A Framework for Designing Green Mathematics Tasks

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Climate change is a complex issue that requires collective actions from various disciplines, including educating future generations about sustainability. Some countries such as Indonesia and Australia have included sustainability into their school curricula, which require teachers to design and implement tasks related to a sustainability issue. However, teachers may not have the capability and understanding to do so. The challenges are difficult for mathematics teachers because designing a mathematics task pertaining to sustainability is rarely explored. Therefore, this paper draws from mathematics education literature to provide and discuss a new framework for designing sustainability-related, mathematics tasks (green mathematic tasks).

Climate change as a global issue accelerated by human activities requires collective actions to address the issue from various disciplines, including education (Abtahi et al., 2017). Some students have not been well-informed regarding climate change, suggesting the importance to educate the future generations (Oliver & Adkins, 2020). Although most students know what climate change is, this does not guarantee that they understand what about the changing climate is problematic, or that they would want to take action to tackle the environmental challenge (Nugroho, 2020). Such actions are more likely to happen if people can comprehend and are aware of the environmental issue as well as the projected effect it has on the earth and living creatures, including humans (Endsley, 2017). Therefore, it is essential to make the future generations aware of the global issue and encourage them to project possible actions to tackle the problem (Barwell & Hauge, 2021).

In terms of educational policy, some countries have included environmental issues and sustainability in their school curricula. For example, Australian Curriculum has cross-curriculum priorities, one of which is sustainability (Dyment & Hill, 2015). The new curriculum of Indonesia called "*Merdeka Belajar*" (*Emancipated Learning*) has co-curricular, thematic projects; one of the themes is related to environmental issues, global change, and sustainability (Anggraena et al., 2022). Both curricula require teachers to integrate sustainability into their teaching plans or to create projects that involve issues around sustainability. However, teachers may currently not have capability and understanding of how to implement this part of curriculum at schools (Dyment & Hill, 2015).

Mathematics education plays a pivotal role to make students aware of real-world issues and contextual mathematics tasks may assist students to understand sustainability-related issues (Barwell, 2018). This is because mathematics can serve both as an interpreting and formatting tool to understand such sustainability challenges as climate change (Steffensen et al., 2021). Unfortunately, designing contextual, authentic mathematics tasks pertaining to sustainability is challenging for teachers (Paredes et al., 2020). Considering the ethical and policy needs to educate the future generations about sustainability as well as the challenge that mathematics teachers may encounter in designing a sustainability task, this paper provides a framework of how to design sustainability-related, mathematics tasks (green mathematics tasks).

Mathematics Tasks Related to Sustainability Issues

A task plays a pivotal role in mathematics classrooms because learning takes place when students are involved in a mathematical activity as required in a task (Ineson & Povey, 2020). There are many different types of mathematics tasks, which have been categorised as abstract, pseudo-contextual, or authentic (Palm, 2007), routine or non-routine (Schoenfeld, 2016), reasoning and proofing (Stylianides, 2009), and problem solving (Schoenfeld, 2016). Therefore, which types of

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mathematics tasks should teachers use when integrating sustainability issues in order to make students aware of such issues?

The most common mathematics tasks used by teachers in classrooms are abstract because mathematics itself is considered an abstract, deductive system that can be used to explain and solve real-world situations (Williamson, 2018). Providing students with more abstract mathematics tasks may give students less opportunity to understand the use of mathematics in real world (Hernandez-Martinez & Vos, 2018). As such, connecting real-world situation and mathematical ideas can assist students to understand the relevance of mathematics out of school; for example, Realistic Mathematics Education was introduced to promote the importance of teaching mathematics in meaningful and useful ways (Freudenthal, 1968).

However, some mathematics tasks designed to be contextual seem unrealistic for students (Palm, 2007). To illustrate, the following water tank task (Figure 1) is considered contextual in a textbook, but students may not interpret themselves as experiencing such a problem as described in the task. They may not see the purpose of figuring out the height that the water will reach. Information is missing or assumed, such as where the water comes from or whether the water tank was empty at the start or not. Despite the lack of purpose and missing information, this task may have sufficient complexity to develop students' reasoning, problem solving, and mathematical thinking (Schoenfeld, 2016).

> The water tank shape cuboid has a width of 2.3 m and a length of 4.6 m. How high will the water reach in the tank if the water flows into the tank at 19 litres of water per second for 52 minutes?
>
> *Source:* Adapted from HackMath (https://www.hackmath.net/en/math-problem/981)

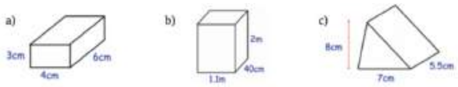
Figure 1. Water tank task.

Sustainability has a realistic purpose, hence the information and situation described in green mathematics tasks should be realistic for students as well. This is important so that students understand not only how mathematical ideas are relevant in real world, but also how to solve a problem in a real situation (Schmidt et al., 2022). Bushnell (2018) designed what he called the *melting ice sheet task* (see Table 1) as an attempt to integrate mathematics and environmental sustainability. He wanted his students to be able to understand the *utility* of mathematical concepts, but the students missed the *purpose* of working on the task (Ainley et al., 2006). Ainley et al. (2006) define *purpose* from students' perspectives of why and how they solve a problem meaningfully and *utility* as to how through the task students perceive the mathematical ideas as useful tools to construct meaning.

When implementing the task, most of Bushnell's students struggled with the concept of prism volume because he deliberately had not taught the topic yet. This made it difficult for his students to find out the volume of water needed to cause a 50 m rise of sea levels. In such situation, some teachers tend to provide their students with explicit, procedural instructions to pave the way to the intended answers (Bushnell, 2018). However, this may lead the students to develop instrumental understanding, instead of relational understanding (Skemp, 1976). Instrumental understanding (knowing how) is easier to demonstrate and assess, but it is also easier to forget while relational understanding (knowing how, when, and why) is more difficult to develop, but the reasoning required strengthens knowledge constructions (Skemp, 1976).

Table 1

Melting Ice Sheet Task Adapted from Bushnell (2018)

Abstract/common tasks	Environment-related tasks
Convert the following lengths into kilometres: 6000 m, 450 m, 125 m, and 90000 m. Calculate the volume of each prism:  Calculate the distance that a train travels in 20 minutes at 90 mph. Calculate the density of a rod of aluminium that has a mass of 575.4 g and a volume of 210 cm^3. Calculate the volume of a 770 g block made of brass which has a density of 8.67 g/cm^3. A cuboid container is used to store boxes. Each box is a cube with side length 1 m. How many boxes can be stored in the container. (The container size is 12 m length, 5 m width, and 2 m height)	Suppose that the ice sheets have melted enough to cause a 50 m rise in sea levels. Convert 50m into kilometres. Given that the global surface area is 361,132,000 km^2, and using your answer to the previous question, calculate the volume of water needed to cause a 50 m rise in sea levels. What is the formula of calculating density? Calculate the mass of water needed to cause a 50 m rise in sea levels (density of water is $1x10^{12}$ kg/km^3). Given the mass of water is the same as the mass of ice, calculate the volume of ice needed to cause a 50 m rise in sea levels (density of ice is $9x10^{11}$ kg/km^3). The volume of an Olympic sized swimming pool 2.5 x 10^{-6} km^3. How many pools would be needed to contain all of the melted ice that causes a 50 m rise in sea levels?

Analysis of the issues raised by these two tasks lead to the following argument. If a task consists of mathematical ideas and problem solving (e.g., water tank task in Figure 1), it may give opportunity for students to develop reasoning and problem solving, but not awareness of sustainability issues (Barwell, 2018). On the other hand, if a task has mathematical ideas and sustainability issues (e.g., melting ice sheet task in Table 1), it may allow students to notice sustainability issues, but not seeing themselves becoming critical and taking actions to solve the sustainability issues (Barwell, 2018; Bushnell, 2018). If a task only includes sustainability issues and problem solving, it cannot show the utility of mathematical ideas (Ainley et al., 2006). Meanwhile, mathematics plays a critical role to format the understanding of sustainability issues (Steffensen et al., 2021). Therefore, all three interdependent elements— sustainability issues, mathematical ideas, and problem solving—are needed in the design of green mathematics tasks.

Designing Green Mathematics Tasks: A Framework

One way to indirectly support students to keep working on a challenging task is providing them with a *purpose* in a task (Ainley et al., 2006), so that they are willing to make mistakes and struggle, which is necessary for them to develop a strategy to solve such tasks (Boaler, 2022). Working on problematic tasks can encourage active engagement in students' construction of mathematical understanding (Schoenfeld, 2016). The following is an example of a challenging, problematic task (designed by the author) called *sea level task* (see Table 2), in which students may find purpose and utility.

Table 2

Sea Level Task

Investigate these two different scenarios by considering the given facts	
Scenario 1: If carbon emissions keep increasing, scientists predict that by 2100 the sea level will raise up to 2 m above that in 2000. *Fact 1:* The global surface area of the ocean is 361,900,000 km^2. *Fact 2*: Antarctic Ice is melting at an average rate of about 150 billion tons per year, and Greenland Ice is losing about 280 billion tons per year.	*Scenario 2*: On the other hand, if carbon emissions are constantly reduced, the sea level will only raise up to 0.3 m by 2100 above that in 2000. *Fact 3:* If 100% of Ice Sheet in Antarctica and Greenland melt, it can raise sea level by 57.9 m and 7.42 m respectively. *Fact 4:* Earth's temperature has risen by 0.08° C per decade since 1880, but the rate since 1981 is more than twice that: 0.18° C per decade.

Note. The facts provided in this task are gathered from official sources such as https://www.climate.gov, https://www.ngdc.noaa.gov, https://climate.nasa.gov, and https://www.antarcticglaciers.org.

For example, from Fact 1 students want to know how much water is needed to cause sea level rises of 2 m and 0.3 m, and eventually utilise mathematical ideas (area and volume) meaningfully to achieve the purpose.

In addition to the elements of sustainability issues, mathematical ideas and problem solving, the sea level task has three salient characteristics (*authenticity, complexity,* and *projection* in the frame of awareness), derived from the main elements (see Figure 2). The task includes real information (see the note in Table 2) and existing environmental issues, which make it authentic from this aspect (Ainley et al., 2006; Palm, 2007). The sea level task is complex and perplexing, which can encourage students to understand the problem and become a confident mathematical problem solver (Schoenfeld, 2016). Finally, the instruction of the sea level task (investigating the scenarios) might also encourage students to project what will happen in the future and what alternatives are available to address the issue, which eventually can awaken students' awareness of an environmental issue (Endsley, 2017). Next, I will explore each characteristic and how they may lead students to take sustainable actions.

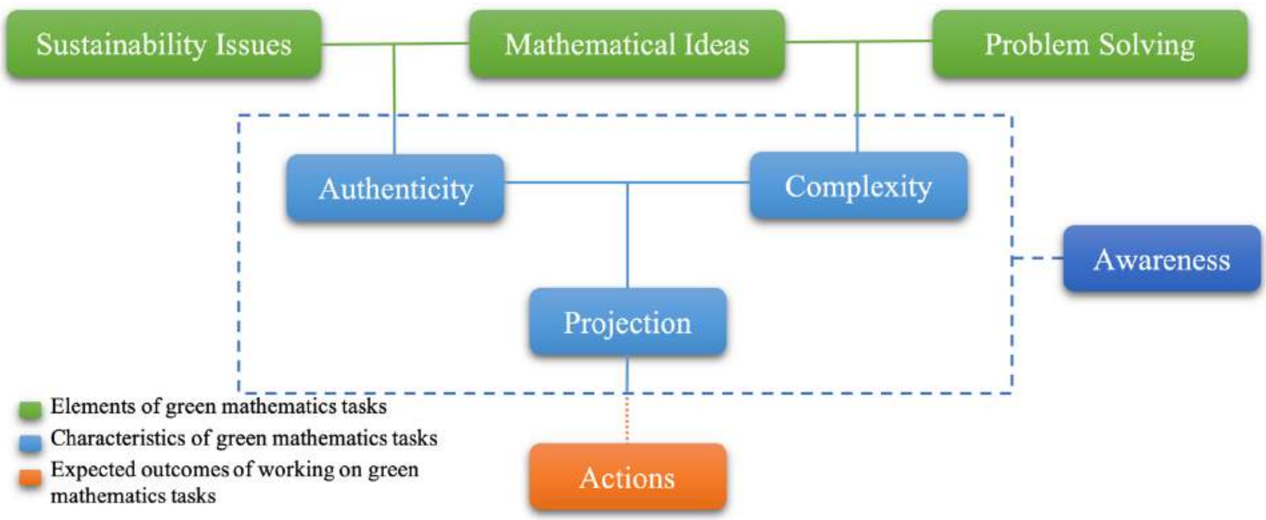


Figure 2. The framework of green mathematics tasks.

Authenticity in Green Mathematics Tasks

The first characteristic of the framework for designing green mathematics tasks is authenticity. The authenticity of contextual tasks refers to real situations that have happened or are predicted to eventuate; authentic tasks are important not only to show the use of mathematics in real world, but also to engage students in solving such tasks (Palm, 2007). To create authentic tasks, the real-world issues are identified first then figuring out mathematical ideas that can be used to work on the tasks (Palm, 2007). However, some contextual mathematics tasks are designed by first identifying the mathematical concepts in the curriculum, then finding real-world issues or contexts where the concepts can be used (Paredes et al., 2020). These different methods of designing contextual tasks result in a 'planning paradox' because the former method may cause unfocused activities whereas the latter can make the tasks inauthentic (Ainley et al., 2006). The designer's intention is therefore to encourage students to see *purpose* and *utility* in green mathematics tasks (e.g., sea level task in Table 1) that can make the task more authentic and avoid such paradox (Ainley et al., 2006).

Palm (2007) proposed eight aspects of authentic tasks: event, question, purpose, information/data, presentation, solution strategies, circumstances, and solution requirements. However, it is challenging for teachers to design a task that meets all these aspects (Paredes et al., 2020). Therefore, de-authenticating some aspects is necessary for educational purposes (Vos, 2018). A question remains; which aspects are essential to be authentic and which ones can be de-authenticated? Vos (2018) suggests the authenticity of the questions and activities are more important than the context of the task. This is because these two aspects can help students to find the purpose and utility of mathematical ideas. However, he refers to mathematical modelling tasks where the context is related to professional work such as planning a city bus schedule, a problem context which would be very challenging for students. Thus, de-authenticating the context of such tasks for educational purposes is likely necessary.

On the other hand, sustainability-related issues as the context in green mathematics tasks should be as authentic as possible so that students can interpret the sustainability issue better (Tran & Dougherty, 2014; Wernet, 2017). Hence, beside the questions and activities, the authenticity of the event and information or data in green mathematics tasks is equally essential. For students to believe the authenticity of a task, these authentic aspects can be validated either by professionals or through official resources, including websites (Vos, 2018) and having the problem described as relevant to students' lives and experiences (Walkington & Hayata, 2017; Wernet, 2017). To conclude, the five salient aspects of Palm's (2007) work for the authenticity of green mathematics tasks in this framework are the event or context, purpose, data/information, questions, and activities.

Complexity in Green Mathematics Tasks

The second characteristic of the framework for designing green mathematics tasks is complexity. The complexity of mathematics tasks refers to the extent to which the problems are unusual (non-routine) for students to solve (Schoenfeld, 2016). One method of exposing mathematical ideas to students is through explicit instruction followed by routine exercises or tasks; another method is through dialogic instruction with problematic, unfamiliar tasks (Clark et al., 2012; Munter et al., 2015). Despite the debates on explicit or dialogic instruction, the role of complexity in mathematics tasks is crucial because it can determine how students perceive mathematical ideas, whether they see the tasks as related to memorising or understanding (Hewitt, 1999). If students memorise the procedures of working on a mathematics task, they tend to become confused when attempting to solve another modified task even if both tasks require the same mathematical ideas (Lubienski, 2000; Salim, 2019). In addition, solving a mathematics problem does not only rely on the ability to memorise mathematical ideas (mathematical knowledge), but also understanding how, when, and whether to use the ideas suggesting mathematics tasks need to be problematic or complex (Schoenfeld, 2016).

Some mathematics tasks explicitly aim to show the use of mathematical ideas in a real-life situation (e.g., melting ice sheet task in Table 1) while other tasks may be designed to develop students' reasoning and problem-solving skills without considering whether they are in a contrived, problematic situation (e.g., water tank task in Figure 1) (Wernet, 2017). While the authenticity of a task helps students to find purpose, working on a complex task challenges students' problem solving as to how mathematical ideas are utilised in a particular context (Lubienski, 2000). As such, the complexity of green mathematics tasks is designed to enhance students' problem solving.

Lubienski (2000) identified difficulties that students may experience when solving a complex, contextual task. First, students can be unfamiliar with vocabulary in the contextualised task; for example, in green mathematics tasks 'carbon footprint' or 'carbon emissions' may be unfamiliar. Providing students with a glossary can assist them to understand the task. Another struggle that students may experience is dealing with uncertainty about what to do, particularly if students are accustomed to learning rules to solve a mathematics problem. To address this issue, sense making as part of problem solving plays a pivotal role to guide students exploring different approaches to comprehend a complex problem (Lubienski, 2000).

When solving a complex task, students are expected to struggle in order to solve the problem using mathematical ideas, and with dialogic, indirect guidance, students can succeed both in problem solving and learning (Kapur, 2016; Simon & Tzur, 2012). In addition, reflecting on their activities of solving a complex task is essential because students can examine their own thinking and connect the task to what they have learned or experienced (Carpenter & Lehrer, 1999). Reflecting on how mathematical ideas are utilised can help students to develop understanding and also realise that similar mathematical ideas can provide different interpretations of data and information pertaining to sustainability (Barwell & Hauge, 2021; Brendefur & Frykholm, 2000). All in all, the complexity of green mathematics tasks in this framework requires students to struggle, develop sense making, and reflect on their activities and mathematical ideas used.

Projection of Sustainability Issues

The third characteristic of the framework for designing green mathematics tasks is projection. Projection refers to the extent to which students relate to and envisage the situation raised by a task. Awareness of a situation proceeds in three levels: Level 1 'perception of elements in a situation', Level 2 'comprehension of a situation', and Level 3 'projection of future status' (Endsley, 2017). The theory of situation awareness is usually illustrated through professional work such as pilots, nurses, and—in education—teachers (Sherin et al., 2011). However, this paper adapts the theory to raise students' awareness of a sustainable-related issue when they work on a green mathematics task.

The authenticity of green mathematics tasks, in relation to the first level of awareness, can help students in perceiving a real sustainability-related event or context, information and data, questions, as well as purpose (Ainley et al., 2006). These authentic aspects can be either provided or gathered by students. The complexity of green mathematics tasks can help students to grasp the given situation through sense making of and reflecting on mathematical ideas used when working on the task (Barwell & Hauge, 2021). These two salient characteristics of green mathematics tasks are essential for students before moving to the next part: projection.

Projection can guide students to envisage the future status of a situation and possibly propose solutions to address the relevant issue (Endsley, 2017). According to the theory of situation awareness, this projection affects students' decision as to what actions they want to take given their perceptions and comprehensions of the situation. Therefore, after working on an authentic and complex task related to sustainability issues (green mathematics tasks), students can project what will happen in the future, so that they become aware of sustainability issues. From this point, they may see themselves taking action to tackle sustainability issues of which they are aware (Endsley, 2017). Actions (as shown in Figure 2) refer to students' behaviours and beliefs regarding sustainability issues and mathematics.

Conclusions

This paper draws from mathematics education literature to provide and discuss a new framework for designing green mathematics tasks. Designing green mathematics tasks that can make students aware of sustainability issues requires three interdependent elements: sustainability issues, mathematical ideas, and problem solving. In addition to these elements, the tasks have three characteristics that can raise students' awareness when working on such tasks. They are authenticity, complexity, and projection. The authenticity of the tasks relies on five important aspects: event or context, purpose, data or information, questions, and activities. The complexity of the tasks allows students to struggle, develop sense making, and reflect on their activities in utilising mathematical ideas. Finally, the projection of what might happen in the future, allows students to decide what sorts of actions to take to address a sustainability issue. Investigations (in progress) are required to justify how teachers design and implement a sustainability-related, mathematics task in classrooms in relation to the framework.

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