Symposium: Big Ideas in School Mathematics

Chair: Yew Hoong Leong
Nanyang Technological University
yewhoong.leong@nie.edu.sg

The "Big Ideas in School Mathematics" (BISM) is a Research Project funded by the Ministry of Education, Singapore, and administered through the Office of Educational Research, National Institute of Education, Nanyang Technological University. The project began in 2020 and its aim is to investigate various areas in relation to teaching towards mathematical Big Ideas in Singapore schools. The study has currency in so far as "Big Ideas" were introduced in the latest Syllabus Revision by the Ministry of Education. There are three sub-studies in the project: the first is on the development of instruments to measure knowledge of BISM among primary- and secondary-level students and teachers; the second is on professional development work for secondary-level teachers on BISM; the third is similar to the second but for primary-level teachers. The papers in this symposium report information and findings on all these sub-studies.

Overview of the Symposium Papers and Presenters

Presenters: Associate Professor Leong Yew Hoong (Chair), Associate Professor Toh Tin Lam (Paper 1), Mr Mohamed Jahabar Jahangeer (Paper 2), Assistant Professor Choy Ban Heng (Paper 3), Professor Berinderjeet Kaur (Paper 4)

Paper 1: Overview of the research project on Big Ideas in School Mathematics

Authors: Toh Tin Lam, Tay Eng Guan, Berinderjeet Kaur, Leong Yew Hoong, Tong Cherng Luen

This paper provides a brief overview of the entire research project and the component sub-studies.

Paper 2: Assessment of Big Ideas in School Mathematics: Exploring an Aggregated Approach

Authors: Mohamed Jahabar Jahangeer, Toh Tin Lam, Tay Eng Guan, Tong Cherng Luen

This paper reports on developments under Sub-study 1. An item from the student BISM instrument will be discussed. It argues for the use of an "aggregated approach" in considering the scores of the student responses.

Paper 3: From Inert Knowledge to Usable Knowledge: Noticing Affordances in Tasks Used for Teaching Towards Big Ideas About Proportionality

Authors: Choy Ban Heng, Yeo Boon Wooi Joseph, Leong Yew Hoong

This paper reports on developments under Sub-study 2. Part of the professional development under this project involved teachers designing their own instructional materials to foreground a targeted Big Idea. Snippets of tasks in these instructional materials will be discussed.

Paper 4: Primary School Teachers Solving Mathematical Tasks Involving the Big Idea of Equivalence

Authors: Berinderjeet Kaur, Tong Cherng Luen, Mohamed Jahabar Jahangeer

This paper reports on developments under Sub-study 3. An item from the teacher BISM instrument will be discussed. Some data on teachers' responses to the item will be shared. There are thus implications to teacher professional development on the Big Idea of Equivalence.

2023. In B. Reid-O'Connor, E. Prieto-Rodriguez, K. Holmes, & A. Hughes (Eds.), *Weaving mathematics education research from all perspectives. Symposium in the proceedings of the 45th annual conference of the Mathematics Education Research Group of Australasia* (pp. 39–55). Newcastle: MERGA.

Primary School Teachers Solving Mathematical Tasks Involving the Big Idea of Equivalence

Berinderjeet Kaur
Nanyang Technological University
berinderjeet.kaur@nie.edu.sg

Cherng Luen Tong
Nanyang Technological University
cherngluen.tong@nie.edu.sg

Mohamed Jahabar Jahangeer
Nanyang Technological University
jahangeer.jahabar@nie.edu.sg

The primary school mathematics syllabuses in Singapore as of the year 2020 reinforces that Big ideas are central to the learning of mathematics. In support of the push to teach for big ideas, a research study is presently underway. A part of it is on the professional development (PD) of primary school mathematics teachers. As part of the PD teachers attempted a mathematical task as measure of the big idea, Equivalence, in an online environment at the start of their PD. Data from the task show that teachers, were generally not cognisant of the big idea of equivalence when solving the task. They were also unable to distinguish between a heuristic (diagrams) and a mathematical idea about relationships, specifically equivalence as in the mathematical task.

The revised school mathematics curriculum, in Singapore, as of 2021 has placed emphasis on learning mathematics as a body of connected knowledge (Ministry of Education, 2019). Four themes, namely properties and relationships, representations and communications, operations and algorithms, and abstractions and applications together with six big ideas have been emphasised for the teaching of mathematics in primary schools. A "big idea is a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole" (Charles, 2005, p. 10). The six big ideas are diagrams, equivalence, invariance, measures, notations, and proportionality. A research study, Big Ideas in School Mathematics (BISM) is presently underway in Singapore and a part of it is on professional development (PD) of primary school mathematics teachers related to the enactment of Big Ideas in their mathematics instruction. Research has documented that teachers' lack of relevant content knowledge of Big Ideas in mathematics translates into their lack of explicit attention to Big Ideas underpinning mathematics taught in schools and results in developing isolated compartments of mathematical knowledge in their students (Askew, 2013). The study reported in this paper draws on part of the data from the BISM project. It attempts to uncover if teachers drew on the big idea of equivalence when solving mathematical tasks that encompass equivalent relationships at the beginning of their PD.

The Study

Participants and Instrument

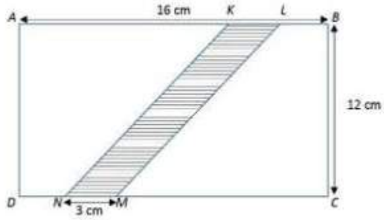
All the mathematics teachers in two primary schools, P1 and P2, participated in the PD (see Kaur et al. 2021; 2022). The PD was spread over two years. In the first year 24 teachers from school P1 and 32 teachers from school P2 and in the second year 23 teachers from school P1 and 33 teachers from school P2 participated in the PD. Due to teacher movement in and out of schools, in the second year there was one less teacher in school P1 and one more teacher in school P2.

Each year during the first session of the PD teachers attempted a set of three mathematical tasks in an online computer environment. These tasks were part of a collection of tasks that were being put together as measures of two big ideas, namely equivalence and proportionality. In the first-year teachers attempted 2 tasks on proportionality and 1 on equivalence, and in the second year they attempted 1 task on proportionality and 2 tasks on equivalence. We limit the data in this paper to the item on equivalence that teachers in School P1 attempted during the first session of their PD in the first year.

Figure 1 shows the equivalence task the teachers attempted in the first session of their first year. The task had 5 parts. Parts 1, 2 and 3 were tasks independent of each other that involved geometrical shapes and measurement. Similar tasks are found in end of school examinations for primary 6 in Singapore schools. Part 4-1 prompted the teachers to review their solutions to Parts 1, 2 and 3 and reflect on any common idea they may have drawn on whilst working on their solutions. Part 4-2 offered some options for teachers to consider about what may have guided their solutions in Parts 1, 2 and 3. Part 5-1 was yet another task on geometry and measurement that teachers had to attempt. Following Part 5-1 was Part 5-2, where teachers were again asked to review their solutions for Parts 1, 2, 3 and 5-1 and consider what may have guided their solution process.

Part 1

The figure below shows a rectangle $ABCD$.
The lines KN and LM are parallel.

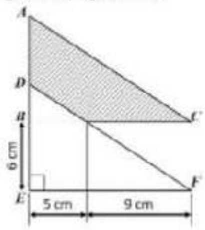


Find the shaded area.

● 18 cm^2 ● 36 cm^2 ● 156 cm^2 ● 192 cm^2 ● Others

Part 2

In the diagram below, ABC and DEF are two identical right-angled triangles. Triangle DEF is partly placed on top of triangle ABC.

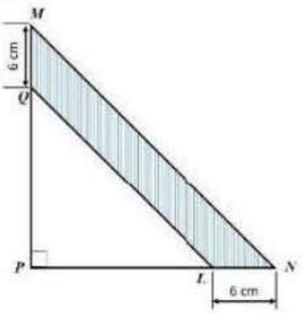


Find the shaded area.

● 42 cm^2 ● 57 cm^2 ● 75 cm^2 ● 84 cm^2 ● Others

Part 3

In the diagram QPL and MPN are two right-angled isosceles triangles. QPL is placed on top of MPN such that $QM = LN = 6$ cm. Area of the shaded part is 144 cm^2.



Find the length of PL.

● 21 cm ● 24 cm ● 27 cm ● 30 cm ● Others

Part 4-1

What is a common mathematical idea in Part 1, Part 2, and Part 3?

To help you recall click on the links below to see the parts		
Part 1	Part 2	Part 3

Enter your response in the space below.

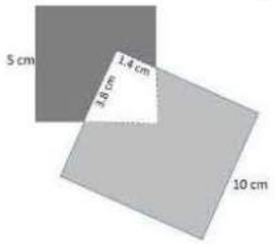
Part 4-2

Which of the following statements best describes a common mathematical idea across Part 1, Part 2 and Part 3?

- ○ I used diagrams for the parts.
- ○ I used equivalence for the parts.
- ○ I used guess and check for the parts.
- ○ I used proportionality for the parts.
- ○ Others (Please elaborate)

Part 5-1

The diagram shows a lighter square with side 10 cm placed on top of part of a darker square with side of length 5cm. Their common region is unshaded.



Find the difference between the area of the lighter region and the area of the darker region? (You may want to consider drawing a model)

● 5 cm^2 ● 5.2 cm^2 ● 5.32 cm^2 ● 70 cm^2 ● Others

Part 5-2

Choose one of the statements that best describes what is common about Part 1, Part 2, Part 3 and Part 5?

To help you recall click on the links below to see the parts			
Part 1	Part 2	Part 3	Part 5-1

- ○ In all these parts I used diagrams.
- ○ In all these parts I used equivalence.
- ○ In all these parts I used guess and check.
- ○ In all these parts I used proportionality.
- ○ Others (Please elaborate)

Figure 1. Example of mathematical task illuminating equivalence as a big idea.

Data and Discussion

Table 1 shows the performance of 24 teachers from School 1 on the mathematical item shown in Figure 1.

Table 1

Performance of Teachers on Mathematical Task Shown in Figure 1

Task	Response	n (%)
Part 1	36 cm^2 (correct answer)	21 (87.5)
Part 2	57 cm^2 (correct answer)	18 (75.0)
Part 3	21 cm (correct answer)	18 (75.0)
Part 4-1	Others*	24 (100)
Part 4-2	I used diagrams for the parts.	7 (29.2)
	I used equivalence for the parts.	3 (12.5)
	I used guess and check for the parts.	2 (8.3)
	I used proportionality for the parts.	9 (37.5)
	Others (Please elaborate)	3 (12.5)
	Use algebra / Cut-outs and diagrams / Use algebra and part-whole relations	
Part 5-1	Others (75 cm^2–correct answer)	7 (29.2)
Part 5-2	In all these parts I used diagrams.	7 (29.2)
	In all these parts I used equivalence.	3 (12.5)
	In all these parts I used guess and check.	2 (8.3)
	In all these parts I used proportionality.	12 (50.0)
	Others (Please elaborate)	0 (0.0)

*Responses of the teachers were coherent with Part 4-2.

It is apparent from the data in Table 1 that at least 18 (75%) of the teachers managed to work through Parts 1, 2 and 3 of the task and arrive at the correct answer. 12 of them mentioned using diagrams, equivalence and part-whole relations as mathematical ideas in their solutions. To resolve Part 1, as shown in Figure 2, one may find the area of the shaded portion by finding the difference between the areas of rectangles with sides 16 cm by 12 cm and 13 cm by 12 cm. Similarly for Parts 2 and 3, teachers may have ‘used diagrams’ to illuminate relationships. It appears that some teachers were using diagrams as a heuristic to illuminate a mathematical idea which many failed to name as equivalence. This may have been due to a lack of ‘vocabulary’ in their mathematics discourse.

However, for Part 5-1 it appears that teachers were challenged when trying to construct a relationship using diagrams. The hint provided could have led them to make equations such as:

- area of lighter region + area of overlap = 100 cm^2
- area of darker region + area of overlap = 25 cm^2

and observe a relationship, but many appear to have failed at it. It is not clear what teachers meant by ‘used proportionality’ in their responses to Parts 4-1, 4-2 and 5-2. As teachers were not interviewed about their responses to the parts of the task, we are unable to decipher what they meant by this.

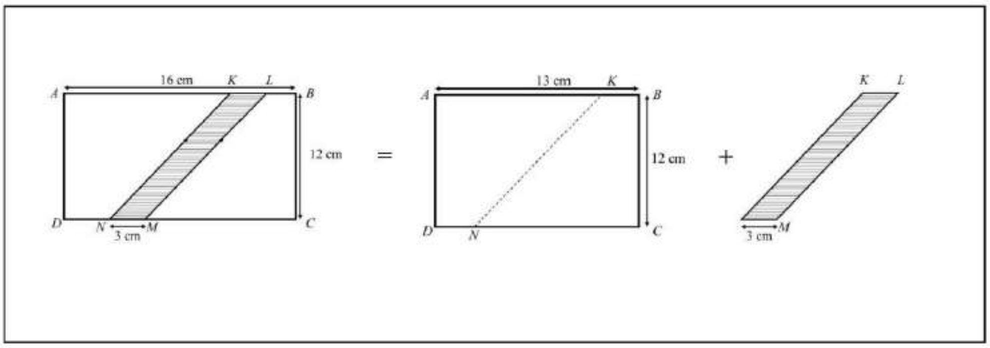


Figure 2. Equivalent relationship of parts in Part 1 of task.

Conclusion

It is apparent from the teachers' responses to the parts in Figure 1 that generally they were not cognisant of the big idea of equivalence which is stated as follows in the mathematics syllabus for primary schools (Ministry of Education, 2019, p. 15):

> Equivalence is a relationship that expresses the 'equality' of two mathematical objects that may be represented in two different forms. The conversion from one form to anther equivalent form is the basis of many manipulations for analysing, comparing, and finding solutions. In every statement about equivalence, there is a mathematical object (e.g. a number, an expression or an equation) and an equivalence criterion (e.g. value(s) or part-whole relationships).

The findings of the study reported here were critical in shaping the following PD sessions as teachers' lack of relevant knowledge of Big Ideas translates into their lack of explicit attention to them in their instruction (Askew, 2013). During the second session of the PD, teachers shared how they had attempted to resolve Parts 1, 2 3, and 5-1. The whole group discourse together with inputs from the experts (University professors) created a shared vocabulary for Big Ideas and specifically—equivalence and how such an idea facilitated solutions of mathematical tasks similar to the ones in Figure 1 and others in the school mathematics curriculum.

Acknowledgements

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References

Askew, M. (2013). Big ideas in primary mathematics: Issues and directions. *Perspectives in Education*, 31, 5–18.

Charles, R. I. (2005). Big ideas and understandings as the foundations for elementary and middle school mathematics. *Journal of Mathematics Education Leadership*, 7(3), 9–24.

Kaur, B., Jahabar, J. M., & Tong, C. L. (2021). Perceptions of big idea of equivalence amongst mathematics teachers in primary schools. In Inprasitha, M., Changsri, M., & Boonsena, N. (Eds.), *Proceedings of the 44th conference of the International Group for the Psychology of Mathematics Education* (p. 152). Khon Kaen, Thailand: Thailand Society of Mathematics Education.

Kaur, B., Jahabar, J. M., & Tong, C. L. (2022). Big ideas of equivalence and proportionality in a grade six mathematics lesson. In Fernandez, C., Llinares, S., Gutiérrez, A., & Planas, N. (Eds.), *Proceedings of the 45th conference of the International group for the Psychology of Mathematics Education* (p. 246). Alicante, Spain: University of Alicante.

Ministry of Education. (2019). 2021 Primary mathematics teaching and learning syllabus. Singapore: Author.