons due to shaky student verbalizations of the checks for multiplication and division and in later lessons when some tutors inconsistently required students to verbalize the checks. Nevertheless, the EA strategy, with its stronger conceptual focus on fraction operations, along with a hint of advantage with the NAEP ES of 0.12 favoring FM+CA+EA over FM+CA, suggests the need to test effects of a refined version of EA in future research.

Main Conclusion

Based on this study’s results, our main conclusions conclusion concerns the efficacy of *Super Solvers*. We conclude that its effects of *Super Solvers* are strong. For the *Super Solvers* FM component, the mean ES was 1.12 across FM measures (including NAEP); for the *Super Solvers* CA component, the ES was 1.72 on calculations. For the third *Super Solvers* component, focused on word problems, we rely on the Malone et al. (2019) randomized controlled trial, in which the ES on fraction word problems was 0.66. In that prior study, the mean ES across FM outcomes was 0.90, as compared to 1.12 *SD*s in the present study. We attribute the present study’s somewhat higher mean ES to key refinements in the present study’s *Super Solvers* FM component, including a coordinated approach to the conceptual and strategic processes across the comparing, ordering, and number line placement FM activities.

Thus, *Super Solvers,* when conducted with strong fidelity to its instructional design, improves the fractions performance of fourth and fifth graders who are at-risk for poor fractions learning, as reflected in their low whole-number performance at the start of the year. As such, *Super Solvers* offers schools the capacity to provide at-risk students the chance for improved progress toward competence with one of the foundational skills necessary for success with advanced mathematics content, as well as high-school graduation and post-school occupational success.

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Table 1

Percentage of Time Spent in Control versus Intervention Representing Fractions and Assessing Fraction Magnitude

Topic	Strategy/Tool	Control % *M*	(*SD*)	Intervention %
Representing Fractions	Fraction tiles	14.17	(9.67)	20.00
	Fraction circles	15.00	(11.08)	10.00
	Pictures with shaded regions	32.50	(16.43)	10.00
	Blocks	14.44	(12.52)	0.00
	Number lines	21.39	(9.00)	60.00
	Other	2.78	(8.15)	0.00
Assessing Fraction Magnitude	Number lines	14.44	(13.41)	20.00
	Drawing pictures	16.94	(9.80)	0.00
	Referencing manipulatives	6.11	(9.80)	5.00
	Benchmark fractions	13.33	(6.76)	40.00
	Defining numerator and denominator	13.06	(7.49)	25.00
	Finding common denominator	21.39	(12.23)	15.00
	Cross-multiplying	18.33	(22.10)	0.00
	Other	0.56	(2.32)	0.00

Note: For each topic, teachers allocated 100 points across the various strategies or tools listed on the survey to indicate relative emphasis each had in their instruction.

Table 2

Percentage of Teachers Reporting Relative Emphasis on Fraction Calculations Problem Types

Problem Type	Did Not Teach	Minimal Emphasis	Moderate Emphasis	Strong Emphasis
Addition/subtraction with like denominators	3.7	18.5	18.5	59.3
Addition/subtraction with unlike denominators	14.3	17.9	21.4	46.4
Addition/subtraction with mixed numbers	0.0	3.6	28.6	67.9
Addition with whole numbers	0.0	18.5	29.6	51.9
Subtraction with whole number as subtrahend	3.6	25.0	28.6	42.9
Subtraction with mixed number as subtrahend	10.7	21.4	32.1	35.7
Multiplication[a] with like denominators	28.5	10.7	21.4	39.2
Multiplication[a] with unlike denominators	32.1	10.7	21.4	32.1
Multiplication[a] with whole numbers	10.7	3.6	21.4	64.2
Multiplication[a] with mixed numbers	39.3	14.2	14.2	28.6
Division[b] with like denominators	60.7	17.9	14.3	7.1
Division[b] with unlike denominators	64.3	7.1	14.3	14.3
Division[b] with whole numbers	60.7	3.6	14.3	21.4
Division[b] with mixed numbers	67.9	7.1	14.3	10.7

Note: Percentages reflect the distribution reported by the 28 (of 36) teachers who taught fraction calculations. [a]One teacher did not teach fraction multiplication; [b]16 did not teach fraction division.

Table 3

ICCs for School, Classroom, and Intervention Dyads

Measure	Condition	ICC (School)	ICC2 (Classroom)	ICC2 (Dyad)
0-2 Number Line[1]	FM+CA	0.038	0.117	0.148
	FM+CA+EA	0.041	0.114	0.108
	Control	0.047	0.094	
Ordering	FM+CA	0.048	0.176	0.272
	FM+CA+EA	0.054	0.189	0.193
	Control	0.076	0.128	
NAEP	FM+CA	0.088	0.149	0.137
	FM+CA+EA	0.079	0.198	0.160
	Control	0.113	0.081	
Calculations	FM+CA	0.095	0.222	0.246
	FM+CA+EA	0.124	0.221	0.086
	Control	0.169	0.054	

[1]0-2 Number Line values are multiplied 10 for greater precision.

Note. FM+CA is fraction magnitude plus calculations intervention; FM+CA+EA is fraction magnitude plus calculations intervention with error analysis. 0-2 Number Line is *0-2 Fraction Number Line* task (Hamlett, Schumacher, & Fuchs, 2011, adapted from Siegler et al., 2011). Ordering is *Ordering Fractions* (Malone & Fuchs, 2017). NAEP is released fraction items from the National Assessment of Educational Progress. Calculations is *Fraction Calculations* (Malone & Fuchs, 2017).

Table 4

Pre- and Posttest Means (Ms) and Standard Deviations (SDs) and Adjusted Means (M_{adj}) by Condition

	FM+CA			FM+CA+EA			Control		
	Pretest	Posttest		Pretest	Posttest		Pretest	Posttest	
Measure	*M* (*SD*)	*M* (*SD*)	M_{adj}	*M* (*SD*)	*M* (*SD*)	M_{adj}	*M* (*SD*)	*M* (*SD*)	M_{adj}
0-2 NL	0.57 (0.14)	0.31 (0.13)	0.31	0.56 (0.12)	0.32 (0.13)	0.32	0.54 (0.14)	0.51 (0.16)	0.52
Ordering	2.11 (1.24)	5.91 (3.23)	5.87	1.96 (1.11)	5.98 (3.28)	6.05	2.09 (1.90)	2.47 (2.13)	2.44
NAEP	4.81 (2.59)	6.23 (3.44)	6.11	4.60 (2.91)	6.50 (3.34)	6.52	4.49 (2.36)	6.05 (3.42)	6.14
Calculations	0.95 (1.03)	6.52 (3.23)	6.53	0.93 (1.02)	6.37 (2.89)	6.38	0.98 (1.07)	2.08 (1.22)	2.06

Note. FM+CA is fraction magnitude plus calculations intervention; FM+CA+EA is fraction magnitude plus calculations intervention with error analysis). 0-2 NL is *0-2 Fraction Number Line* task (Hamlett, Schumacher, & Fuchs, 2011, adapted from Siegler et al., 2011). Ordering is *Ordering Fractions* (Malone & Fuchs, 2017). NAEP is released fraction items from the National Assessment of Educational Progress. Calculations is *Fraction Calculations* (Malone & Fuchs, 2017).

Table 5

Results of Bayesian Estimates with Credible Intervals[1]

Measure	Contrast[2]	Mean Difference	95% Credible Interval Lower Limit	95% Credible Interval Upper Limit	Significant	Condition with > value
0-2 NL[3]	FM+CA v FM+CA+EA	-0.175	0.542	0.906		
	Intervention v Control	2.018	2.498	1.493	*	Intervention
Ordering	FM+CA v FM+CA+EA	-0.021	-1.743	1.572		
	Intervention v Control	3.448	2.374	4.416	*	Intervention
NAEP	FM+CA v FM+CA+EA	0.486	-1.222	2.155		
	Intervention v Control	0.264	-0.877	1.395		
Calculations	FM+CA v FM+CA+EA	-0.270	-1.847	1.229		
	Intervention v Control	4.336	3.318	5.281	*	Intervention

[1]With Bayesian estimation, a 95% CrI has a 95% probability of containing the parameter (this is preferred to *p*-values and frequentist confidence intervals). [2]For contrasts, *intervention* refers to combined intervention conditions (FM+CA and FM+CA-EA). [3]Number Line values are multiplied by 10 for greater precision and multiplied by -1 such than higher values indicate stronger performance. *Note.* FM+CA is fraction magnitude plus calculations intervention; FM+CA+EA is fraction magnitude plus calculations intervention with error analysis. 0-2 NL is *0-2 Fraction Number Line* task (Hamlett, Schumacher, & Fuchs, 2011, adapted from Siegler et al., 2011). Ordering is *Ordering Fractions* (Malone & Fuchs, 2017). NAEP is released fraction items from the National Assessment of Educational Progress. Calculations is *Fraction Calculations* (Malone & Fuchs, 2017).

Table 6

Effect Sizes

Measure	INT vs. Control	FM+CA vs. Control	FM+CA+EA vs. Control	FM+CA+EA vs. FM+CA
0-2 Number Line	1.47	1.42	1.35	0.08
Ordering	1.22	1.27	1.31	0.05
NAEP	0.05	-0.01	0.11	0.12
Calculations	1.72	1.88	1.98	-0.05

Note. INT = combined intervention conditions. FM+CA is fraction magnitude plus calculations intervention; FM+CA+EA is fraction magnitude plus calculations intervention with error analysis. 0-2 Number Line is *0-2 Fraction Number Line* task (Hamlett, Schumacher, & Fuchs, 2011, adapted from Siegler et al., 2011). Ordering is *Ordering Fractions* (Malone & Fuchs, 2017). NAEP is released fraction items from the National Assessment of Educational Progress. Calculations is *Fraction Calculations* (Malone & Fuchs, 2017).