Symposium: Big Ideas in School Mathematics

Chair: Yew Hoong Leong
Nanyang Technological University
yewhoong.leong@nie.edu.sg

The "Big Ideas in School Mathematics" (BISM) is a Research Project funded by the Ministry of Education, Singapore, and administered through the Office of Educational Research, National Institute of Education, Nanyang Technological University. The project began in 2020 and its aim is to investigate various areas in relation to teaching towards mathematical Big Ideas in Singapore schools. The study has currency in so far as "Big Ideas" were introduced in the latest Syllabus Revision by the Ministry of Education. There are three sub-studies in the project: the first is on the development of instruments to measure knowledge of BISM among primary- and secondary-level students and teachers; the second is on professional development work for secondary-level teachers on BISM; the third is similar to the second but for primary-level teachers. The papers in this symposium report information and findings on all these sub-studies.

Overview of the Symposium Papers and Presenters

Presenters: Associate Professor Leong Yew Hoong (Chair), Associate Professor Toh Tin Lam (Paper 1), Mr Mohamed Jahabar Jahangeer (Paper 2), Assistant Professor Choy Ban Heng (Paper 3), Professor Berinderjeet Kaur (Paper 4)

Paper 1: Overview of the research project on Big Ideas in School Mathematics

Authors: Toh Tin Lam, Tay Eng Guan, Berinderjeet Kaur, Leong Yew Hoong, Tong Cherng Luen

This paper provides a brief overview of the entire research project and the component sub-studies.

Paper 2: Assessment of Big Ideas in School Mathematics: Exploring an Aggregated Approach

Authors: Mohamed Jahabar Jahangeer, Toh Tin Lam, Tay Eng Guan, Tong Cherng Luen

This paper reports on developments under Sub-study 1. An item from the student BISM instrument will be discussed. It argues for the use of an "aggregated approach" in considering the scores of the student responses.

Paper 3: From Inert Knowledge to Usable Knowledge: Noticing Affordances in Tasks Used for Teaching Towards Big Ideas About Proportionality

Authors: Choy Ban Heng, Yeo Boon Wooi Joseph, Leong Yew Hoong

This paper reports on developments under Sub-study 2. Part of the professional development under this project involved teachers designing their own instructional materials to foreground a targeted Big Idea. Snippets of tasks in these instructional materials will be discussed.

Paper 4: Primary School Teachers Solving Mathematical Tasks Involving the Big Idea of Equivalence

Authors: Berinderjeet Kaur, Tong Cherng Luen, Mohamed Jahabar Jahangeer

This paper reports on developments under Sub-study 3. An item from the teacher BISM instrument will be discussed. Some data on teachers' responses to the item will be shared. There are thus implications to teacher professional development on the Big Idea of Equivalence.

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Overview of the Research Project on Big Ideas in School Mathematics

Tin Lam Toh
Nanyang Technological University
tinlam.toh@nie.edu.sg

Eng Guan Tay
Nanyang Technological University
engguan.tay@nie.edu.sg

Berinderjeet Kaur
Nanyang Technological University
berinderjeet.kaur@nie.edu.sg

Yew Hoong Leong
Nanyang Technological University
yewhoong.leong@nie.edu.sg

Cherng Luen Tong
Nanyang Technological University
cherngluen.tong@nie.edu.sg

Big Ideas in school mathematics can be seen as overarching concepts that occur in various mathematical topics in a syllabus. Although there has been much interest recently in the understanding of Big Ideas, there is little research done in the assessment of Big Ideas thinking. In this paper, we discuss our research on Big Ideas in School Mathematics. The study consists of three sub-studies: the first sub-study on developing an instrument to measure Big Ideas; two sub-studies on measuring students' and teachers' Big Ideas at test-points before and after a professional development on Big Ideas for primary and secondary school teachers and students.

In the recent mathematics curriculum revision conducted by the Singapore Ministry of Education (MOE), there is a new emphasis on the disciplinarity of mathematics and Big Ideas that are central to the discipline so as to bring coherence and connections between different topics. The objective of this new emphasis is to develop in students a deeper and more robust understanding of mathematics and better appreciation of mathematics (MOE, 2018; MOE, 2019). Each Big Idea connects various concepts and understanding across topics, strands and levels.

The definition of a Big Idea was proposed by Charles (2005) as "a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole" (p.10). Prior to Charles' definition, the notion of Big Ideas in mathematics education became prominent when it was highlighted by the National Council of Teachers in Mathematics (NCTM) in 2000 that "[t]eachers need to understand the Big Ideas of mathematics and be able to represent mathematics as a coherent and connected enterprise. Their decisions and their actions in the classroom—all of which affect how well their students learn mathematics—should be based on this knowledge." (p. 17).

From our collective classroom experience, the presentation in school mathematics syllabuses as discrete strands and topics could have led teachers and students to view mathematics as a collection of topics with weak connections. Thus, Big Ideas illuminate the interconnectedness between topics across strands and this aids the robustness of understanding mathematics. The depth of understanding is dependent on the number and strength of the connections (Hiebert & Carpenter, 1992, p. 67).

Challenges in Teaching for and Measuring Big Ideas

Researchers have affirmed the existence of real challenges in the mathematics classroom for teaching Big Ideas in schools from both teachers' and students' perspectives (e.g., Hsu, Kysh, Ramage & Resek, 2007; Askew, 2013; Schoenfeld, 2019). Teachers in schools may not possess the relevant content knowledge pertaining to Big Ideas in mathematics. Lack of such knowledge is manifested in their teaching, for example, in their inability to realize that the generation of the exponent rules is traceable to the definition for positive integral exponents and that the distributive

property is a Big Idea understanding for combining like terms and multiplying binomials (Hsu, Ramage & Resek, 2007).

Their deficiency of such knowledge often translates into their lack of explicit attention to Big Ideas underpinning mathematics taught in schools. Consequently, this results in students' acquisition of compartmentalized mathematical content knowledge (Askew, 2013). Lack of appropriate professional development for teachers associated with Big Ideas in mathematics, coupled with lack of time for professional development add to the challenges of teaching for Big Ideas (Hsu, Ramage & Resek, 2007; Askew, 2013).

To date, there has been little research on the assessment of Big Ideas. This could be attributed to three major reasons: firstly, researchers have different classifications of Big Ideas (e.g., Charles, 2005; Niemi et al., 2006; Singapore Ministry of Education, 2018, 2019). Secondly, the lack of clarity on the intent of the assessment. Furthermore, any additional instrument to measure Big Idea would mean an additional load to the already heavy high-stake national examinations. Thirdly, it is difficult to create items that assess thinking which link numerous mathematical understandings that cut across topics.

Conceptualization of the Research Project Big Ideas in School Mathematics

In addressing the challenges of teaching and measuring Big Ideas, a team of researchers (the authors of the papers in this symposium) conceptualized a research project Big Ideas in School Mathematics (BISM). Broadly, the aim of BISM is twofold: firstly, to develop assessment items to measure of Big Ideas in school mathematics for assessing how teachers and students connect numerous mathematical understandings into a coherent whole over multiple points of their respective developments. To date, there is a dearth of such an instrument. The second aim is to study the development of Primary and Secondary mathematics teachers' and students' knowledge of BISM across a period of time during which teachers participate in professional development about BISM. The research project consisted of three sub-studies: (1) Measures of Big Ideas in School Mathematics (BISM Measures); (2) Big Ideas in Secondary School Mathematics; and (3) Big Ideas in Primary School Mathematics.

Sub-study 1: Measures of Big Ideas in School Mathematics. This sub-study involved the development of instruments for use in sub-studies (2) and (3). The aim of this sub-study was to develop, pilot and validate instruments to measure the knowledge of Big Ideas in School Mathematics (BISM) for primary / secondary school teachers and students.

Initially, we studied the few existing instrument for the measure of Big Ideas by Niemi et al. (2006). Their items consist of three main types of tasks to measure Big Ideas in mathematics: basic computation tasks, partially-worked problems (with or without explanations), and explanation tasks. Basic computation tasks aim to assess whether students could recognize tasks representing specific Big Ideas. They could then apply the relevant Big Ideas and successfully complete the task. The designed tasks are simple and well-defined. Partially worked problems require students to fill in one to three boxes for missing numbers or symbols in the problem solution, or fill in a complete problem solving step. For an explanation task, a fully worked example is given before those partially worked examples. The selected worked example usually involves no more than 3 to 4 steps, and the fully worked examples are from similar mathematics topics but not the same topic used for assessment. The explanation tasks are based on partially worked examples with justifications. Students, in this case, need to understand the steps solved by others, and must be able to provide the principles for one of the steps. Just like the partially worked example tasks, the explanation tasks follow a fully worked example which covers a similar topic but not the same topic for real assessment.

Our approach to the assessment of Big Ideas draws on the PISA experience of assessing mathematical literacy (Stacey & Turner, 2015) in general and in Tout and Spithill's (2015) writing

of items to test mathematical literacy in particular. Our overarching principle in the development and validation of items or tasks is fitness-for-purpose because because the notion of Big Ideas can be contentious at its boundaries. Also, we expected the conceptualisation of Big Ideas to be complex and cut across school mathematics content. As such, the assessment items must be accessible to students and teachers. In addition, all the assessment items are designed for computer-based testing. For details about the instrument, refer to Jahangeer et al. (2023), which occurs as a research paper in this conference proceeding.

In this study, we focused on two Big Ideas Equivalence and Proportionality. Each item, consisting of five parts, tests on only one of the two Big Ideas. Part 1 to Part 3 each consists of a selected response question focusing on the same Big Idea and are from the same topic. To facilitate thinking beyond topical content and procedural knowledge, Part 4 seeks to assist participants to look for the link connecting the three parts. Part 4 also seeks to trigger students' Big Idea concepts, if any. The participants then attempt Part 5, a question that focuses on the same Big Idea but based on a different topic. Part 5 assessed the participant's ability to transfer the knowledge of Big Idea across a different topic. We also rode on the affordance of this sub-study to address the real issue of assessment fatigue among students. This is our attempt to balance between maintaining the validity of the instrument (students must answer sufficiently many types of problems); and not over-testing the students (to avoid assessment fatigue of students, aligned to the increasing emphasis on the mental well-beings of students). This will be reported in Paper 2.

Sub-study 2: Big Ideas in Secondary School Mathematics. This sub-study aimed to study the trajectory growth in (a) secondary school teachers' knowledge of BISM in relation to their involvement in professional development related to BISM; and (b) lower secondary school students' knowledge of BISM through their two years' schooling at the lower secondary level. The findings we have obtained so far for this sub-study is presented in Paper 3.

Sub-study 3: Big Ideas in Primary School Mathematics. This sub-study is an analogue of Sub-study 2, with the focus on primary school mathematics teachers and upper primary students at Primary 5 and Primary 6. The findings we have obtained so far for this sub-study is presented in Paper 4.

The instrument developed in sub-study 1 was administered to the teacher and student participants in sub-studies 2 and 3 at various chronological points between the two years' schooling. The first test-point, administered prior to the commencement of the teachers' professional development, provided the baseline information on the state of the teachers' knowledge of BISM prior to formal participation in professional development, and the students' knowledge of BISM prior to their teachers being officially cognizant of BISM. It also guided the researchers in designing the professional development interventions for the participating teachers.

Conclusion

This study will inform how teachers understand the rationale for teaching towards Big Ideas, their belief and appreciation in the value to teach towards Big Ideas, and how these are translated into their teaching practices in their efforts to develop in students a greater awareness of the disciplinarity of mathematics, the ideas that are central to the discipline, and bring coherence and connection between different topics and across levels. In view of this, most importantly, the study will inform how students are able to better learn new mathematical knowledge with an appreciation of Mathematics as a discipline and its applications in the world.

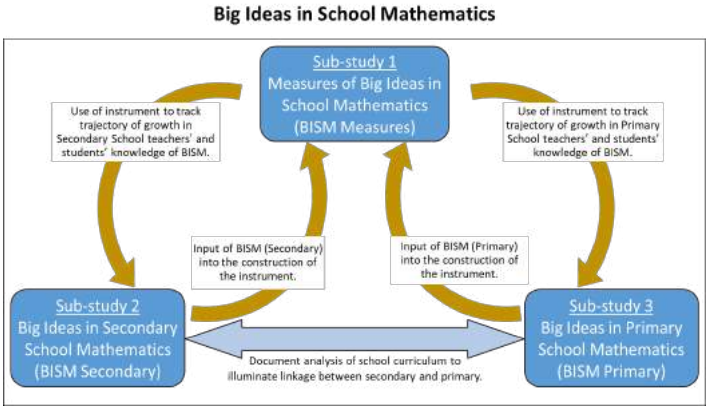

Figure 1. The relation between the three sub-studies in BISM project.

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