A Tri-Nation Comparative Study of Place Value in Early Years Curricula Documents

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In this paper we compare the early years mathematics curricula of Germany, South Africa, and Australia in relation to the place value concept. Place value is an important topic as it underpins much of the number work completed by learners in the early years of schooling. We found that there were differences between the three curricula that could be summarised using five themes: namely, number range, place value structure, role of the zero, influence of language, and use of materials. We argue that how the different curricula deal with these five themes influences the quality of learning provided and we highlight key areas of concern. In concluding we identify three important implications for our future research project.

Introduction

The genesis of this article occurred in conversations that the authors held preparing a grant application, which aims to support the development of place value (PV) content knowledge (CK) and pedagogical content knowledge (PCK) for in- and pre-service teachers in Germany, South Africa, and Australia. It became immediately apparent that our respective national curricula differed greatly in the way PV is represented and in the way each national curricula is conceptualised. Consequently, it was necessary to commence our project with a comparative study of the three curricula. In this paper we provide a brief theoretical background in relation to teaching PV and outline our methodology in analysing the curricula. We then provide a brief overview of each curricula and identify five themes that differ between the respective curricula. We conclude by highlighting areas of concern within each curricula and with an outline of important areas for further investigation within our ongoing research project.

Theoretical Background

PV involves the learning of several “big ideas” (Van de Walle, 2015, p. 247) and fundamental to these is the integration of early experiences of *counting* and *grouping* amounts in tens (and in tens of tens, and in tens of tens of tens, and so on). Building upon this knowledge is the activity of *bundling*, that is recognising a group of ten as a new object with a new name (e.g., one *ten* instead of a group of ten ones). As there are different ways to partition an amount into parts; that is bundles of different sizes, the acquisition of the *part-whole concept* is also necessary. Based upon the foundational knowledge of counting, grouping, bundling, and the part-whole concept is the concept of *positional notation*, which comprises the sub-concepts of *place value*, (the value of the bundle unit), the value of *digits*, (based both on the symbol and on its position), and finally the *number value*, (the application of the part-whole concept to each of the digits in a number).

Rogers (2012) notes that “despite the unchanging and recursive nature of our base-ten system, it seems some students never manage to fully unravel the hidden code that underlies place value”

(2023). In B. Reid-O’Connor, E. Prieto-Rodriguez, K. Holmes, & A. Hughes (Eds.), *Weaving mathematics education research from all perspectives. Proceedings of the 45th annual conference of the Mathematics Education Research Group of Australasia* (pp. 533–540). Newcastle: MERGA.

(p. 648). Kortenkamp and Ladel (2013) reinforce this view, identifying that PV is a difficult topic to teach effectively. These findings are evident in the South African context, with research by Graven and Venkat (2021) indicating that poor mathematics performance was in part due to learners' "lack of understanding of PV and the base 10 number system" (p.24). Learners is the South African term used to refer to students or to children.

Given the difficulty for many learners in understanding PV, it is concerning that several researchers have indicated that understanding of PV is also problematic for some teachers. Research by Hopkins and Cady (2007) indicated teachers' difficulty in using different bases, the use of expanded notation, making conversions, and pictorial representations. The problem of a lack of CK and PCK of PV is often evident with pre-service teachers. Thanheiser and Melhuish (2019) indicate that many pre-service teachers come into mathematics content courses with knowledge of how to implement arithmetic procedures but without understanding the conceptual PV knowledge underlying them.

The availability of new digital tools in supporting learners in understanding PV is a more recent development. Cognisant of the need to utilise appropriate digital materials (see Larkin, 2016), we argue that the use of quality digital materials is beneficial in supporting the development of learners' conceptual understanding of PV. At the core of our argument is the knowledge that digital manipulatives provide learners with opportunities to interweave pictorial and symbolic representations, with the actions that they perform on them, to emphasise the underlying mathematical concepts by, for example, linking multiple external representations (Ladel, 2009).

Based on the review of the literature above, and on our analysis of the curricula from the three countries, the following research question guided our analysis.

- How do the analysed curricula differ in terms of what PV content is to be taught and how this content progressively builds?

Answering this question should enable the authors to identify implications for research and practice in the future.

Method

We analysed the Australian Curriculum (Mathematics F-10 Version 9.0) (ACARA, 2022), the South African Curriculum and Assessment Policy Statement: Foundation Phase (Grades 1-3) Mathematics (SA.DBE, 2011) and the Bildungsstandards im Fach Mathematik für den Primarbereich. Beschluss der Kultusministerkonferenz vom 15.10.2004, i.d.F. vom 23.06.2022 (KMK, 2022). As Germany has different curriculum documents for each of the 16 federal states, we chose to analyse the two that are relevant for the in- and pre-service teachers we work with, namely, the Bildungsplan der Grundschule. Mathematik (MKJS BW, 2016) of Baden-Württemberg and the Rahmenlehrplan Teil C Mathematik (MBJS BB, 2015) of Brandenburg. Prior to the analysis, the sections pertaining to place value of the German curriculum were analysed by the German members of our team and translated into English by one of the German-speaking authors for further analysis. The Australian and South African curricula are published in English.

In the first step of analysis, each separate curriculum was analysed by a team member and relevant text was mapped to acknowledged key PV concepts (See Ladel et al., 2023 in work), including pre-concepts (part-whole, counting, and grouping) and PV sub-concepts (bundling [Base Ten], the decimal part-whole concept, and positional notation) as these are critical for flexible PV understanding. In the second step, a different team member (from a different country) also reviewed each curriculum and made comments for consideration. In the third step we discussed the comments as an entire research team and collated our findings in an Excel spreadsheet (See Figure 1 for an excerpt).

		p. 215	p.216	p. 217
grade/term			Grade 2 Term 1	
number range			up to 25	
Decimal Part-Whole	Standard partitioning	Place Value T4 - Recognise the place value of numbers 11 to 19. (decompose two-digit numbers into ten and ones e.g. 18 is 10 and 8) "In Term 1, learners work with a higher number range and continue to: - count and group to make a group of tens and loose ones; and write 18 = 1 ten and 8 loose ones 13 = 10 and 3." "Ten is 1 ten that contains 10 ones. Regular ten and one words (24 is 2 groups of 10 and 4 ones or 2 tens and 4 ones) need to be used regularly to establish a language that symbolises decomposing and composing." Building up and breaking down This is one of the most important techniques in the Foundation Phase (learners will also use decomposing frequently in the Intermediate Phase). Using this technique allows learners to split (decompose) and recombine numbers to help make calculations easier. during this term learners will: • break up numbers using place value; • break up numbers using multiples of 10; and • break up into number pairs e.g. pairs that make 20. (p. 227)	"Learners should continue to manipulate concrete apparatus by grouping to form ten ones and understanding that 10 is one group of ten loose ones" "Base tens blocks (part of the Dienes blocks) can be introduced to develop the idea of a ten as a single entity and that: 10 ones make 1 ten; 20 ones make 2 tens; and 16 ones make 1 ten and 6 loose ones"	"Learners should be engaged in written activites that build and consolidate: - the concept of groups of ten and loose ones/units; and - the value of a digit." **"During this term learners will break up numbers using place value" (p. 226)**
	Non Standard partitioning strong			"Equivalent representations During this term the focus is showing equivalent representations for the same number. **Twenty should be described as 2 tens (using the bundles or groups of objects) or 2 groups of tens (Kev - what is difference?).** It is important to show learners that 20 can look different. So show 20 loose objects, one group of ten and 10 loose ones and 2 groups of ten."
	Non Standard partitioning not strong			**NOTE (Lise)**: The focus is the language to be used - "Ten is 1 ten that contains 10 ones. Regular 'ten and one' words (24 is 2 groups of 10 and 4 ones or 2 tens and 4 ones) need to be used regularly to establish a language that symbolises decomposing and composing".

Figure 1 Excerpt from the South African Curriculum Excel spreadsheet.

Brief Overview of the Three Curricula

The nationwide German curriculum (KMK, 2022) briefly lists the competencies learners should acquire till the end of Grade 4. This is a result of a reorientation of the curriculum design from an *input orientation* to an *output orientation* (Klieme et al., 2003). PV is only directly mentioned as "Understanding number representations and number relationships". In addition, there are also implicit mentions that we will discuss later; however, these might not be readily identified by teachers. As indicated earlier, the nationwide German curriculum is substantiated in special curricula within each of the 16 federal states. The curriculum of Baden-Württemberg (MKJS BW, 2016) defines the competences for Grade 1/2, with the number range up to 100, and for Grade 3/4, with the number range up to 1 000 000. The curriculum of Brandenburg (MBJS BB, 2015) refers not to grades, but to levels that span several years of schooling, however, the Brandenburg curriculum refers to the same number ranges. As with the national curriculum, the curricula of each state remains superficial, for example, "children are able to use the decimal place value system and recognize its structure (ones, tens, hundreds, bundles, unbundles)" (MKJS BW, 2016). Special didactical material is barely mentioned in this part of the curricula ("number and operation"), but is seen instead as an extra competence—"working with mathematical objects and tools".

South Africa has a highly prescriptive curriculum. Each phase of the schooling system has its own curriculum document for mathematics. The Foundation Phase (Grades R to 3) document specifies what needs to be taught in each of the four terms in the school year. Although the number range increases in each term, the concepts and skills remain the same. PV is taught from Grade 1 and these learners are required to decompose numbers in accordance with the number range for the grade; namely "decompose numbers into multiples of 10 and ones/units" (SA.DBE, 2011) up to 20.

In Grade 3 learners are required to "decompose three-digit numbers up to 999 into multiples of 100, multiples of 10 and ones/units … [and] identify and state the value of each number" (SA. DBE, 2011). Clarification notes provide guidelines on how to teach PV and are included in the curriculum. The curriculum suggests that teachers use concrete and semi-concrete materials (e.g., bundles of sticks, PV cards and number lines) to support learners in developing an understanding of PV; however, no mention is made of the use of digital resources.

The Australian curriculum (ACARA, 2022) is quite prescriptive in terms of PV content descriptors, indicating the range of numbers that learners work with, and then less prescriptive in providing elaborations that teachers may utilise. In terms of the range of numbers that learners are expected to work with, in Foundation learners "name, represent and order numbers including zero to at least 20"; however, there is no mention of PV. In years 1, 2 and 3 learners "recognise, represent, and order numbers to at least (120, 1000, beyond 10 000) using physical and virtual materials, numerals, and number lines (Year 1 and 2) and naming and writing conventions in Year 3. The use of digital (virtual) materials is also recommended throughout the early school years.

This brief synopsis suggests that there is significant variability in terms of curricula structures, curricula specifications, number ranges, and materials used to teach PV across the three countries.

Cross Curricula Comparison of the Three Curricula

In this section we compare the three curricula documents according to the five distinguishing features that emerged from the analysis: namely number range; PV structure; the role of zero; the influence of language in learning PV; and the use of materials in PV learning.

Number Range

Of initial note across the first four years of the three curricula is the difference in the number range that teachers are required to focus on each year (See Table 1).

Table 1

Number Range Per Year Group Across the Three Countries

Australia	Germany	South Africa
Foundation: to at least 20		Reception: up to 10
Year 1: to at least 120	Grades 1-2: up to 100	Grade 1: Up to 19 Term 3: 11-15; Term 4: 11-19
Year 2: to at least 1 000		Grade 2: Up to 99 Term 1: 11-25; Term 2: 11-50; Term 3: 11-75; Term 4: 11-99
Year 3: 10 000 and beyond	Grades 3–4: up to 1 000 000	Grade 3: Up to 999 Term 1: up to 99; Term 2: up to 500; Term 3: up to 700; Term 4: up to 999

The number range in both the Australian and South African curricula is significantly lower than Germany (Year 3/4 learners working up to 1 000 000) with Year 3 learners in Australia working beyond 10 000 and Grade 3 learners in South Africa working with numbers up to 999. In both the Australian and South African curricula learners are effectively "adding a place each year"; however, the South African curriculum is more prescriptive as it specifies the number range for each term in each grade, for example, in Grade 2, the number range is up to 25, 50, 75 and 99 in terms 1, 2, 3 and 4 respectively. The adding of a "place" each year is in our view problematic for the development of learners' PV understanding. Limiting learning to one new place each year means that it is not easy for learners to develop an understanding that the decimal PV system is based on groupings of 10 (i.e., 10^0, 10^1, 10^2, 10^3 etc.) and that it has a repeated naming pattern (e.g., 10 ones is the same as 1 ten, ten tens are the same as 1 hundred…). As the PV principle can be repeated at any place, including decimals, restricting PV understanding to a fixed number of places seems counter intuitive.

There are several inconsistencies regarding the number range in the curricula of South Africa, for instance, while Grade 2 learners are required to calculate up to 99, examples in the clarification notes for teachers often exceed this upper limit with Grade 2 learners required to "partition two-digit numbers in multiples of tens and ones" (SA. DBE, 2011). With the examples provided including "12 tens and 8 ones; 18 tens and 4 ones" (SA. DBE, 2011). These notes imply that South African Grade 2 learners should recognise the PV of numbers beyond 99. The different language used to specify the number range in the curricula is also worth noting. In the South African and German curriculum, the words "up to" are used to indicate the end point for each grade, while in the Australian curriculum, the words, "to at least", suggest that teachers are encouraged to go beyond the specified number range.

Place Value Structure

Greater emphasis is given to understanding base-ten PV patterns in the Australian curriculum. For example, in Year 2, learners are required to read and write "numerals, and saying and ordering two-, three- and four-digit numbers using patterns in the number system", while in Grade 3 learners are required to use "the repeating pattern of place value names and spaces within sets of 3 digits to name and write larger numbers: ones, tens, hundreds, ones of thousands, tens of thousands, hundreds of thousands, ones of millions, tens of millions" (ACARA, 2022). This is strengthened by requiring learners to predict and name "the number that is one more than 99, 109, 199, 1009, 1099, 1999, 10 009 … 99 999 and discussing what will change when one, one ten and one hundred is added to each" (ACARA, 2022). Likewise, the structure of the base-ten PV system is emphasised in Germany where learners "are able to use the decimal place value system and recognise its structure (units, tens, hundreds, bundles, unbundles)" (MKJS BW, 2016).

In the German curriculum there is much attention given to bundling and unbundling as it is repeatedly referred to throughout the curriculum document. There is also explicit attention given to "establishing the relationship between unbundling and transferring" (MKJS BW, 2016) in which the written algorithms for the four operations are linked conceptually to PV concepts. Multiple decompositions of numbers are also included in the curriculum from as early as Grade 1-2 in the German curriculum, allowing learners the opportunity to engage with the concepts of part-whole relationships. It is problematic, though, that many aspects of teaching PV are hidden in arithmetic strategies and are not mentioned explicitly in the curriculum, as not all teachers will be aware of the opportunities to teach PV when teaching arithmetic strategies.

Partitioning in non-standard and standard ways is explicitly stated in the Australian curriculum: "partition, rearrange, regroup and rename two- and three-digit numbers using standard and non-standard grouping" (ACARA, 2022). This includes "renaming numbers in different ways for example, renaming 245 as 24 tens and 5 ones or 2 hundreds and 45 ones" (ACARA, 2022). In the elaborations in the curriculum documentation, it is mentioned that learners should use the PV chart and move materials from one place to another to show this renaming. Learners thus have multiple opportunities to explore PV through both standard and non-standard partitioning of numbers. It is to us astonishing that, in the German curriculum, non-standard partitioning is not mentioned explicitly at all. Implicitly it is included (e.g., in "understand oral and semi-written calculation strategies for the four basic arithmetic operations and use them flexibly") (KMK 2022); however, the need for non-standard partitioning remains largely unacknowledged, and again teachers might miss this aspect in their teaching.

In the South African curriculum, there is mention of both standard and non-standard partitioning. As an example, in the Grade 2 curriculum it is mentioned that learners should "show different arrangements of numbers, for example, 35 can be shown as 35 loose ones, 3 tens and 5 loose ones and 2 groups of tens and 15 loose ones". This is, however, only one of two examples of non-standard partitioning. Learners are exposed very early to standard PV partitioning of numbers into tens and ones (Grade 1, Term 3), and later into hundreds, tens, and ones (Grade 3, Term 2) and this standard partitioning is emphasised in the curriculum and national workbooks. The dominant strategy used for calculating in the South African curriculum is 'breaking down and building up', for example 346 + 154 is broken down into 300 + 40 + 6 + 100 + 50 + 4. Compared to Australian learners, South African and German learners are exposed to less exploration around the concept of PV and are required to use standard partitioning almost exclusively from Grade 1. This limits their opportunity to fully conceptualise the power of the decimal system.

The Role of the Zero

It is important that the role of the zero is clearly understood in PV. In the Australian and South African curricula, the zero is introduced as a number in the Foundation and Grade R years respectively, whilst in Germany (Baden-Württemberg) it is only mentioned in Grade 3. In South Africa, Grade R teachers are encouraged to, "point out that zero means 'nothing'". The special meaning of the digit zero in the number 10 is not made explicit, even though learners in Grade R work with numbers to 10. This is a missed opportunity, and the emphasis on the zero meaning "nothing" may introduce misconceptions related to PV understanding later where the digit zero plays a significant role rather than being 'nothing'. In Year 2 (Australia), learners "recognise the role of a zero digit in place value notation" (ACARA, 2022) and Grade 3 (Germany) learners engage with "the special meaning of the number 0" (MKJS BW, 2016). In South Africa, learners only encounter this in the final term of Grade 3 (the end of the Foundation Phase), when they are required to "recognise 0 as a place holder in two and three-digit numbers". In all three curricula it is interesting to note that the emphasis on the special role of the zero in PV does not appear earlier when learners first encounter numbers beyond ten and multiples of ten.

Language in the Learning of Place Value

Both the German and Australian curricula make explicit mention of language-related issues in the teaching and learning of PV. In the German curriculum, teachers are asked to pay attention to "which errors in speech or spelling are due to misconceptions about place value or linguistic difficulties (for example, language of origin, mixing up tens and ones)" (MKJS BW, 2016). In the German language, two-digit numbers are expressed by naming the ones and then the tens, for example, *einundzwanzig* (one and twenty) and thus this so-called 'number inversion' of tens and ones could be understandable if a learner's language of origin has a different system of naming numbers. This note in the curriculum is relevant and necessary. The Australian curriculum makes explicit mention of language in an elaboration note in which it is explained that Year 1 learners should come to recognise that "numbers are used in all languages and cultures but may be represented differently in words and symbols" as well as including that in Year 3 learners should compare "the Hindu-Arabic numeral system to other numeral systems" (ACARA, 2022). In contrast there is no mention of the influence of language on PV understanding in the South African curriculum. This is a missed opportunity given that South Africa has eleven official languages, and that these languages have several differing number-naming conventions regarding PV (including ones that follow the German convention of naming ones before tens). This is particularly the case given that in the Foundation Phase, mother tongue instruction is promoted; however, from the Intermediate Phase, Grades 4-6 onwards, most learners transition to learning in English despite it being the home language of less than 10% of learners (Robertson & Graven, 2019).

Learning Materials

It is interesting to note the different teaching and learning materials that are mentioned in the three curricula and the differences in what is emphasised when comparing them. In the Australian curriculum there is explicit mention of virtual materials, which is absent in the South African curricula and only mentioned as "the students use mathematical tools appropriately (e.g., drawing tools, digital tools)" (KMK 2022) in the German curriculum. The other physical materials that are mentioned in the Australian curriculum include hundred charts, number lines and large collections of recycled materials to represent large quantities. There is also mention made of PV charts. In Germany, the PV chart is also mentioned, as well as the number line. In addition, base ten blocks are also mentioned, and the requirement that learners "represent numbers up to 1 000 000 in different ways" (MKJS BW, 2016) regarding the use of these materials. The materials mentioned in the South African curriculum include sticks (to create bundles of 10), connecting cubes, an abacus, PV cards and base ten blocks. Interestingly, base ten blocks are only mentioned for use when learners are working with numbers smaller than 100, which means that it is only the tens and ones that would be used. It is therefore no different to using bundles of sticks and, consequently, the power of the representation of ones, tens, hundreds, and thousands, offered by these blocks, is lost.

Conclusion and Implications

Above we have noted several 'missed opportunities' and areas of concern. The German and Australian curricula are similar in providing a greater number range, while the South African curriculum is very prescriptive and limits the number range that learners are expected to work with. This restriction of number ranges regarding the learning of PV may reduce learner opportunities to see the patterns of PV and the structure of our decimal number system. The curricula also vary in terms of recommending when learners should be introduced to zero as a place holder, the use of physical or virtual materials to support learners, and in the approach they take towards the importance of language in teaching PV. In the case of the South African curriculum there is no mention of digital resources and, given that the covid pandemic introduced many teachers and learners to the use of digital resources for teaching and learning, we consider this as a missed opportunity. In terms of moving forward, our analysis of the curricula suggest these avenues for research and practice.

- What are the implications for research, and for teacher education and practice, based on how PV is presented in the respective curricula?
- Investigating ways in which in-service and pre-service teachers can be provided opportunities to develop understanding of the prerequisite concepts and sub-concepts underpinning the teaching of PV and how these are presented in various curricula.
- How to authentically embed the use of appropriate materials, including virtual manipulatives, to support PV learning, including opportunities to experiment with these materials.
- Further research into the crucial role language plays within PV learning, particularly in contexts where the structure of the PV system is different between the mother tongue and the language of instruction.

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