Developing Student Teacher Knowledge of Instructional Strategies for Teaching Proportions: The Important Role of Practicum

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This paper reports on the development of six student teachers' knowledge of instructional strategies (KOIS) for teaching proportions during a 2-month practicum in China. Development of four subcomponents was explored through Content Representation (CoRe) questionnaires and follow-up interviews. Data was analysed deductively and levels of each subcomponent determined based on a scoring rubric. Implications include that practicum is capable of developing KOIS effectively as final scores were at the maximum level with some participants showing greater initiative within their KOIS than others. Implications include that a further level in the rubric could be considered to reflect when creativity is shown in PCK.

Knowledge of instructional strategies (KOIS) is widely accepted as an important component of pedagogical content knowledge (PCK), which is crucial professional knowledge for mathematics teachers for effective teaching and student learning. Research has emphasised that teachers' mathematical PCK is significantly associated with student gains in mathematical understanding (Baumert et al., 2010; Hill et al., 2005), leading to the imperative of cultivating teachers' PCK in all stages of their teaching career. However, evidence shows that student teachers' PCK is not usually at a desired level at the end of initial teacher education and still needs support both in China (e.g., Bao, 2016; Li, 2016), as well as internationally (e.g., Callingham et al., 2012; Şahin et al., 2016). As the final chance for Chinese student teachers to enrich their PCK, practicum provides student teachers opportunities for their PCK to be explored and expanded upon in authentic classroom contexts with support from mentor teachers (Hume & Berry, 2013). Based on Ball et al.'s (2018) Mathematical Knowledge for Teaching (MKT) framework and Hanuscin et al.'s (2018) conceptualization of PCK components, this study reports on six student teachers' development of one PCK component (i.e., KOIS) during their final practicum. KOIS was chosen for this study as this aspect of PCK had been developed more than other PCK components (i.e., knowledge of curriculum, knowledge of instructional strategies) in a large study investigating the development of student teachers' PCK in practicum. The research question is:

- To what extent is KOIS developed during practicum?

Background Literature

Knowledge of Instructional Strategies

Since Shulman (1986) introduced the notion of PCK (i.e., teachers' knowledge of transferring subject matter into comprehensible forms for all learners) and its two central domains of instructional strategies and understanding of students, many researchers have drawn inspiration to conceptualise their own PCK models. Among the various components identified as constituting PCK, KOIS and knowledge of students (KOS) have consistently been identified as the most important by researchers (e.g., Park et al., 2011; Sæleset & Friedrichsen, 2021). As a component of PCK, KOIS has been interpreted in different forms for teaching different subject areas. For teaching science, Park and Oliver (2008) speculated two components of KOIS, namely topic-specific activities and representations, and subject-specific strategies. This was almost consistent with the conceptualization of Hanuscin et al.'s (2018) work, which added a subdomain of "strategies for

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adapting instruction for diverse learners" for science education and identified subcomponents of KOIS as including:

> Knowledge of topic-specific and science-specific strategies for teaching. Teachers should have knowledge of the conceptual power of a particular activity and/or representation, the extent to which it facilitates student learning, and how they might adapt that instruction to better facilitate the learning of diverse students. (p. 668)

Many mathematics education scholars have contributed to the development of ideas relating to PCK, including KOIS. For instance, Ball et al.'s (2008) MKT model, Krauss et al.'s (2008) three dimensions of PCK, and Chick et al.'s (2006) Clearly PCK framework. The MKT model has been used as a theoretical framework by researchers investigating teachers' or student teachers' mathematics PCK (e.g., Jacob & McConney, 2013; Livy & Downton, 2018) and has become a foundational tool in mathematics education (Melhuish et al., 2021). It divides mathematics knowledge for teaching into two aspects: subject matter knowledge and PCK. KOIS in the MKT model is referred to as knowledge of content and teaching, and combines with two other domains—knowledge of content and students and knowledge of content and curriculum—to constitute the entire MKT framework. According to Ball et al. (2008), teachers must know a series of strategies about how to design instruction, including being able to arrange specific instructional content, choose appropriate representations and activities, be aware of the advantages and disadvantages of representations and activities in particular contexts, and identify instructional affordances of different methods and procedures. "Instruction" is another appellation of KOIS in the three dimensions of PCK in Krauss et al.'s (2008) model (i.e., tasks, student, and instruction), representing knowledge of multiple representations and explanations of mathematical problems. In the Clearly PCK framework, KOIS represents the strategies and representations appropriate for teaching specific mathematics concepts (Chick et al., 2006).

Overall, literature suggests the following aspects should be included in KOIS for mathematics teaching: strategies for teaching mathematics, activities and representations for specific mathematics topics, and strategies for adapting the activities, representations or other instructional strategies. Development of the four subcomponents of KOIS- subject-specific strategies, topic-specific activities, topic-specific representations, and strategies for adapting instruction for diverse learners—have therefore been investigated in this research.

Developing KOIS in Practicum

Research has focused on the effect of various factors on developing student teachers' KOIS in practicum. For example, investigating the contribution of one task (e.g., classroom observation) in the practicum (Livy & Downton, 2018), and the impact of the relationship with mentor teachers (Msimango et al., 2020), or interventions (Sæleset & Friedrichsen, 2021). Livy and Downton (2018) reported a study of developing student teachers' PCK in which they observed student teachers teach a single geometric reasoning lesson during practicum. Data from the observations and fieldnotes contributed to researchers' understanding of what student teachers noticed and learnt about teaching geometric reasoning. However, their research investigated student teachers' PCK development in the observation of one lesson and did not seek to explore their development over time. Similarly, Sæleset and Friedrichsen (2021) explored the integration of knowledge of strategies and knowledge of learners in the intervention of stimulated recall, where student teachers watching video recordings of their lessons and reflected on their teaching. Their findings indicated that student teachers' instructional strategies and knowledge of learners were frequently integrated in the process, with topic-specific strategies being developed most. Although the two studies mentioned above were conducted in practicum, they focused on just one task within the practicum. In contrast, Msimango et al. (2020) reported on how mentoring relationships might impact student teachers' PCK development during practicum. Data from separate interviews of student teachers and their mentor teachers showed inconsistencies in communication had acted as barriers for developing PCK, while harmonious communication with mentor teachers was helpful for PCK development, particularly for instructional strategies. These researchers investigated the influence of mentoring on PCK development through interview data only, again without systematically measuring changes in student teachers' PCK.

Methods

In this study, the researcher explored the development of the four aspects of student teachers' KOIS (i.e., subject-specific strategies, topic-specific activities, topic-specific representations, and strategies for adapting instruction for diverse learners), which together comprise one component of their PCK, over the entire practicum. Triangulation of multiple data sources from interviews and questionnaires were used. By analysing all the data sources in relation to the different levels specified in the scoring rubric developed by Hanuscin et al. (2018), strong evidence of the development of KOIS during practicum was revealed.

Participants

Participants were six student teachers who were studying a four-year undergraduate program in a university in China. The program comprises four semesters of foundational learning including university-based learning, plus a two-month practicum in a local primary school as the final component of the qualification. Student teachers spend this practicum in schools solely under the supervision of school mentor teachers, experiencing a range of school-based educational activities (e.g., planning lessons, observing classroom teaching, and teaching lessons to the whole class). The six participants were recruited due to the abundant learning opportunities relating to teaching proportions within their practicum, and their degree theses are on a topic outside of mathematics education, making a high level of prior knowledge of mathematics unlikely to contribute to PCK development in practicum. There were five student teachers working in year four of primary school, experiencing the instruction of decimal-related topics, and one student teacher working in year five, experiencing the instruction of fraction-related topics. They are referred to using pseudonyms.

Instruments

This was a qualitative multiple case study (Yin, 2016) with multiple data sources from questionnaires and interviews. The CoRe questionnaire (Loughran et al., 2012) is a matrix that includes a series of topics about a particular content area and a set of eight pedagogical questions corresponding to the components of PCK, which has been widely utilized by PCK researchers (e.g., Hume & Berry, 2013; Nilsson & Karlsson, 2019). The present study also used the CoRe questionnaire, which was adapted by adding prompts and two further questions to make it as suitable as possible for participants to understand the questions explicitly and to maximise the usefulness of the collected data. The adaption was informed by the trial of data collection tools when the author piloted the tool with six primary mathematics teachers at other schools. In this study, all student teachers completed the questionnaire on three topics. There was a unified CoRe topic over the three topics, "the meaning of decimals (or fractions)", because in China it is the first topic that primary school children need to be taught when they start learning about decimals (or fractions) and it can be a challenging topic for teaching and learning within primary education.

The CoRe questionnaire was followed by a supplementary interview to explore the reasons for participants' questionnaire responses and to add to the content of the questionnaire, thus ensuring rich and detailed data. All student teachers' changes on their PCK level were explored using two CoRe questionnaires and follow-up interviews, once at the beginning and the other at the end of the practicum.

Data Analysis

Data was systematically analysed in a deductive way, where all participants' CoRe and interview responses corresponding to the identified KOIS subcomponents were identified and evaluated with a four-level scoring rubric (Hanuscin et al., 2018). With this rubric, each participant's responses in relation to the KOIS subcomponents were scored from Level 1 (limited knowledge) to Level 4 (robust knowledge) considering the quality of their strategies, the reasonableness of their choices on particular strategies, and their understanding of the strengths and weaknesses of these strategies in specific contexts. Through this process, the development of student teachers' KOIS in practicum was able to be made explicit.

Findings

Findings from the CoRe questionnaires and interviews indicated that practicum had improved student teachers' understanding of instructional strategies for teaching proportions, as all student teachers' level of each subcomponent of KOIS had been developed to the highest level of Level 4 when most initial scores were Level 1 with the rest Level 2. Student teachers' initial understanding of different topics was varied, as there were differences in their initial scores among topics within one subcomponent domain (e.g., topic-specific activities). It seemed that due to their varying understandings of the individual topics, there was thematic variability in the level of KOIS subcomponents reflected in their initial responses. However, this variability was not evident in their final data, according to the rubric. Although most student teachers had developed their KOIS to the highest level of the CoRe by the end of practicum, it seemed that for most, their understanding of teaching strategies was based on imitating the strategies from their mentor teachers and key resources as the strategies identified by them were aligned with their mentor teachers' lessons and the textbook. Only Xue showed creativity on the instructional strategies, rather than imitating others.

Initial KOIS

In the initial data, student teachers showed a limited understanding of instructional strategies, especially strategies that are specific to particular topics and strategies for adjusting their teaching. They provided general instructional strategies and seemed to lack confidence about answering the CoRe and interview questions. When answering questions on the activities or representations that they would like to apply while teaching specific topics, they simply mentioned the category of activities or representations, but did not seem to be able to explain further detail about the timing and purpose of the strategies. For example, Shang noted that she would use a ruler to help children understand decimal counting units, but she did not seem able to explain specifically how or when she would use the ruler. Moreover, student teachers did not show appropriate awareness on adjusting their instructional strategies. They seemed to be inclined to teach as they had planned, because they had some trepidation about the failure of varying their teaching from the plan. As Wangrn said, "The instructions should be the same to all children in one lesson, for the purpose of achieving the instructional goals quickly and smoothly".

Final KOIS

In student teachers' final data, their understanding of instructional strategies had noticeably improved. They were able to identify appropriate strategies that are specific to mathematics learning in general or specific to particular topics, and the strategies of adapting activities or representations, as well as the importance of applying these strategies and potential barriers in particular contexts.

Subject-specific strategies. Several subject-specific strategies featured in student teachers' responses, including group inquiry activities and reasoning, in-class games, and math note-booking (i.e., organising mathematics concepts in a particular form, such as a mind map). Although their explanations for using these strategies were specific to mathematics, they were not specific to particular topics. Group inquiry activities had been cited by all student teachers for all of their three topics. The group inquiry activities were usually combined with reasoning, where children were guided to generalize conclusions from specific contexts presented by teachers in the process of group cooperation and discussion. According to Xue, group inquiry activities contribute to not only enhance children's understanding of new concepts as they are working out the principles for themselves, but also exercise and improve children's abilities of independent learning and cooperating in the process, rather than simply to learn certain content knowledge. The limitations of group inquiry were indicated from the perspectives of teacher and children respectively. From the perspective of teachers, according to Chen, the in-class time might be tight for novice teachers, as group activities are time consuming. From the perspective of children, according to Song and Xue, some children may not pay attention to the discussion or to the tasks given to them by the teacher, but just chat or play with others:

> Some individual students may not be focused in class and not discuss with peers carefully. The teacher may ignore them. (Song)

> It may be a bad design for children who need to be supervised deliberately by teachers, or children with poor learning autonomy and concentration. Because he/she usually can not concentrate on the class independently. (Xue)

Topic-specific activities. The topic-specific activities that were identified by student teachers included hands-on activities, topic-specific scenarios (e.g., a scenario of "ranking four children's long jump scores" that two student teachers who included the "size comparison of decimals" topic noted), and "reactivating" activities (i.e., review the key prior knowledge and understanding of it at the beginning of the new lesson, before introducing the new concept). All five student teachers who worked in year four noted a hands-on measurement activity for the topic of "meaning of decimals", which is an example activity in the textbook (PEP, 2016, p. 32). The activity encourages children to use hands-on tools (e.g., ruler) to measure the length of objects in the classroom (e.g., desk and blackboard) at the beginning of teaching the meaning of decimals, aiming at introducing the creation of decimals (i.e., to represent numbers that are not integers). According to Chen, measurement is an interesting activity that enables children to operate with their hands, which motivates children's enthusiasm for mathematics learning and engages them in the classroom, as well as being a good introduction to the new concept. In addition, children could also adopt the measurement results as general knowledge, as was indicated by Wangrn. However, the barriers of the practical measurement activity identified by student teachers included that it may distract children from the learning that follows, and it is time-consuming. According to Chen, it is difficult for the teacher to pull children who are engrossed in the measuring activity back into the normal classroom, which is not conducive to the smooth progress of the whole lesson. This was similar with Wangxy's statement:

> This group measurement activity actually takes longer and is less likely to ensure effective teaching. Firstly, there is some variability in the learning ability of each group, and secondly, some students may not be very motivated to learn, which may further affect the teaching progress. (Wangxy)

Topic-specific representations. Multiple topic-specific representations were pointed out by student teachers, including physical objects (i.e., meter stick, $1dm^2$ squared paper, and $1dm^3$ cubes), analogy (i.e., life-related analogy and prior knowledge related analogy), and online interactives and simulations (i.e., animations and graphic representations). Student teachers discussed representations for facilitating children's learning consistent with textbooks and literature, as well as the strengths and weaknesses of the representations for particular children. For example, physical objects were referred to as the best pedagogical tools to introduce the decimal-related concepts. All five student teachers who worked in year four identified the physical object of a meter stick for the "meaning of decimals" topic and some of them further included $1dm^3$ cubes ($1cm^3$ units, $1mm^3$ units). The three student teachers' whose CoRe involved the "properties of decimals" topic identified meter stick and $1dm^2$ squared paper ($1cm^2$ units, $1mm^2$ units). According to them, the meter stick was used as a visual display of length, enabling children to understand abstract concepts of the meaning of decimals and decimals' counting units through observing the conversion of different length units. The meter stick was applied first because it is a carrier of length units and is intuitive and visual, as was indicated by student teachers, and units of length are more familiar and easier than units of area or weight to primary school children. The visuals of squared paper and cubes were applied following the meter stick in the topic of "meaning of decimals" and "properties of decimals" respectively, as they were verification of the conclusions drawn from the units of length, from the perspective of area and volume respectively:

> The last session was about the law of the properties of decimals derived from length units, and children might wonder, is it possible that the law derived from this particular context could applied to other context? So we take this conjecture from the particular to the general, by the area of the square paper. (Wangxy)

Information that fits the analogy can be extracted from four student teachers' responses in their CoRe questionnaire. Features of the analogy that was presented by student teachers were that it was life-related or existing knowledge-related. For example, for the "meaning of fraction" topic, Xue indicated that he would like to use common objects that are normal in children's real-life to represent the concepts of "unit 1" and "fractional unit" (e.g., consider a class of 30 students as unit 1, and the fraction unit is $\frac{1}{30}$). The application of analogies allows children to understand abstract proportion-related concepts in a simple and familiar way, according to the student teachers, as well as deepening children's enthusiasm for learning mathematics. As Xue said, "children will find mathematics existing everywhere in their lives".

Strategies for adjusting teaching for diverse learners. Student teachers identified several strategies for adapting instruction for diverse learners, which were represented by several strategies for children of different knowledge levels and for different classes. They were able to explain the ways of adapting strategies, as well as the strengths and limitations of the adjustment. The adjustment strategies shared by all student teachers included a strategy of adjusting group inquiry activities and a "little teacher" pattern. The former was increasing or decreasing the amount of inquiry, based on the overall level of children in one classroom. For example, for the class of lower level, according to student teachers, the teaching design should be simpler and the group inquiry should be reduced as they felt that children with poor comprehension skills are better suited to a teacher-led or teacher-explained approach, whereas the collaborative inquiry approach would not be conducive to their learning or stimulate their divergent thinking, and would be time-consuming. The "little teacher" pattern refers to children acting as teachers to instruct their classmates in concepts. According to student teachers, children think in a more similar way to each other than to the teacher. Therefore, it may be more effective for children to understand a concept when a peer instructs it. It can exercise children's logical thinking skills at the same time, as the "little teachers" have to find a clear and logical way to organize and represent their ideas:

> First of all, it can test whether students have understood the concept and can express it correctly. Secondly, it can exercise the students' expression ability, where teachers are able to see whether they can express it clearly in a logical way. (Chen)

Discussion and Conclusion

Findings of this study include that practicum is effective in cultivating student teachers' KOIS, as they were developed from initial scores at lower levels to all final scores being at the maximum level. Due to the characteristics of topic and context specificity of PCK (Park & Chen, 2011), it was reasonable that student teachers did not perform well on their initial score of topics that they did not know and in unfamiliar contexts, even though they had completed almost 4-year university-based coursework before practicum. Therefore, this study raises a question for further investigation about how student teachers had made such huge progress in only 2-months of practicum. The reasons for the efficiency may be related to student teachers in China having strong understanding of the concepts of proportions and only needing to learn how to teach these concepts to primary children. Their mathematics content knowledge entry to initial teacher education programme is high (equal to around the year 13 level), or it may be related to assistance from mentor teachers or utilisation of other sources as was indicated by Xue (e.g., expert teachers' classroom videos). Further research could help probe what aspects of practicum have most effect on KOIS development.

An important implication is that incorporating a higher level in the scoring rubric could be useful. Although student teachers' KOIS had been developed to the maximum level identified by the rubric, their understanding of strategies was seemed consistent with the textbooks and their mentor teachers' lessons but lacked independent innovation, especially the representations and activities specific to particular topics. Although all student teachers' KOIS were developed to the same highest level, the example of one student teacher (i.e., Xue) who is more confident to be creative with strategies rather than only adapting ideas from the textbook and mentor teachers indicated that there could be a higher level (i.e., Level 5) included in the CoRe rubric to measure and report on further levels of competence, such as more innovation and creativity. It would also be useful for future research to investigate ways of cultivating student teachers' creativity in designing effective teaching strategies, rather than imitating others.

Limitations of this study include that student teachers might have felt anxious or not confident to share things that they were not certain of in the first interview because they felt their PCK would be judged, but they felt more confidence or trusted the researcher more as the study progressed. For future research investigating student teachers' PCK, it would be useful for the researcher to have established relationships with participant student teachers before the practicum. Despite the study limitations, the findings add to understanding of PCK development in that they show that KOIS—especially the development of knowledge of representations and activities that are specific to particular topics—can be effectively developed during practicum.

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