Students' Errors in Solving Sequences and Series Word Problems Based on Problem-Solving Steps of Polya

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Abstract: Problem-solving is one of the 21st-century skills. However, students still have difficulty solving sequences and series word problems. The purpose of this research is to analyze students' errors in solving sequences and series word problems based on problem-solving steps of Polya. The research method is descriptive qualitative. The research subjects were six students of XI-B SMA Plus Ar-Rahmat Bojonegoro who were given written tests and interviews. The written test consists of two sequences and series word problems. The results show that the percentage of students who made mistakes in the step of understanding the problem is the smallest and in the step of looking back is the largest. The research findings show that there are four types of student errors; misunderstanding the meaning of the keywords, incorrectly relating what is known and asked to the previous knowledge, incorrectly distinguishing concepts and strategies for solving real-world context problems due to positive interference, and miscalculation due to pseudo-covariational reasoning from "wrong" answers.

Keywords: Students' Errors, Word Problems, Sequences and Series, Problem-Solving, Polya's Steps

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Introduction

Many organizations and educators argue that to cope with the rapidly evolving world because of globalization, students must develop 21st-century skills that include creativity, critical thinking, problem-solving, communication and collaboration, and technological fluency (Bray & Tangney, 2016; Voogt & Roblin, 2012). Mathematics education is the right domain to develop 21st-century skills by preparing students to be able to apply mathematics in real-world problem-solving (Gravemeijer et al., 2017). Problems are tasks that cannot be solved by direct effort and will require some creative insight to solve (Liljedahl, 2015; Mason et al., 2010; Polya, 1965).

Problem-solving in mathematics means that students apply their knowledge and frequently develop new mathematical understandings to find solutions to problems for which there was previously no known solution (NCTM, 2000). Problem-solving ability, according to Polya (2004), is identified as the ability to (1) understanding the problem, (2) devising plan, (3) carrying out the plan, and (4) looking back. According to In'am (2014), understanding the problem is a necessary step before beginning problem-solving activities, devising plan was to make a direction for developing appropriate strategies to solve the problem, carrying out the plan was to carry out the problem-solving following the selected approach, strategy, and model, and looking back was an effort that needed to be made during problem-solving to assess the results.

PISA is an international-level assessment study, which measures students' problem-solving abilities. A scenario-based approach was used in the assessment of problem-solving at PISA in 2012, the assessment of collaborative problem-solving at PISA in 2015, and the assessment of mathematical literacy which includes problem-solving abilities at PISA in 2018 (OECD, 2019a). The results of the PISA study from 2000 to 2018 in the mathematics section show that Indonesian students always perform very poorly. The average score of 15-year-old Indonesian students in the 2012 PISA study was second to last out of 64 countries (OECD, 2014). In PISA 2015 (OECD, 2016), the average score was better but still far below the average score. In PISA 2018 (OECD, 2019b), the average value fell again and was ranked 72nd out of 77 countries.

To develop problem-solving skills and provide practical application in real-world situations, teachers can give word problems to students (Verschaffel et al., 1999). Word problems are a combination of numbers and words in which students apply mathematics instruction in a problem-solving context (Pfannenstiel et al., 2015; Wyndhamn & Säljö, 1997). However, research shows that word problems are the most difficult type of problem faced by students (Verschaffel et al., 2020).

One of the mathematics problems that is considered difficult is the problem of patterns, for example, problems that require the ability to determine sequence patterns to calculate series (Kurniati et al., 2015). The results of observations at SMA Plus Ar-Rahmat Bojonegoro showed that some students still had difficulties in solving the word problems of sequences and series. This is evident from the fact that 52% of students received scores below average on the previous sequences and series test that included word problems.

Based on the background of the problem above, this study aims to analyze students' errors in solving the word problems of sequences and series based on problem-solving steps of Polya.

Method

The method used in this study is a qualitative method with a qualitative descriptive approach. The study was conducted on February 24, 2022. Of the 21 students of class XI-B SMA Plus Ar-Rahmat Bojonegoro who were given a written test, six students were chosen as research subjects using purposive sampling. It is intended that the subjects are chosen based on their information's relevance to the research's goals or specifically by those who are knowledgeable and have effective communication skills (Creswell, 2012).

The identification process is carried out by categorizing the written test results based on the errors made by students in the problem-solving steps of Polya. Data collection techniques in this study were tests and interviews. The test instrument used consisted of two word problems. Problem number (1) is an arithmetic series problem, while number (2) is a geometric series problem as shown in Figure 1.

1. The sum of paper produced by a factory follows the rules of an arithmetic series and is expressed as $P = \frac{5}{2}h^2 + \frac{3}{2}h$, where P is the sum of production (in tons) and h is the number of days. The number of papers produced by the factory on the 10th day is...
2. A child plays on a swing with his father in the park. The father swung his son back, then let go and let the swing stop on its own. If the length of the first arc formed is 2 meters and in each subsequent swing the length of the arc becomes 3/4 of the previous arc, then the length of the swing path until it stops is ...

Figure 1. Test Instrument

Data were analyzed using the triangulation method. Triangulation is the process of strengthening data from various types of sources (Creswell, 2012). The study's supporting evidence is strengthened by examining the accuracy of the findings and interpreting the data from the same source with different ways, specifically through student work and interviews. Data reduction, data presentation, and conclusion drawing were all steps in the triangulation process (Miles & Huberman, 1984).

Based on the purpose of this study, students' errors in solving the word problems of sequences and series were analyzed based on problem-solving steps of Polya with indicators presented in Table 1.

Table 1. Error Indicators Based on Problem-Solving Steps of Polya

Solving Steps	Error Indicators
Understanding the problem	1. The student misreads the problem given. 2. The student misinterprets math words or sentences in the problem. 3. The student is wrong in determining information about what is known and asked in the problem.
Devising plan	1. The student is wrong in remembering the formula or concept he has received to devise a plan. 2. The student is wrong in associating what is known and asked in the problem with the knowledge that they mastered.

Solving Steps	Error Indicators
Carrying out the plan	1. The student is wrong in carrying out the completion plan he has made. 2. The student is wrong in doing calculations.
Looking back	The student is wrong in re-checking the truth of the answer.

Results

Of the 21 students who were given the test, the percentage of students who made errors in solving the word problems of sequences and series based on problem-solving steps of Polya is shown in Table 2.

Table 2. Percentage of Students Who Made Errors in Problem-solving Steps of Polya

Problem-solving Steps of Polya	Percentage of Students Who Made Errors		Average
	Problem I	Problem II	
Understanding the problem	38%	24%	31%
Devising plan	62%	52%	57%
Carrying out the plan	76%	52%	64%
Looking back	76%	52%	64%

Then six students who represent errors in each step of problem-solving steps of Polya are selected shown in Table 3.

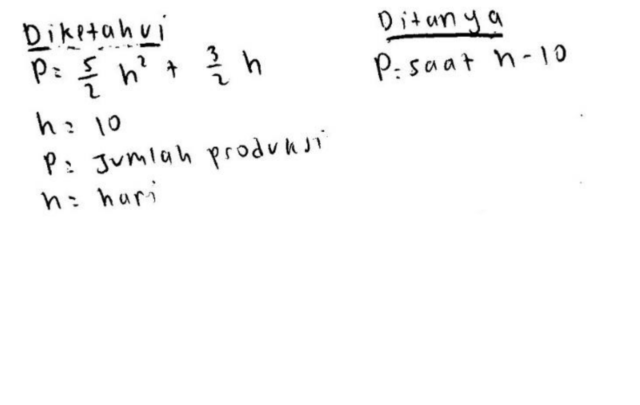
Table 3. Subjects' Errors

Problem-solving Steps of Polya	Subject					
	AHA	MRAH	SAF	ASW	SMHW	RAF
Understanding the problem	√	√	–	–	–	–
Devising plan	√	√	√	√	√	–
Carrying out the plan	√	√	√	√	√	√
Looking back	√	√	√	√	√	√

Description:
(√) The subject made an error in that step
(–) The subject did not make a mistake in that step

Error in the Step of Understanding Problem

Figure 2 shows student AHA misinterpreted the math word or sentence in problem number (1). Student AHA did not understand the difference between "the number of" and "the sum of". What is known in the problem is "the sum of production" which is denoted by P. Meanwhile, what is asked in the problem is "the number of productions". However, the student wrote that what is asked is also P.



Translation:

Known:

$P = \frac{5}{2}h^2 + \frac{3}{2}h$

$h = 10$

P = the sum of production

h = days

Asked:

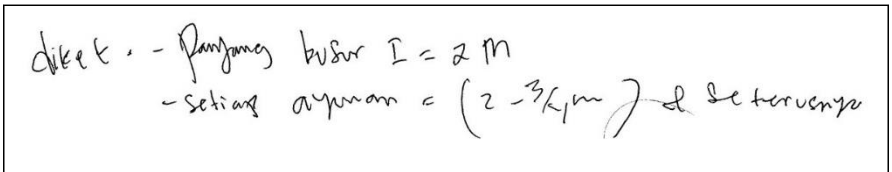
P when *h* = 10

Figure 2. Error in Understanding Problem Number (1)

To ensure this, the researcher conducted interviews with student AHA as follows.

Researcher	: what is asked in the problem?
Student	: the number of paper production, Ustaz.
Researcher	: what do you write here?
Student	: P when h = 10.
Researcher	: what is P?
Student	: the sum of paper production.
Researcher	: what is the difference between "the number of" and "the sum of"?
Student	: they are the same, Ustaz.

Figure 3 shows that student MRAH misinterpreted the math word or sentence in problem number (2). Student MRAH misunderstood the sentence "in each subsequent swing the length of the arc becomes 3/4 of the previous arc". The sentence should mean a multiplication operation, but the student performed a subtraction operation.



Translation:

Known:

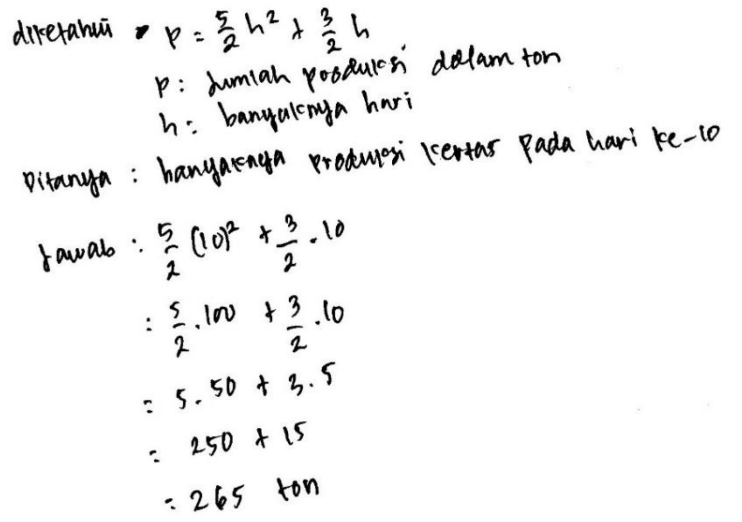
Length of first arc = 2 m

Each swing = (2 - 3/4 m) and so on

Figure 3. Error in Understanding Problem Number (2)

Error in the Step of Devising Plan

Figure 4 shows student SAF was wrong in relating what the known and the asked in problem number (1) with the knowledge he mastered. Student SAF can write down what the known and the asked correctly but used the formula P to find the Un.



Translation:

Known:

$P = \frac{5}{2}h^2 + \frac{3}{2}h$

P: the sum of production in tons

h: the number of days

Asked:

The number of paper production on the 10th day

Answer:

$\frac{5}{2}(10)^2 + \frac{3}{2} \cdot 10 = \frac{5}{2} \cdot 100 + \frac{3}{2} \cdot 10$

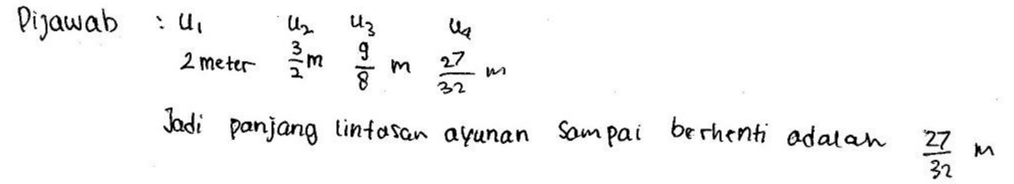
$= 5 \cdot 50 + 3 \cdot 5$

$= 250 + 15$

$= 265$ ton

Figure 4. Error in Devising Plans Problem Number (1)

Figure 5 shows that student ASW was wrong in relating what the known and the asked in problem number (2) with the knowledge he mastered. Student ASW did not know that the problem in the problem should be solved using the formula for an infinite geometric series.



Translation:

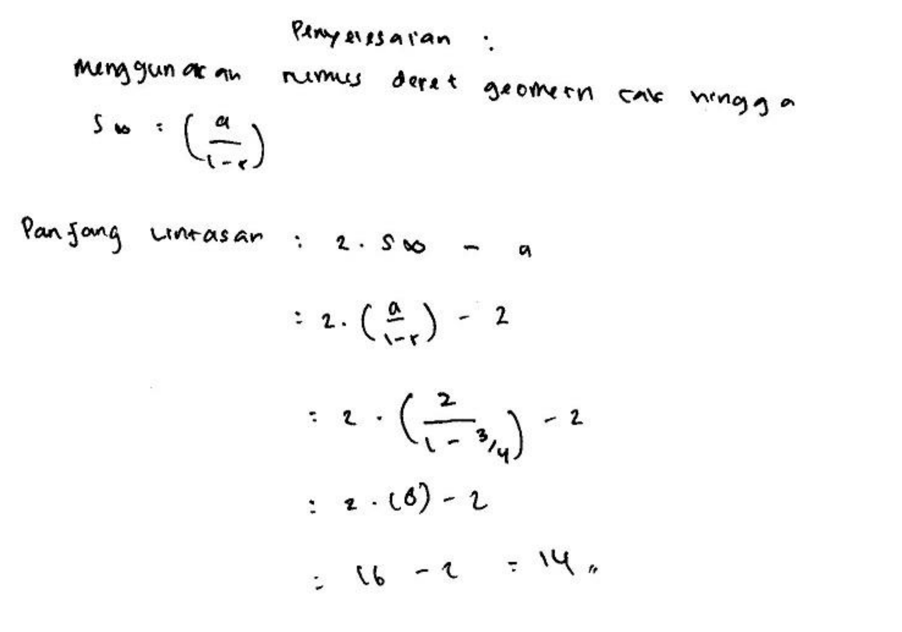
Answer:

U_1	U_2	U_3	U_4
2 meters	$\frac{3}{2}$ m	$\frac{9}{8}$ m	$\frac{27}{32}$ m

So the length of the swing path until it stops is 27/32 m

Figure 5. Error in Devising Plans Problem Number (2) Student ASW

While Figure 6 shows student SMHW was wrong in remembering the formulas or concepts that he has received to make plans for solving problem number (2). Student SMHW already knew that the problem can be solved using the formula for an infinite geometric series. However, when calculating the trajectory, student SMHW multiplied an infinite geometric series by 2, then subtracted it by the first term.



Translation:

Using the formula for an infinite geometric series

$S_\infty = \left(\frac{a}{1-r}\right)$

Length of the path:

$2 \cdot S_\infty - a = 2 \cdot \left(\frac{a}{1-r}\right) - 2$

$= 2 \cdot \left(\frac{2}{1 - {}^{3}/_{4}}\right) - 2$

$= 2 \cdot (8) - 2$

$= 16 - 2$

$= 14$

Figure 6. Error in Devising Plans Problem Number (2) Student SMHW

To find out the reason, the researcher conducted interviews with student SMHW as follows.

Researcher : why did you multiply S with 2?

Student : because the path is back and forth, Ustaz.

Researcher : then why did you subtract it with a?

Student : because the first one is only counted once.

Researcher : have you ever worked on a similar problem?

Student : I have.

Researcher : what about?

Students : find the length of the path of the bouncing ball.

From the interview results, it is known that student SMHW thought that the strategy for solving the swing problem is the same as the strategy for solving the bouncing ball problem which both use an infinite geometric series.

Errors in the Step of Carrying Out the Plan

Figure 7 shows the student RAF was wrong in calculating problem number (1). The student RAF miscalculated $3 \times 9 = 37$, it should be $3 \times 9 = 27$.

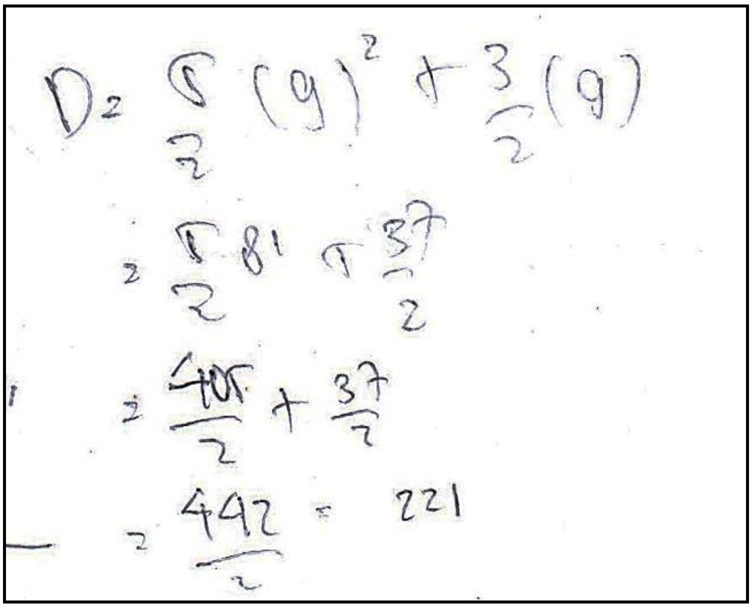


Figure 7. Error in Carrying Out the Plan Problem Number (1)

To find out the reason, the researcher conducted interviews with student RAF as follows.

Researcher : how much is 3/2 × 9?

Student : (recounting) 27/2, Ustaz.

Researcher : Then why do you write 37/2 here?

Student : At that time, I counted it in the air.

From the results of the interviews, it is known that student RAF made a mistake in doing calculations because he did not write down the process on paper and only counted in the air. However, when re-confirmed the student RAF was able to give the correct answer.

Discussion

According to Table 2, the proportion of students who made errors in the understanding of the problem was the smallest, and the proportion who made errors in the step of looking back was the biggest. This is consistent with earlier research by Son et al. (2019), which found that the bigger the percentage of student errors, the further the step in problem-solving steps of Polya.

Students who made mistakes in understanding the problem are wrong in interpreting words or mathematical sentences in the problem. This is in accordance with previous research conducted by Boonen et al. (2014) that one of the sources of students' difficulties in solving word problems is understanding the readings and the meaning of the keywords used. Mathematical vocabulary is the main factor that helps students in understanding math word problems (Powell et al., 2017).

Students who made mistakes in the step of devising plan are wrong in remembering the formula or concept that they have received to devise a plan and are wrong in relating what is known and asked in the problem with the knowledge that they mastered. Students who were wrong in remembering the formulas or concepts that they have received to devise a plan cannot distinguish problem-solving strategies using infinite geometric series. According to Makovski & Jiang (2008), this is called proactive interference. Proactive interference is old information that interferes with recalling new information (Sternberg & Sternberg, 2017).

Students who made mistakes in carrying out the plan are wrong in doing calculations. However, when confirmed again, the student can correct the answer. This according to Subanji (2011) is called pseudo covariational reasoning from “wrong” answers, namely students give wrong answers, but after reflection, they can fix them so that they become correct answers.

Students who made mistakes in looking back are wrong in re-checking the truth of their answers. In fact, according to Pratikno & Retnowati (2018) writing a conclusion sentence is one of the indicators to see if someone checks the results of their work. In addition, the conclusion sentence also shows a person’s understanding of a problem (Saygılı, 2017). Students who are wrong in the step of looking back in this study are the same students who are wrong in the step of carrying out the plan. This is in accordance with previous research conducted by Sukoriyanto et al. (2016) that students who make mistakes in the step of understanding the problem also make mistakes in the steps of devising plan, carrying out the plan, and looking back.

Conclusion

Based on the results and discussion above, it can be concluded that in the problem-solving steps of Polya, there are four types of errors made by students, namely misunderstanding the meaning of the keywords, incorrectly relating what is known and asked to the previous knowledge, incorrectly distinguishing concepts and strategies for solving real-world context problems due to positive interference, and miscalculation due to pseudo-covariational reasoning from “wrong” answers.

Recommendations

For further researchers, it is recommended to conduct further research regarding the four types of errors.

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References

Boonen, A. J. H., Van Wesel, F., Jolles, J., & Van der Schoot, M. (2014). The role of visual representation type, spatial ability, and reading comprehension in word problem-solving: An item-level analysis in elementary school children. *International Journal of Educational Research*, *68*, 15–26. https://doi.org/10.1016/j.ijer.2014.08.001

Bray, A., & Tangney, B. (2016). Enhancing student engagement through the affordances of mobile technology: a 21st-century learning perspective on Realistic Mathematics Education. *Mathematics Education Research Journal*, *28*(1), 173–197. https://doi.org/10.1007/s13394-015-0158-7

Creswell, J. W. (2012). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research (4th edition)*. Pearson Education, Inc.

Gravemeijer, K., Stephan, M., Julie, C., Lin, F. L., & Ohtani, M. (2017). What Mathematics Education May Prepare Students for the Society of the Future? *International Journal of Science and Mathematics Education*, *15*, 105–123. https://doi.org/10.1007/s10763-017-9814-6

In'am, A. (2014). The implementation of the Polya method in solving Euclidean geometry problems. *International Education Studies*, *7*(7), 149–158. https://doi.org/10.5539/ies.v7n7p149

Kurniati, K., Kusumah, Y. S., Sabandar, J., & Herman, T. (2015). Mathematical Critical Thingking Ability Through. *IndoMS-JMS*, *6*(1), 53–62. https://files.eric.ed.gov/fulltext/EJ1079602.pdf

Liljedahl, P. (2015). Numeracy task design : a case of changing mathematics teaching practice. *ZDM - International Journal on Mathematics Education*, *47*(4), 625–637. https://doi.org/10.1007/s11858-015-0703-6

Makovski, T., & Jiang, Y. V. (2008). Proactive interference from items previously stored in visual working memory. *Memory and Cognition*, *36*(1), 43–52. https://doi.org/10.3758/MC.36.1.43

Mason, J., Burton, L., & Stacey, K. (2010). *Thinking Mathematically Second Edition*. Pearson Education Limited.

Miles, M. B., & Huberman, A. M. (1984). Drawing Valid Meaning from Qualitative Data: Toward a Shared Craft. *Educational Researcher*, *13*(5), 20–30. https://doi.org/10.3102/0013189X013005020

NCTM. (2000). *Principles and Standards for School Mathematics*. The National Council of Teachers of Mathematics, Inc. https://www.researchgate.net/publication/269107473_What_is_governance/link/548173090cf22525dcb61443/download%0Ahttp://www.econ.upf.edu/~reynal/Civil wars_12December2010.pdf%0Ahttps://think-asia.org/handle/11540/8282%0Ahttps://www.jstor.org/stable/41857625

OECD. (2014). *PISA 2012 Results: What Students Know and Can Do – Student Performance in Mathematics,* *Reading and Science* (Issue February 2014). OECD Publishing. https://doi.org/http://dx.doi.org/10.1787/9789264208780-en

OECD. (2016). *PISA 2015 Results (Volume I): Excellence and Equity in Education*. OECD Publishing. https://doi.org/https://doi.org/10.1787/9789264266490-en

OECD. (2019a). *PISA 2018 Assesment and Analytical Framework*. OECD Publishing.

OECD. (2019b). *PISA 2018 Insights and Interpretations*. OECD Publishing.

Pfannenstiel, K. H., Bryant, D. P., Bryant, B. R., & Porterfield, J. A. (2015). Cognitive Strategy Instruction for Teaching Word Problems to Primary-Level Struggling Students. *Intervention in School and Clinic*, *50*(5), 291–296. https://doi.org/10.1177/1053451214560890

Polya, G. (1965). *Mathematical Discovery, On Understanding, Learning, and Teaching Problem-solving.* Wiley. https://doi.org/10.2307/2315372

Polya, G. (2004). *How to solve it: A new aspect of mathematical method*. Princeton University Press. https://doi.org/10.1017/cbo9780511616747.007

Powell, S. R., Driver, M. K., Roberts, G., & Fall, A. M. (2017). An analysis of the mathematics vocabulary knowledge of third- and fifth-grade students: Connections to general vocabulary and mathematics computation. *Learning and Individual Differences*, *57*(May), 22–32. https://doi.org/10.1016/j.lindif.2017.05.011

Pratikno, H., & Retnowati, E. (2018). How Indonesian Students Use the Polya's General Problem-solving Steps. *Southeast Asian Mathematics Education Journal*, *8*(1), 39–48. https://doi.org/10.46517/seamej.v8i1.62

Saygılı, E. ; (2017). Examining The Problem-solving Skills and The Strategies Used by High School Students in Solving Non-routine Problems. *E-International Journal of Educational Research*, *8*(2), 91–114.

Son, A. L., Darhim, & Fatimah, S. (2019). An analysis to student error of algebraic problem-solving based on polya and newman theory. *Journal of Physics: Conference Series*, *1315*(1). https://doi.org/10.1088/1742-6596/1315/1/012069

Sternberg, R. J., & Sternberg, K. (2017). *Cognitive Psychology, Seventh Edition*. Cengage Learning.

Subanji. (2011). *Teori Berpikir Pseudo Penalaran Kovariasional*. Penerbit Universitas Negeri Malang.

Sukoriyanto, S., Nusantara, T., Subanji, S., & Chandra, T. D. (2016). Students' Errors in Solving the Permutation and Combination Problems Based on Problem-solving Steps of Polya. *International Education Studies*, *9*(2), 11–16. https://doi.org/10.5539/ies.v9n2p11

Verschaffel, L., De Corte, E., Lasure, S., Van Vaerenbergh, G., Bogaerts, H., & Ratinckx, E. (1999). Design and evaluation of a learning environment for mathematical modeling and problem-solving in upper elementary school children. *Mathematical Thinking and Learning*, *1*, 195–230.

Verschaffel, L., Schukajlow, S., Star, J., & Van Dooren, W. (2020). Word problems in mathematics education: a survey. *ZDM - Mathematics Education*, *52*(1), 1–16. https://doi.org/10.1007/s11858-020-01130-4

Voogt, J., & Roblin, N. P. (2012). A comparative analysis of international frameworks for 21 st century competences: Implications for national curriculum policies. *Journal of Curriculum Studies*, *44*(3), 299–321. https://doi.org/10.1080/00220272.2012.668938

Wyndhamn, J., & Säljö, R. (1997). Word problems and mathematical reasoning - A study of children's mastery of reference and meaning in textual realities. *Learning and Instruction*, *7*(4), 361–382. https://doi.org/10.1016/S0959-4752(97)00009-1