

# PREREQUISITES FOR STEM CLASSES USING AN EXAMPLE OF LINEAR ALGEBRA FOR A COURSE IN MACHINE LEARNING: INTERACTIVE ONLINE VS TRADITIONAL CLASSES

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## ABSTRACT

Any advanced class in Science, Technology, Engineering, and Mathematics fields requires prerequisite knowledge. Typically, different students will have different levels of knowledge in these prerequisite areas. A prerequisite (Linear Algebra for Machine Learning course) was implemented as an interactive online course using Jupyter Notebooks and nbgrader and compared with traditional classroom mode. Post-assessment test shows that traditional class provides a better level of understanding. However, a survey shows a preference by students and instructors for interactive implementation compared to traditional class.

## KEYWORDS

Prerequisites, Machine Learning, Linear Algebra, Interactive Self-Study Course, Traditional Course Delivery, Jupyter Notebooks

## 1. INTRODUCTION

One of the characteristics of Engineering, as well as advanced Science and Mathematics, classes is that they are based on a wide spectrum of knowledge, study of which is usually spread among different subjects and fields. While a deep level of knowledge of each subject is highly beneficial, in many cases we lean to “understanding main concepts” level of knowledge as sufficient for the foundational or pre-requisite subjects.

For example, while studying in depth the main mathematical facts; mathematical classes targeting Engineering and Science students have a tendency to skip on methodology of proof, etc.

Similar situation exists for all major vs applied courses. The standard approaches, dealing with addressing such content dependencies, are either creating specialized applied courses coupled with proper rigid class schedule addressing these dependencies or creating highly specialized list of pre-requisite facts in the form of either self-study mini-courses or required material, which is covered in the recitations.

Some of the courses, so called “buzz” courses, have tendency to be taken by students of many specializations and levels. One of such courses in our university is a course in Machine Learning (ML), due to ML applicability in almost any current science and engineering major.

Our previous paper describes and analyzes the Interactive Online Concept Inventory (IOCI) system, see (Grabarnik, Kim-Tyan, & Yaskolko, 2020)

An ML course relies on knowledge of linear algebra, multi-dimensional calculus and probability. Another approach to handling pre-requisites, in addition to the above-mentioned two, is to provide material for student self-study on top of the refresher material and/or crash course material given during the course. The advantage here is that students get at least the minimum amount of the required material, with an option for additional self-learning if desired. We encounter multiple disadvantages, however, with such an approach. For one, time needed for the main subject is spent on prerequisites. Review time for prerequisites should be limited as it is very challenging to cover necessary material at a sufficiently high level. While students have the option to self-study, learning with an instructor is significantly more effective and efficient. Another disadvantage:

neither students nor instructors could verify whether the necessary level of understanding and ability to apply the prerequisite material had been achieved. This may be remedied with quizzes or tests, which in turn require additional precious instruction time.

We implemented the IOCI (Interactive Online Concept Inventory) course using iPython Notebook (Perez & Granger, 2007) software with additional course management support provided by the nbgrader plugin (Jupyter Notebooks, 2020). The course was developed on Amazon's c9 cloud and is available to students online. The course works in an automated or semi-automated way, allowing the instructor to see test results by topic and, if necessary, intervene and comment on student answers.

During COVID time it was difficult to overestimate the timing and usefulness of such IOCI course, however, as we return to normal classes, we pay more attention to the quality of instructions outcomes, and as our initial estimation shows, there is an instruction quality gap in the self-study vs traditional classes.

Due to finally ending COVID restrictions, we thought that it is important to return to this topic and spend more time on comparison of the traditional classes vs IOCI classes. This paper is devoted to describing our system(s) and courses, steps taken to make sure that we compare as close systems as possible, and the result of our initial analysis, that confirmed our previous, very restricted, observations.

This paper proceeds as follows. In section 2, we describe existing CIs and state-of-the-art Interactive Online Systems and work on a comparison of the traditional vs online classes. In subsection 3.1, we proceed to a description of LA (Linear Algebra) as a prerequisite material for the Machine Learning course. We show how CI addresses the requirement of the specific prerequisite material. In subsection 3.2, we describe the cloud system used for the initial implementation of the course as well as hardware requirements for running a test experiment of about 200 software simulated test students. In section 4, we provide a preliminary (proof of concept) evaluation of our approach. We end our paper with a conclusion and discussion of future work.

## **2. INTERACTIVE ONLINE CLASSES: STATE OF THE ART**

### **2.1 Overview**

The purpose of a prerequisite class differs from a “normal” class. It prepares a student for another class, not directly for a future career. Hence, it is often perceived as something less necessary. As observed in (Sato & et al., 2017), (Grabarnik, Kim-Tyan, & Yaskolko, 2018) students often see prerequisites as a waste of time and avoidable. If handled appropriately, a prerequisite course would solve motivational issues. One way to minimize time and resources spent is to make it self-paced so that a student goes through it at a comfortable pace and when time is available.

The first part of the outlined program – teaching only the material needed - is course specific and should be addressed on case-by-case basis.

The second part about level and form of material taught, however, can be answered in general, at least for Science, Technology, Engineering, and Mathematics (STEM) classes.

### **2.2 Notion of Concept Inventory**

While teaching STEM classes, as we observed in most cases, a conceptual understanding and an ability to apply the prerequisite material are sufficient. Students are not expected to know details, such as proofs, etc. The CI is the best existing approach to assessing conceptual understanding rather than memorization of a set of facts. CI, as a form of an assessment, is based on checking if a student understands basic concepts of a given subject as opposed to reciting a number of subject specific facts, equations, etc. As David Hestenes states in his paper, *Force Concept Inventory*, (Hestenes, 1992) CI Assessment is “not a test of intelligence” but rather, “it is a probe of belief systems”.

An immediate advantage of CI is that it can be used for any student. That is, it does not matter what the subject specific background of the student is, since, as stated above, CIs do not test formal knowledge but rather understanding of basic concepts. For example, as was demonstrated in (Epstein, 2013), there is no significant difference observed between the test results even if the class time, class readiness, or type of class are different. That includes even classes that lack traditional lectures, such as Mathematica-based classes.

Typically, CIs are created and delivered as multiple-choice tests. However, as opposed to standard tests CIs are not comparison tests but norm-referenced tests.

The main goal of CIs, as stated above, is to test the students' understanding of basic concepts. However, a typical CI test also checks for typical misconceptions.

The first CI was developed and published by David Hestenes in 1992 (Hestenes, 1992). It is known now as the Force Concept Inventory, or FCI and covers Newtonian Mechanics concepts. It was an immediate success and was recognized and accepted by thousands of educators.

Hestenes coined the term “modeling” to describe the conceptual approach to teaching – as opposed to the traditional factual approach. By now “modeling” approach covers well over 100,000 students each year.

As a result of CI's popularity, the American Modeling Teachers Association (AMTA) was created and grew into a nationwide community. Moreover, CIs began in various fields of engineering, science and mathematics.

CI assessment in introductory and prerequisite classes was studied, in (Grabarnik, Guysinsky, & Yaskolko, 2014), (Grabarnik & Yaskolko, 2013), (Sands, Parker, Hedgeland, Jordan, & Galloway, 2018), (Madsen, McKagan, & Sayre, 2017) (ALEKS), and (Krause, Decker, & Griffin, 2003). With CI the subject specific background of a given student is not significant as stated above because CIs do not test formal knowledge but rather test the student's understanding of related concepts, that is the student's working knowledge.

An understanding of related concepts is exactly what is needed in prerequisite classes: Mastering prerequisite material at a working knowledge level to apply it to the upcoming class.

Another advantage of using CIs: they are already developed for a wide variety of subjects in such areas as Natural Sciences, Engineering, Life Sciences, Mathematics & Statistics.

Therefore, there already exist large depositories of test problems for many subjects in case a need to create a prerequisite class for one of such subjects.

The last aspect – the interactive, self-paced form of the class – can be addressed only using technology.

### 2.3 Existing Interactive Online Systems

Interactive Online Systems are now widely used in both purely online and mixed-mode programs. The most popular ones are ALEKS™ (ALEKS), Cengage WebAssign (WebAssign), Knewton (Knewton), Pearson MyMathLab Study Plan (MyMathLab), Acrobatiq (Acrobatiq), Adapt (Adapt), etc. All these systems offer self-paced automatically graded classes for various subjects. Typically, each such class offers an Initial Assessment and then, based on the output each student gets, activities and learning material to work on with regular re-assessments to check on progress. Such re-assessment outputs in turn are again used to adjust the assigned activities and learning material.

As stated in (Lockee, 2021) the flexibility and learning possibilities that have emerged from necessity are likely to shift the expectations of students and educators, fading more away the line between classroom-based instruction and virtual learning.

The largest summary of online vs. classroom comparison research (Means & et al., 2010) concludes that students in online conditions perform modestly better, on average, than those learning the same material through traditional instruction. Learning outcomes for “students in online learning exceeded those of students in traditional classrooms, with an average effect size of +0.20 favoring online conditions.”

However, “mixed-mode approach had a larger advantage relative to purely face-to-face instruction than did purely online instruction.” The mean effect size in studies comparing mixed mode with traditional instruction was +0.35,  $p < .001$ . The existing systems, however, all emulate traditional classes in terms of curricula and syllabi. The only difference is the form in which the material and assessment are presented.

On one hand it makes the comparison quoted above reliable since there is an objective expected output for each curriculum – and the only difference is the form of presenting the material. Indeed, according to the study itself “analysts examined the characteristics of the studies in the meta-analysis to ascertain whether features of the studies' methodologies could account for obtained effects. Six methodological variables were tested as potential moderators: (a) sample size, (b) type of knowledge tested, (c) strength of study design, (d) unit of assignment to condition, (e) instructor equivalence across conditions, and (f) equivalence of curriculum and instructional approach across conditions. Only equivalence of curriculum and instruction emerged as a significant moderator variable ( $Q = 6.85$ ,  $p < .01$ ).”

On the other hand, simply emulating the existing traditional classes does not allow the online interactive form to use completely its intrinsic advantages. We do believe that prerequisite classes can benefit more from advantages than the online interactive form offers.

Using these three aspects together facilitates the creation of prerequisite classes that cover only the material really needed and taught in a conceptual form, assessed using the CI approach and put in a form of a self-paced interactive online class using Jupyter Notebook, or a similar platform.

## 2.4 Comparison of IOCI and Traditional Courses

The key aspects of IOCI classes are use of CI, asynchronous access and self-paced learning. While traditional classes can use, and many do use CI, two other aspects are of inherently online nature. Therefore, these two are mostly responsible for the differences in performance between the two learning modes.

The number of online classes has grown fast in recent years and will continue growing, and that creates multiple access issues, both technical and on a personal level (Lockee, 2021). Asynchronous self-paced classes obviously greatly alleviate the access issues and therefore make IOCI classes useful and valid solutions for post-COVID era.

There is a lot of research of online classes vs traditional classes based on learner characteristics and engagement, while influence of course design and development was examined to lesser degree (Martin, Sun, & Westine). That makes it important to compare learner perspective and actual performance in a close conjunction with the specific classes' designs.

E-learning is effective in increasing knowledge and is highly accepted. However, it is important not to focus only on increasing knowledge, but also on a field specific and social skills. E-learning should not only be based on the delivery of content, but students should be able to work with the materials and receive feedback. Successfully implementing online learning into the curriculum requires a well-thought-out strategy and a more active approach (Bączek & et al.). IOCI classes do offer both the field specific material to learn and constant proactive feedback, allowing to make the learning process more effective emulating traditional classes in that aspect, while keeping all online learning advantages listed above.

## 3. IOCI LINEAR ALGEBRA (LA) VS TRADITIONAL LA AS PREREQUISITE COURSES FOR ML

### 3.1 Required LA and IOCI content

The LA prerequisite class for Machine Learning class is an interactive online self-paced class built on the Jupyter Notebook platform.

The lectures are based on “Linear Algebra Review and Reference” by Zico Kolter and consist of four chapters:

1. Basic Concepts and Notation
  - 1.1. Matrix Multiplication
  - 1.2. Operations and Properties
  - 1.3. Matrix Calculus

The material presents basic definitions and concepts of LA necessary for studying Machine Learning.

Each chapter is divided into smaller sections. For example, the “2 Matrix Multiplication” chapter is divided as follows:

- 2.1 Vector-Vector Products
- 2.2 Matrix-Vector Products
- 2.3 Matrix-Matrix Products

Each section is supplemented by an auto-graded assessment based on CI principles.

A typical problem for Basic Concepts would be:

Find the dimensions of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

1.  $2 \times 3$  (\*)
2.  $3 \times 2$
3.  $1 \times 6$
4.  $6 \times 1$

Option A is a key since the matrix has two rows and three columns.

Option B is a distractor that checks for a misconception that mixes rows with columns.

Option C is a distractor that checks for a misconception that considers a matrix as one long row with six elements.

Option D is a distractor that checks for a misconception that considers a matrix as one long column with six elements.

Another typical example would be

Matrix

$$\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

has eigenvalues:

A. -1 and 0

B. -1 and 2 (\*)

C. 0 and 2

D. It has no eigenvalues

Option B is a key since  $(-1-x)(2-x)-0*0=0$  has two roots, -1 and 2.

Option A is a distractor that checks for a misconception of the eigenvalues being the values of first row elements.

Option C is a distractor that checks for a misconception of the eigenvalues being the values of second row elements.

Option D is a distractor that checks for a misconception of considering a characteristics polynomial being  $-1*2-(0-x)(0-x)$ .

IOCI assessments are based on a sufficiently large pool of problems and are randomly generated for each student and for each attempt. IOCI course provides final assessment after completion of the course material.

A student is able to take this class any time before taking the Machine Learning class, at a pace that fits her or his schedule and degree of prior knowledge. In addition to the lectures, we include the option of having students ask the instructor questions or discussing any aspect of the class with other classmates. Each assessment is auto graded but also can be graded by the instructor in case a student challenges the grade.

Traditional LA class covers material of the regular LA class, which, in turn covers all required pre-requisites and content of the IOCI course.

Pre-ML course evaluation for the students taking ML course after traditional LA course is taken from the final evaluation for the IOCI LA course. Comparison results of the Pre-ML evaluation and IOCI final are given in section IV. It shows a small but statistically significant advantage of the traditional course.

## 3.2 Organization of Classes and System Implementation

The system supporting IOCI was implemented on AWS Cloud 9 virtual machines with 20 Gb. hard-drive and 2 Gb RAM running Ubuntu v. 14, with Python 3.6, miniconda and installation of JupyterHub with nbgrader.

The system had some performance issues. To deal with performance the system was moved to a Lenovo P-520C workstation with Intel Xeon 6 core W-2133 Processor with vPro, 32 Gb. of RAM with dual hard-drive 512 Gb SSD and 2 Tb. HDD and 2 GB Nvidia P2000. This PC configuration proved to be sufficient to run up to 200 test students. We run IOCI on that workstation now.

The IOCI system was heavily used during COVID years 2020-21. Starting with Fall of 2021, we are in a process of returning to normal schedule and running regular pre-requisite classes. It is interesting to compare students' performance and satisfaction from regular classes and IOCI system. For that we had classes taught by the same faculty, using a similar style and methodology that were used in IOCI. Students were offered pre-course tests similar to the IOCI tests.

#### 4. EVALUATION OF THE APPROACH

We evaluated traditional classroom and IOCI approaches by running two classes in parallel. for about 20 graduate students each taking the Machine Learning course. Half of the students studied the LA prerequisite material in the form of traditional class. Another half used the IOCI class we created. For both groups regular CI based assessments were used to compare the objective output. These assessments included pre- and post- preparation CI-based tests that check the quality of the required comprehension of the LA material. We also offered one-question survey for both instructors and students. The survey seeks to discover if the student/instructor prefers traditional classroom form or an IOCI form. An outline of the measurements approach may be found in (Means & et al., 2010), (Sands, Parker, Hedgeland, Jordan, & Galloway, 2018), (Evans, Howson, & Forsythe), (Gossman & Powell, 2019).

Both classes offered a sample that shows prerequisite materials used by their counterparts. Both the tests and the survey showed a statistically significant preference by the students of IOCI class over traditional class with 5% significance level.

Test results analysis is summarized in Table 1 below and uses standard t – test with a different standard deviation for testing if one of the means is larger than the other. The value of the test t shows statistical significance with a confidence level of  $\alpha = 5\%$ . Here the value  $df$  is degree of freedom,  $d$  is value of statistics,  $t$  is value of t-test corresponding values  $d$  and  $df$ .

Survey preference is analyzed in Table 2 using small samples t-test for population proportion (see D'Agostino et. Al., 1988, Upton 1982). A summary of analysis is offered below in Table 2. Here, the value of  $N-2$  is the degree of freedom, the value  $d$  is calculated (see D'Agostino et. Al., 1988, Upton 1982) as

$$d = (ae - bc) \left( \frac{N-2}{N(nac+mbe)} \right)^{\frac{1}{2}} \quad (1)$$

and values of the variables  $a, e, b, c, N, n, m$  used in the formula are corresponding ones in the numerical data below.

Table 1. One sided two means T-test for grades IOCI vs Traditional

	IOCI	Traditional
N	20	19
mean	82	88
std	6.3	6.6
df (degree of freedom)		29.0407
d (see formula (1))		1.738422
t		0.046237

Figure 1. One sided two means T-test for grades IOCI vs Traditional (for data see Table 1)

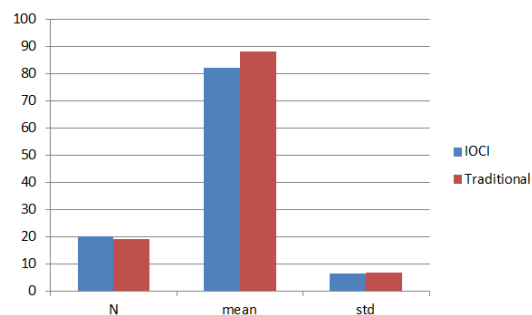


Table 2. Small samples T- test for population proportion comparison

	IOCI Users	Traditional Users	Total
Prefer IOCI	a = 17	b = 10	s = 27
Prefer Traditional	c = 3	e = 9	f = 12
Total	m = 20	n = 19	N = 39

N-2	37
d	2.196271215
t	0.018612316

Table 1 confirms our intuition that the average tests results for traditional classes is greater than the average tests results for IOCI classes – by about 6%. The difference is significant with 5% confidence level.

Table 2 shows that benefits and convenience of the IOCI classes are still preferred by users by vast majority of IOCI classes' users and by marginal majority of Traditional classes' users. This result is also statistically significant with 5% confidence level.

## 5. CONCLUSION

In our previous paper on the topic of prerequisites for the STEM courses, in particular, courses in Machine Learning, we analyzed option of providing prerequisites in the form of suggested reading only or, as an alternative, interactive online concept inventory (IOCI) form. Due to the new, COVID, reality we encountered the situation when regular scheduling of the courses was disturbed, and we had to rely only on IOCI for prerequisites. We observed that the general level of understanding as well as pre-course grades were lower than usual. We compared pre-COVID, normal scheduling pre-course assessment and COVID time assessment involving IOCI approach. Our limited in size study showed that while students show better satisfaction with IOCI approach, assessment shows statistically significantly better results for regularly scheduled courses.

We plan to run the LA prerequisite course with larger numbers of instructors and students and incorporate comments and suggestions from all participants. In a future we intend to offer the IOCI course as open source available to anyone.

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