Identifying and Evaluating Upper Primary School Students' Mental Computation Strategies

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This conceptual paper discusses two frameworks, developed independently by the lead author, that will provide the conceptual foundation for the identification and evaluation of mental computation strategies students demonstrate during an upcoming research project entitled Mental Computation in Year 5. These frameworks will be used by the lead author during an intervention to investigate the application of mental computation strategies in problem solving tasks involving duration of time. It is an intended outcome of the project that the two frameworks will be useful for teachers and students in upper primary school to provide feedback regarding the teaching and learning of mental computation.

The research project, *Mental Computation in Year 5*, is a qualitative research study designed to investigate mental computation strategies used by Year 5 students when engaged in additive and multiplicative calculation tasks. The aim of this research is to contribute to educators' understanding of the mental computation strategies used by upper primary students. This research is important as mental computation is recognised in curriculum documents as an important component of Numeracy.

The research project involves documentation and analysis of the mental computation strategies used by Year 5 students in one school. The lead author will identify mental computation strategies by interviewing students individually to determine different types of strategies that are demonstrated by the students, the range of strategy types used by each student, and the ability of the students to be flexible in their use of mental computation. During these interviews, students will complete additive and multiplicative tasks using mental computation. The students will be invited to articulate their mental computation process by providing verbal reasoning, allowing the lead author to document the strategies that are being used.

To support the research, two frameworks have been designed by the lead author: *Mental Computation—Strategy Type Framework* (MC-STF) and *Mental Computation—Efficiency and Flexibility Framework* (MC-EFF). The MC-STF is designed to identify and name the strategies that students use and provides the foundation for coding student strategies identified in the research. The MC-EFF is designed to determine the effectiveness of the strategies identified, and to provide a simple mechanism, which supports provision of effective feedback to students. Both frameworks will be trialled by the lead author during an intervention where participants will develop their ability to use mental computation strategies to solve additive and multiplicative tasks and then use these strategies to solve problems involving duration of time.

In addition to the frameworks being used in the research project, we argue that both frameworks are useful for classroom teachers and will enhance the teaching and learning of mental computation in upper primary school and in the provision of feedback to students regarding their mental computation strategies. Hattie and Timperley (2007) indicate the importance of providing feedback to students in ways that help them identify their current knowledge (in this case their mental computation strategies) and then provide them with some concrete steps as to how their knowledge can be further developed. Therefore, the MC-EFF is an important pedagogical tool.

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In this paper we will establish a definition of mental computation, including strategies, using evidence from the literature. Next, we will discuss the two frameworks created for the research project and indicate how the frameworks provide a conceptually robust way to identify and evaluate the strategies used by the participants in the study. We conclude by identifying several pedagogical implications regarding the use of the two frameworks by classroom teachers to support the teaching and learning of mental computation.

Defining Mental Computation

A significant advantage of mental computation is the development of conceptual understanding of number, as research has shown that students who engage in using flexible mental computation strategies develop greater number sense (May, 2020; Vincent, 2013). In addition, calculation is a mathematical skill used by adults in their daily life, 86% of which are done using mental strategies (Northcote & Marshall, 2016). Therefore, we argue that the teaching and learning of mental computation is an important part of the mathematics curriculum. Mental computation, or mental arithmetic as it is also commonly referred to, has been defined as the computation of numbers in the head (Heirdsfield, 2002; Maclellan, 2001), without the use of external aids such as calculators or pen and paper (Maclellan, 2001). Lemonidis (2016) claims that mental computation can also include some recording of symbolisation to assist with memory; however, for the purposes of this research, our definition of mental computation does not include any use of external aids. Establishing a clear definition of mental computation from the literature is difficult due to differences in the qualities that define the concept itself and the terminology used by various researchers in this domain (Ruiz & Balbi, 2019). In addition, different terms are used to refer to the same ideas or concepts and multiple interpretations are often given for the same term (Lemonidis, 2016). In this paper the following definition of mental computation is used—*Mental computation is the computation of numbers in the head using flexible strategies*.

To further complicate matters, researchers have also sought to identify the parameters of what constitutes or does not constitute mental computation. For example, Russo (2015) suggests that mental computation is not the recall of basic facts, that is, calculations involving single-digit numbers, which students learn to recall over time. Although, the quick-fire recall of these facts may assist with the mental computation process (Maclellan, 2001), we agree with Russo and do not consider the knowledge of simple basic facts as indicating an ability to complete mental computation tasks. Likewise, mental computation also does not include standard written algorithms, which Maclellan (2001) describes as being an example of an inflexible strategy given that they follow a standardised form in which numbers are treated as single digits, without any identification of their place value, and are then acted upon uniformly. Indeed, in our view, the use of written algorithms can be detrimental to the development of a range of flexible mental computation strategies.

Most current curriculums require students to use flexible mental computation strategies as well as encouraging students to invent and use their own strategies. For example, in the Australian context, The General Capabilities of the Australian Curriculum Version 9 describes a range of flexible strategies that students are expected to use (ACARA, 2023). Given the complexities of current classrooms, where students exhibit a wide range of mathematical abilities, and given the encouragement for students to develop and use flexible strategies, it can often be difficult for teachers to easily identify the steps that individual students are following when completing mental computation tasks.

Mental Computation Strategies

As indicated earlier, mental computation, however defined, clearly involves the use of strategies (Lemonidis, 2016). However, due to the diversity in strategy names and processes, developing a similarly clear understanding of the range of strategies is difficult (Ruiz & Balbi, 2019).

Nevertheless, there are three broad categories of strategies evident in the literature: *Jump*, *Split*, and *Compensate* (Heinze et al., 2018; Lemonidis, 2016), which we will initially use to identify and describe students' mental computation strategies. General descriptions of each strategy, as presented in the literature, are provided below. These descriptions provide the basis for an initial identification and classification of the wide range of strategies that will likely be identified in the research project—*Mental Computation in Year 5*. The frameworks are an initial suggestion, based on the literature, as to the likely strategies that students will use in solving mental computation tasks. Should alternative strategies be identified during the project, the lead author will adapt and refine the frameworks. All three strategies apply to addition, subtraction, multiplication, and division, for the purposes of this research paper, we will only provide one example of each operation.

Jump Strategy

The term *Jump* describes the strategy where the student jumps from one number to another. This strategy closely resembles a counting strategy. Over the last three decades this strategy has variously been labelled using the terms: sequential counting (Beishuizen et al., 1997); aggregation (Clark, 2008); bit by bit (Money, 2010); and stepwise (Csikos, 2016). Essentially, in each of the descriptions provided, the strategy involves starting the calculation process at one of the given numbers and jumping to the next number. Lemonidis (2016) provides examples of this strategy for multiplication and division. The expression 15 * 5 is solved by repeatedly adding (or jumping) 15, five times: 15, 30, 45, 60, 75. Repeated addition may also be used to solve the expression 75 ÷ 5 by repeatedly adding 15: 15, 30, 45, 60 then 75.

Split Strategy

The term *Split* is used to describe the strategy where numbers are partitioned, or split, to make calculations more manageable. For the purposes of this research, splitting a number constitutes a change to that number. This strategy has been variously labelled using the terms: decomposition (Beishuizen et al., 1997; Torbeyns & Verschaffel, 2016); separation (Clark, 2008); break up numbers (Hartnett, 2008); place value right to left or place value left to right (Money, 2010); number splitting (Russo, 2015); and split 10s (Chesney, 2013). Numbers can be split using either standard or non-standard partitioning. Torbeyns and Verschaffel (2016) describe this strategy in relation to subtraction. The example they provide is 457 - 298 where both the minuend and the subtrahend are split according to place value (standard partitioning). The process follows 400 - 00 = 200; 50 - 90 = -40; 7 - 8 = -1. Therefore 200 - 40 - 1 is 159.

Compensate Strategy

The final strategy, *Compensate*, again involves the manipulation of numbers to make the calculation more manageable. The strategy involves compensating for that manipulation or change to accurately complete the calculation. The compensate strategy has been variously labelled using the terms: varying strategies (Torbeyns & Verschaffel, 2016); holistic (Lemonidis, 2016); adjust and compensate (Hartnett, 2008); and round then compensate (Money, 2010). Heinze et al. (2018) demonstrates compensate for addition: 527 + 398. This strategy involves converting the number that is close to a multiple of 10 into a multiple of 10. In this instance, two is added to 398, treating it as 400. The new equation 527 + 400 is easy to do mentally, with a result of 927. The number then must be adjusted to compensate for the original change, so two is taken away from 927. Therefore 527 + 398 (+2) = 927 (-2) = 925.

Mental Computation—Strategy Type Framework (MC-STF)

Due to the mathematical nature of the strategies, the effectiveness of the strategy differs according to the calculation task (Heinze et al., 2018). For example, the *Jump* strategy is the most efficient strategy when calculating amounts that bridge 10. *Compensate* is usually the most efficient strategy to use when calculating with a number near a multiple of 10. Torbeyns and Verschaffel, (2016) state that 963 - 499 is most efficiently calculated using the *Compensate* strategy because 499 is near 500.

As indicated earlier, many current curricular goals promote the development of students' ability to evaluate the characteristics of the task and determine which strategy will most effectively solve the equation (Torbeyns & Verschaffel, 2016). Students need to be able to compare different strategies and identify the one that would be most appropriate to use (Graven & Venkat, 2019). To achieve this, students need a common language to use. The MC-STF aims to provide teachers and students with a common language to use when discussing strategies used.

Although *Jump*, *Split*, and *Compensate* are three strategies that we expect to see used heavily by students in the research, as explained above, the identification and naming of strategies is diverse (Ruiz & Balbi, 2019) and it is expected that more than just these three strategies will be used by students. Therefore, the lead author has developed an initial conceptual framework, based on the Jump, Split and Compensate strategies, that will be an initial starting point for classifying students' strategies in a more fine-grained way. This approach considers that students will invent their own strategies, and that these invented strategies will likely involve a blending of the three strategies named above. The MC-STF will classify strategies, which use a combination of *Jump*, *Split* or *Compensate*, by linking the names. For example, a strategy that uses both *Jump* and *Split* would be named Jump-split.

For initial coding purposes, any blended strategies will also provide an indication of the major component of the strategy i.e., *Jump* or *Split* or *Compensate*. In any instance of a blended strategy, the major component of the strategy will be classified using an upper-case letter and the minor component (or components) will be classified using a lower-case letter(s). By way of example, the equation 38 + 17 may be done by partitioning 17 by place value (10 and 7), then starting the calculation at 38 and jumping 10, making 48, then jumping 7, making 55. This would be identified as Js. By recording the differences in strategies used by Year 5 students, the research aims to identify the range of strategies used, the effectiveness of these strategies and any common misconceptions that are evident.

Mental Computation—Efficiency and Flexibility Framework (MC-EFF)

As we explored earlier, computational fluency is an essential skill in mathematics (NCTM, 2000) and is one of the proficiencies in the Australian Curriculum. ACARA (2023) states that fluency involves students carrying out procedures flexibly, accurately, efficiently, and appropriately; and that students are fluent when they "choose and use computational strategies efficiently" (ACARA, 2023, F-10 Curriculum Version 9: Mathematics). The definition of computational fluency used in this research aligns with the definition in the Australian Curriculum. Computational fluency describes the use of efficient strategies; and applying these strategies flexibly and with accuracy (Dole et al., 2018). Another component of computational fluency is students' ability to choose the most appropriate strategy. We now briefly define how flexibility, efficiency, accuracy, and appropriateness are defined in the literature.

Flexibility refers to the skill of using number sense knowledge and recall of basic facts to manipulate numbers to complete a calculation. Students require a rich understanding of number sense to be able to calculate flexibly (Graven & Venkat, 2019). This is seen in contrast to the use of inflexible strategies (i.e., algorithms), where students are merely following a set sequence of steps to do the calculation (Heirdsfield, 2002), with no requirement for flexible thinking. An efficient strategy is one that can be carried out easily by a student. This will be evident during the process as the student should manage the tracking of sub-problems (Russell, 2000), manage the cognitive load on their working memory, and navigate the changes they make to the numbers. Efficiency considers the number of performed solution steps and the mental effort needed to perform the solution steps (Heinze et al., 2018).

When developing computational fluency, significant emphasis is placed on accuracy. Most assessments of computation fluency are designed to only measure speed and accuracy (Hopkins et al., 2019), with little emphasis placed on the identification of strategies used. For the purposes of this research, the provision of accurate answers is critical to the results, and only the strategies that generated accurate calculations will be documented. The time taken to complete a calculation (i.e., speed), will not be measured in this project, but may be a consideration for future research. Finally, students need to be able to make appropriate decisions regarding the choice of strategies they use. They need to be able to recognize a variety of strategies to solve a computation (Dole et al., 2018), compare these strategies (Graven & Venkat, 2019), and then choose the most appropriate strategy (Heinze et al., 2018).

In this paper we suggest that the MC-EFF framework (See Fig 1) is a tool that researchers and teachers can use to evaluate the efficiency and flexibility of different strategies, and then provide feedback to students on the overall effectiveness of the strategy they are using. One of the aims of the project is to encourage teachers to teach, and students to learn, strategies that fall in the high flexibility-high efficiency quadrant. The MC-EFF will be used to measure the students' ability to choose the most appropriate strategy. In the research project, each strategy identified will also be coded according to their flexibility and efficiency and this will help determine the overall effectiveness of a strategy.

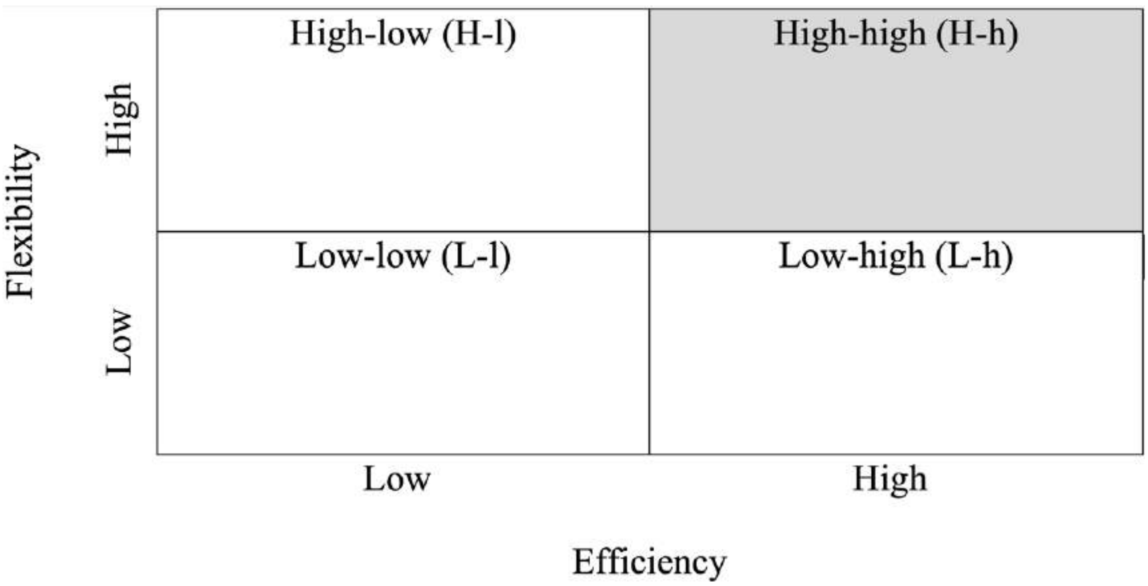

Figure 1. Mental computation—efficiency and flexibility framework. (MC-EFF) (Author 1)

For the purposes of this study, we have developed the following definitions of flexibility and efficiency. Flexibility involves modifying numbers, for example, standard partitioning, non-standard partitioning, or changing a number to make it more manageable. The number and types of changes used within the strategy determine its flexibility. If a strategy involves multiple changes to the numbers, and the use of different types of modifications, it will fall in the high flexibility category. Strategies using a low number of changes will fall in the low flexibility category. Efficiency involves the number of steps required to execute a strategy, for example, using a known fact to determine the calculation of an unknown equation, using a friendly number to assist with the calculation, and the amount of short-term memory required to complete the operation. Working memory is limited (Ding et al., 2021), so strategies that require less working memory are paramount. In this criterion, the minimum number of steps required, the use of known facts or friendly numbers, and having less short-term memory load, the higher the efficiency of the strategy. The number of steps and the types of modifications differs according to the complexity of the equation. For example, the steps used to efficiently solve 31 + 25 and 267 + 328 would differ.

Using the expression 28 + 13, we provide examples of the MC-EFF coding in use. These examples represent the possible mental strategy steps that a student may articulate when doing mental computation. Using one approach, determining the answer could follow these steps:

- 20 + 10 is 30 (both addends use the *Split* strategy)
- 8 + 3 is 11 (both addends use the *Split* strategy)
- 30 + 11 is 41

This approach to solving the expression would be identified as High-low (H-l)—demonstrating high flexibility, due to the multiple changes to the numbers, but low efficiency because there were 3 steps.

However, the same expression may be solved using the following approach:

- Move 2 from 13 to 28 to make 30 + 11 (2 numbers changed using the *Compensate* strategy)
- 30 + 11 is 41

This strategy would be identified as High-high (H-h)—demonstrating high flexibility because two numbers were changed and high efficiency as there were only two steps involved and thus putting less load on working memory.

Another approach could include the following steps:

- 13 - 2 = 11
- 28 + 2 = 30 (*Compensate* for the 2 removed from 13)
- 30 + 11 is 41

This strategy, like the first one, would be identified as High-low (H-l) demonstrating high flexibility due to multiple changes and low efficiency because of the higher number of steps.

Implications for Pedagogy

Reasoning is the action of thinking about mental computation in a logical way. When students reason they build new knowledge as they create and validate their mathematical ideas (Herbert, 2019). However, teachers find it difficult to identify when students are reasoning in a mathematics lesson (Jazby & Widjaja, 2019) and these authors recommended that teachers plan carefully for reasoning, which includes designing appropriate tasks to increase the chance of students being able to reason. The MC-STF and MC-EFF frameworks, created by the lead author, have been designed to achieve this by assisting teachers and students in developing the knowledge and vocabulary needed to explain, analyse, and evaluate the strategies they use, identify the gaps or weaknesses in these strategies, identify more efficient ways of completing the computation (if possible), and then to justify why the strategy selection reflects the most efficient way to complete the mental computation. These are some of the skills identified by the Australian Curriculum when students are reasoning (ACARA, 2023).

Feedback is the information provided by a person or experience regarding someone's performance (Hattie & Timperley, 2007). It is proposed that both frameworks, but particularly the MC-EFF, can enhance the quality of feedback that teachers can provide their students as they complete mental computation tasks and as they make decisions about which strategies to use in future tasks. The structure of the MC-EFF provides teachers and students with the information they require to improve their strategy selection choice by increasing awareness of the flexibility and efficiency of their current mental computation strategies.

Conclusion

The MC-STF and the MC-EFF frameworks, designed by the lead author, and explained in this paper, will initially be used to code the observations in the research project—*Mental Computation in Year 5*. The framework MC-STF will be used to name the strategies identified as students complete a variety of additive and multiplicative tasks and discuss their reasoning with the researcher. The MC-EFF will be used to identify the effectiveness of each strategy. At the completion of the project, both these frameworks will be available to assist teachers in identifying the mental computation strategies used by students and then assist them in the provision of feedback, that supports students identifying the strategies they are using and to determine their effectiveness in terms of flexibility and efficiency. This will add to our understanding of how best to support the development of fluency in mental computation.

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