Evidence of Young Students' Critical Mathematical Thinking

Chrissy Monteleone
Australian Catholic University
chrissy.monteleone@acu.edu.au

Jodie Miller
The University of Queensland
jodie.miller@uq.edu.au

In this study, the authors investigate the ways in which young students demonstrate their critical mathematical thinking (CMT). Students aged 5-6 who are beginning their first formal year of education participated in the study. Data is presented from individual clinical interviews undertaken with 16 students. These interviews were analysed using the Critical Mathematical Thinking for Young Students (CMTFYS) framework to identify common patterns in the responses. The findings suggest that these young students beginning school, most often rely on providing explanations and more specifically, justifying, to demonstrate their CMT.

Critical thinking has been identified as an essential skill both for education and future employability, as established by several studies, policy, and curriculum directives (ACARA, 2016; Urib-Enciso et al., 2017). It is a globally recognised term that is emphasised across various subjects, including mathematics, and is considered a crucial skill for preparing students for the 21st century (Urib-Enciso et al., 2017). However, a study conducted in 2018, evidenced that only 50.9% of Australian teachers of middle school classes (104/355) help students to think critically in mathematics lessons (Dix et al., 2018). This is not an easy result to interpret because at present there are no generally accepted definitions of what constitutes critical thinking, especially for young students, nor are there clear practices that can support teachers to develop critical thinking in mathematics. Despite the lack of a clear definition, it is well acknowledged that the development of critical thinking is important for all learners, and it is essential to start developing these skills at the start of formal schooling.

The process of critical thinking involves analysing, evaluating, and making informed judgments or decisions about information or ideas (Urib-Enciso et al., 2017). It can be argued that some of these processes also form part of mathematical thinking, which involves the application of logical reasoning and problem-solving skills to comprehend mathematical concepts and solve problems (Wood et al., 2006). While there are similarities between the two terms, there are also discrepancies. To better understand this intersection, an analysis was undertaken of both sets of literature as part of a larger study to establish the term, Critical Mathematical Thinking (CMT) (Monteleone, 2021) and develop a conceptual framework Critical Mathematical Thinking for Young Students (CMTFYS) that supports the definition and conceptualisation of how young students evidence CMT (Monteleone, 2021).

Despite the push in curriculum direction for teachers to engage students in critical thinking, it is unclear how much this is supported in mathematics education for young students. With little literature to draw on in CMT, it is important to examine the plethora of studies focusing on developing mathematical thinking for primary school students. Examining the research, it appears there are five approaches that can support teachers to guide young students to engage in mathematical thinking: (i) students engaging with strategies that support sense making (Wood et al., 2006); (ii) students displaying reasoning and justifying during learning experiences (Warren et al., 2013); (iii) students making known connections to mathematical ideas and transferring their thinking (Clements & Sarama, 2007); (iv) students progressing in trajectories and displaying their mathematical thinking (Siemon et al., 2017), and (v) students engaging in problem solving (Wood et al., 2006). It is important to note that, the majority of these studies have been conducted with students that have already been attending formal schooling, therefore, little is still known about how young students entering formal schooling, demonstrate their CMT.

(2023). In B. Reid-O'Connor, E. Prieto-Rodriguez, K. Holmes, & A. Hughes (Eds.), *Weaving mathematics education research from all perspectives. Proceedings of the 45th annual conference of the Mathematics Education Research Group of Australasia* (pp. 363–370). Newcastle: MERGA.

The purpose of this paper is to address this problem by examining what CMT young students display as they enter formal schooling, underpinned by the CMTYS framework.

Critical Mathematical Thinking for Young Students Framework

Critical Mathematical Thinking (CMT) is a term that focuses on the application of critical thinking within a mathematical context (Monteleone, 2022; Monteleone et al., 2023). As mentioned above, CMT was conceptualised as part of a larger study, through a review of seminal literature pertaining to the broad areas of critical thinking and mathematical thinking. This led to the development of a conceptual framework titled, Critical Mathematical Thinking Framework for Young Students (CMTFYS) (Monteleone, 2021). The identified themes and sub-themes that underpin the CMTFYS framework emerged from both the critical thinking (Ellerton, 2018; Facione, 1990) and mathematical thinking literature (Cengiz et al., 2011; Wood et al., 2006). Figure 1 presents the CMTFYS themes and sub-themes.

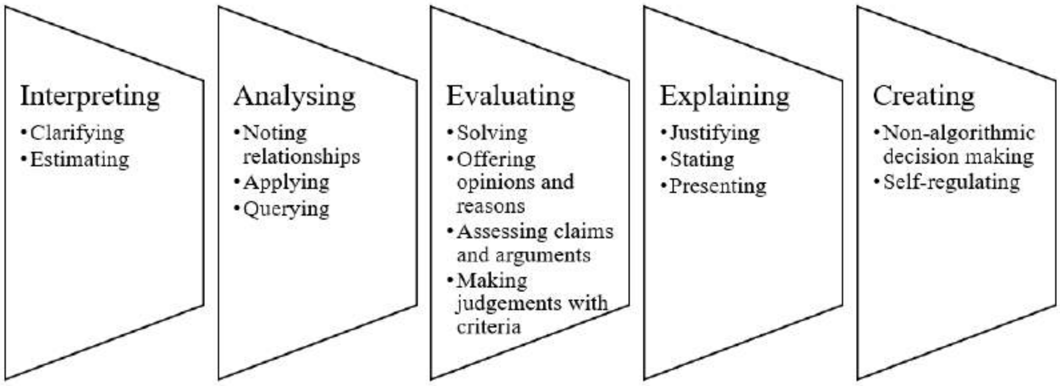

Figure 1. Critical mathematical thinking conceptual framework for young students (CMTFYS).

The Five CMTFYS Themes

The following presents the literature for the five themes presented in the framework.

Interpreting is considered an essential part of critical thinking, as it involves the formation of logical judgments or conclusions (Ellerton, 2018). According to Facione (2011), critical thinkers who make decisions may also engage in interpretation. The literature on interpreting that is part of the CMTFYS include clarifying (Facione, 2011) and estimating (Lipman, 2003).

Analysing is recognised as an important component of critical thinking, with Facione (2011) incorporating it as a core skill in his definition. He describes analysing as both a cognitive skill and an affective disposition. The sub-themes that represent analysing in critical thinking, emerging from the American Philosophical Association's systematic review on critical thinking (Facione, 1990), include applying, questioning, and noting relationships.

Evaluating claims and thought processes has been identified as an essential practice for promoting mathematical thinking (Cengiz et al., 2011). Similarly, Williams (2000) and Wood et al. (2006), present a series of increasingly complex categories that enable students to evaluate their mathematical thinking. The literature on mathematical thinking also identifies sub-themes of evaluating, including making judgments based on criteria (Cengiz et al., 2011), solving problems (Francisco & Maher, 2005), and providing opinions supported by reasoning (Cengiz et al., 2011). The critical thinking literature identifies assessing claims and making judgements (Facione, 1990) as a common disposition found when individuals evaluate.

Explaining is when an individual provides reasons for decisions made, as well as depth and detail of the explanation (Halpern, 2013). To better understand how explaining fits within critical thinking, sub-themes were identified primarily from the seminal literature of Facione (1990; 2011). These sub-themes, such as stating, presenting, and justifying, help individuals develop their critical thinking skills by enabling them to explain their thought processes and how they arrived at their judgments (Facione, 2011).

Creating involves generating new and innovative ideas as noted by Lipman (1995). Sub-themes associated with creating and critical thinking are related to evaluation and decision-making. One such sub-theme is self-regulation, which involves an individual's ability to evaluate their own inferences. Non-algorithmic decision-making, which involves mental processes, strategies, and representations that people use to solve problems and make decisions, is also identified by Sternberg (1986) as a critical thinking element.

Thus, to ascertain evidence of CMT presented by young learners, the study was underpinned by the following research question:

- What CMT capabilities are evidenced by young students as they begin formal schooling?

Research Design

The findings presented in this paper are from a larger study (Monteleone, 2021) that utilised an explanatory mixed methods design (Creswell, 2013) to investigate how young students elicit their CMT. This design involved collecting and analysing both quantitative and qualitative data to provide a comprehensive understanding of the research topic. The focus of this paper is on the qualitative interview data collected as part of the study.

Participants and Context of the Study

The larger study involved a total of 161 Kindergarten students (5 years 1 month—6 years 8 months) who were in their first six months of formal schooling, from three urban primary schools located in New South Wales, Australia. All three participating schools had similar demographic features, with the Index of Community Socio-educational Advantage (ICSEA) levels ranging from 1092 to 1112. Additionally, the schools had similar above-average results in the National Assessment Program—Literacy and Numeracy (NAPLAN) assessments.

In total, 16 beginning Kindergarten students were selected to participate in the interviews, which included nine male and seven female students. These students were selected after a set of week-long classroom observations of all 161 students using a designed protocol based on the CMT framework, and analysis of quantitative measures (Raven's Progressive Matrices, Slosson Intelligence Test, and the Patterns and Structure Assessment). The 16 students selected represented each of the three participating schools, with 4 of the selected students coming from School A, 5 from School B, and 7 from School C.

Data Collection Methods

All 16 students participated in individual video recorded task-based one-on-one clinical interviews, consisting of eight learning experiences (Table 1). This method follows Piaget's methode Clinique (Hunting & Doig, 1997), which aims to identify the cognitive capabilities of a child in a social learning context.

The Eight Learning Experiences

In total, eight learning experiences were designed to identify young students' CMT. The learning experiences were designed to: (i) begin with an open-ended question, which allows for a wide range of possible responses and encourages students to think creatively and critically (Nicol & Bragg, 2009); (ii) provide multiple entry points for students, meaning that there were different ways for students to approach the learning experience depending on their prior knowledge (Jorgensen et al., 2010); (iii) use physical manipulatives (e.g., blocks or counters), to help students visualise and make sense of mathematical concepts (MacDonald & Lowrie, 2011); and, (iv) cover a range of mathematical content appropriate for the age group. Table 1 presents an overview of learning experiences from the interview including the types of tasks and questions that were used.

Table 1

Example Learning Experiences (LE) from the Clinical Interview

LE	Description of the learning experience
LE1	Framed photo finding the middle: This is a framed photograph of Joey (hold up frame). I would like to hang this frame in the middle of a wall. Now, imagine this piece of paper is a blank wall (hold up A3 paper) and this is the picture frame I need to hang (hold up smaller frame). How can I hang this frame in the middle of the wall?
LE2	Counting unseen items: This is a mini bean bag (show mini bean bag). It is filled with little beans like these (show zip lock bag with some beans). It's too tricky to count them one by one. Can you think of another way to find out how many beans are in this mini bean bag?
LE3	Why is 3 + 3 the same as 4 + 2?: Can you tell me why 3+ 3 is the same as 4+ 2? If appropriate, change the numbers to 2-digit numbers. Ask students to provide two reasons why they are equal. Can you tell me another way you can work this out
LE4	Towers—identifying which tower is taller: Here are two towers that I built earlier (show readymade towers built with different sized blocks). Which tower do you think has more blocks?
LE5	Teddy Bears—real like number sentences: I had some bears in my pocket. Emily gave me some more. I counted and found I have 11 bears altogether. How many did I start with and how many did Emily give me?
LE6	Cubby house—identifying number of tiles required: I have just finished building a cubby house for my children at home (show picture of the cubby house). I would like to put these tiles down on the floor of the cubby house (show square tile). How can I work out how many tiles I need?
LE7	Sandwich—cutting and sharing equally: How many different ways can you cut a sandwich in half? (Provide several pieces of paper shaped as a sandwich).
LE8	Shapes—replicating: How many different ways, using the cut out shapes, can you re-create this shape? (Provide students with the cut out shapes).

Data Analysis

Deductive analysis, drawing on the CMTFYS framework, was undertaken on transcripts of the 16 video recorded interviews. The transcripts were analysed in iterative cycles focusing on a singular aspect of the CMT themes and sub-themes and coding the students dialogue, gestures (e.g., use of resources) and work samples. Each coded instance was discussed between the researchers to contest and critique during the analysis to ensure limited subjectivity. Table 2 displays an example of the coding undertaken for student 19 (S19) in learning experience six.

Table 2

Example of Data Analysis and Coding for Learning Experience Six

Summary of student response	Speaker	Extract from transcript	CMTFYS
Student described using one tile, drawing around it to determine how many floor tiles are required altogether.	S19	You can count and measure the tile, you can buy one tile and then measure it and then draw around it and then do the same on the others and then count the squares.	Explaining—Justifying (student justified to determine the number of tiles required)
	R	You're saying to take a tile and draw around it, trace it and keep tracing to see how many tiles we need?	

Results: Evidence of Young Students CMT

Table 3 displays the occurrence of CMT themes across all eight learning experiences. The table is ordered to display the most frequent occurrence of CMT. Each number represents one student exhibiting CMT in that learning experience. For example, 10 students explained their thinking in learning experience one, while five students evaluated in learning experience six.

Table 3

Young Students CMT Themes Across Learning Experiences

CMT Theme	Learning Experiences								Theme Frequency
	LE1	LE2	LE3	LE4	LE5	LE6	LE7	LE8	
Explaining	10	1	1	2		4	4		22
Evaluating	1		3			5	2	1	12
Analysing	1	1	2		4				8
Interpreting	6	1	1						8
Creating	1		1			4			6
	19	3	8	2	4	13	6	1	56

From the analysis it is evident that these young students engaged in all forms of CMT as identified in the CMTFYS. Learning experience one and three appear to have provided opportunity for young students to display a range of CMT. Explaining appears to be the most frequent CMT displayed by these young students. This type of thinking was evident across almost all (6/8) learning experiences. To better understand the specific explaining sub-themes evidenced by the students, further analysis was conducted. Table 4 displays the number of occurrences of the explaining sub-themes across each learning experience in order of frequency.

Table 4

Young Students' CMT of Explaining Sub-Themes Across Learning Experiences

Explaining Sub-themes	Learning Experiences								Sub-Theme Frequency
	LE1	LE2	LE3	LE4	LE5	LE6	LE7	LE8	
Justifying	6			1		4	4		15
Stating	2	1	1	1					5
Presenting	2								2
Frequency across LE	10	1	1	2	0	4	4	0	22

Analysis of the interview transcripts revealed that for these 16 young students, all sub-themes within the explaining theme were displayed at various points of the interview. Explaining-justifying appeared to be the most commonly displayed CMT theme and sub-theme. This type of CMT (explaining-justifying) also occurred across a range of learning experiences (LE1, 4, 6 and 7). While specific data analysis was not conducted on the learning experiences, it is important to note that it appears that LE1 (framed photo finding the middle) and LE6 (cubby house—identifying the number of tiles required) provided opportunity for these young students to demonstrate more instances of CMT.

The following excerpts are offered to better illustrate the ways in which these young students engaged in explaining-justifying across the learning experiences in the interview.

Excerpt One. While participating in LE1 Student 9 (S9) determined where the midpoint of the paper was by drawing lines (intersecting: vertically, horizontally, and diagonally). The conversation between the researcher (R) and S9 included:

R: How do you know?

S9: You can't fold a wall so you can't fold this paper. I'll draw a line here and here and just to prove it to you. I will draw another line this way and another line this way, that is the middle.

S9 displayed the CMT of justifying by also using gestures to show where lines might go and drawing lines to justify the location of the middle.

Excerpt Two. While participating in LE4 Student 23 (S23) broke apart the two towers (connected blocks) to demonstrate how the blocks were different in size and that the height of the tower would differ due to the different sized blocks. The statement made during the conversation with the researcher that supported S23's justification included:

S23: They both have the same amount of blocks. What I'm thinking right now is, you know how these blocks are more thicker and taller? If I break one off, you'll see the difference. If I put these together it makes a long tower and you see, if I break all of these off, it's small.

S23 displayed the CMT of justifying by explaining that the difference in the towers was not the number of blocks but the length of the blocks.

Excerpt Three. While participating in LE6, Student 1 (S1) used one tile as a repeated unit of measure to determine the tiles required for the cubby house floor. The conversation between the researcher (R) and S1 included:

S1: You can measure and put the square. You can draw the squares.

R: Can you show me what you mean?

S1: You can put tiles from the floor. If you're missing one, you can put one more.

S1 displayed the CMT of justifying by providing reasons to support his strategy.

Excerpt Four. While participating in LE7, Student 11 (S11) considered the real life shape of a slice of bread to ensure two people receive the same amount of bread. The conversation between the researcher (R) and student demonstrates how S11 provided a justification for their response.

S23: If you wanted to have two pieces of toast, you could do this. Two for me and two for you.

R: Which two would you get?

S23: I'll get those two and you'll get those two. Let's see how... One, two, three, four.

S11's justification for the actions taken ensures that each person is to receive the exact same amount of bread.

Discussion and Conclusion

The findings revealed that young students can evidence CMT across all themes of the CMTFYS framework. This shows that CMT is evident in early schooling and young students have CMT capability. If teachers can continue to develop CMT capabilities in students from a young age, this may equip students with the necessary skills for later education and future employability. However, to be able to do this, teachers will need support to understand how to foster CMT in their mathematics classroom (e.g., Dix et al., 2018). The CMTFYS framework may address this issue by supporting teachers to recognise CMT and consider how they provide opportunity for these types of thinking in their mathematics classrooms.

The most common CMT displayed by these young students was *explaining-justifying*. This finding is consistent with earlier research and may have been prevalent for three reasons: (i) research has shown that young students are more likely to explain and justify their mathematical reasoning (Warren et al., 2006); (ii) promoting the ability to explain thinking processes is important for young students and therefore the researcher's questions may have prompted the students to explain their thinking more often; and, (iii) explaining is crucial for learning and understanding mathematical concepts (Facione, 1990; Halpern, 2013) and therefore young students may have had more experience explaining mathematical ideas than engaging in other forms of CMT in prior to school settings. While evaluating, creating, interpreting, and analysing were observed less frequently, these CMT skills should not be ignored. Reasons for the lower occurrences may be that young students have had a lack of opportunity to engage and develop these forms of CMT, and the types of learning experiences in the interview may not have provided enough opportunity to display these forms of CMT.

The study aimed to examine how young students displayed CMT capabilities as they begin formal schooling and has been addressed through the results in alignment with the CMTFYS. The results presented in this paper can be used to inform the teaching strategies that facilitate the development of CMT in young learners. We note the limitations of the study including the small sample size, which may affect generalisability to learners of different ages and the lack of consideration of contextual factors such as teacher practices. Therefore, further research is required including; (i) a larger sample of students to continue to evidence the CMTFYS framework, (ii) an investigation of long-term development of CMT, and (iii) the teaching methods required to foster CMT in young learners.

References

Australian Curriculum, Assessment and Reporting Authority. (2016). *Critical and creative thinking*. ACARA. https://www.australiancurriculum.edu.au/f-10-curriculum/general-capabilities/critical-and-creative-thinking/

Cengiz, N., Kline, K., & Grant, T. J. (2011). Extending students' mathematical thinking during whole-group discussions. *Journal of Mathematics Teacher Education, 14*(5), 355–374. https://doi.org/10.1007/s10857-011-9179-7

Clements, D. H., & Sarama, J. (2007). Effects of a preschool mathematics curriculum: Summative research on the building blocks project. *Journal for Research in Mathematics Education, 38*(2), 136–163. https://doi.org/10.2307/30034954

Creswell, J. W. (2013). *Research design: Qualitative, quantitative, and mixed methods approaches.* (4th ed.). Sage publications.
Dix, K., Hollingsworth, H., & Carslake, T. (2018). *Thinking maths: Learning impact fund evaluation report*. ACER.
Ellerton, P. (2018). *On critical thinking and collaborative inquiry*. NSW Department of Education. https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/education-for-a-changing-world/media/documents/Peter-Ellerton_Critical-Thinking.pdf
Facione, P. A. (1990). *Critical thinking: A statement of expert consensus for purposes of educational assessment and instruction*. The California Academic Press.
Facione, P. A. (2011). *Critical thinking: What it is and why it counts*. The California Academic Press.
Francisco, J. M., & Maher, C. A. (2005). Conditions for promoting reasoning in problem solving: Insights from a longitudinal study. *The Journal of Mathematical Behavior*, *24*(3–4), 361–372. https://doi.org/10.1016/j.jmathb.2005.09.001
Halpern, D. F. (2013). *Thought and knowledge: An introduction to critical thinking*. Psychology Press. https://doi.org/10.4324/9781003025412
Hunting, R. P., & Doig, B. A. (1997). Clinical assessment in mathematics: Learning the craft*. Focus on Learning Problems in Mathematics, 19*(3), 29–48.
Jorgensen, R., Grootenboer, P., Niesche, R., & Lerman, S. (2010). Challenges for teacher education: The mismatch between beliefs and practice in remote Indigenous contexts. *Asia Pacific Journal of Teacher Education*, *32*(2), 161–175. https://doi.org/10.1080/13598661003677580
Lipman, M. (2003). *Thinking in education*. Cambridge University Press.
MacDonald, A., & Lowrie, T. (2011). Developing measurement concepts within context: Children's representations of length. *Mathematics Education Research Journal, 23*(1), 27–42. https://doi.org/10.1007/s13394-011-0002-7
Monteleone, C. (2021). *Critical mathematical thinking in young students* [Doctoral thesis, Australian Catholic University]. Australia. https://acuresearchbank.acu.edu.au/item/8x1z9/critical-mathematical-thinking-in-young-students
Monteleone, C. (2022). Teacher questioning to support young students to interpret and explain their critical mathematical thinking. In N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys. Proceedings of the 44th annual conference of the Mathematics Education Research Group of Australasia* (pp. 402–409) MERGA.
Monteleone, C., Miller, J., & Warren, E. (2023). Conceptualising critical mathematical thinking in young students. *Mathematics Education Research Journal*, 1–21. Advance online first. https://doi.org/10.1007/s13394-023-00445-1
Nicol, C., & Bragg, L. (2009). Designing problems: What kinds of open-ended problems do preservice teachers pose? *Proceedings of the 33rd annual meeting of the International Group for the Psychology of Mathematics Education* (pp. 225–232). PME.
Siemon, D., Horne, M., Clements, D., Confrey, J., Maloney, A., Sarama, J.,Tzue, R., & Watson, A. (2017). Researching and using learning progressions (trajectories) in mathematics education*. Proceedings of the 41st conference of the International Group for the Psychology of Mathematics Education* (pp. 109–136). PME.
Sternberg, R. J. (1986). *Critical thinking: Its nature, measurement, and improvement*. National Institute of Education.
Uribe-Enciso, O. L., Enciso, D. S. U., & Daza, M. d. P. V. (2017). Critical thinking and its importance in education: Some reflections. *Rastros Rostros, 19*(34), 78–88.
Warren, E., Miller, J., & Cooper, T. J. (2013). Exploring young students' functional thinking. *PNA, 7*(2), 75–84. https://doi.org/10.30827/pna.v7i2.6131
Williams, G. (2000). Collaborative problem solving and discovered complexity. *Mathematics Education Beyond, 2*, 656–663.
Wood, T., Williams, G., & McNeal, B. (2006). Children's mathematical thinking in different classroom cultures. *Journal for Research in Mathematics Education*, *37*(3), 222–255.