EMBODYING COVARIATION THROUGH COLLABORATIVE INSTRUMENTATION

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Covariational reasoning and the creation and interpretation of graphs of covariational situations are important skills in math and science. Unfortunately, research shows that students often struggle to make meaningful connections between graphs and the covariational situations they represent. Educational activities designed to help students overcome this struggle tend to use either student-generated or automatically-generated graphs, and have students either act out covariational situations or more passively observe them. In this paper, we present the design of a tool and task that enabled two students to simultaneously embody both the creation of a graph and the covariational actions that the graph represents. Through a process of collaborative instrumentation, the students made meaningful connections between their motions and the embodied traces they created as they reasoned about the covarying quantities of height and time.

Keywords: Algebra and Algebraic Thinking, Middle School Education, Technology, Instructional Activities and Practices

Covariational reasoning, or the ability to coordinate changes in two covarying quantities (Carlson et al., 2002; Thompson & Carlson, 2017), is an important part of how students understand and use graphs. However, research shows that students of all ages often struggle to make connections between graphs and the covariational situations they represent (Oehrtman et al., 2008). For example, students commonly view graphs more as wireframe images of a situation rather than records of covariation, seeing static shapes rather than emerging traces (Moore & Thompson, 2015).

Many different educational activities have been designed and tested that attempt to support students in making these connections. One of the many decisions faced by the designers of these activities is a choice about whether to use student-generated or automatically-generated graphs. Students may create their own graphs and thus see how they are built, but many students struggle with this process and the underlying covariational meaning is lost (e.g., Mevarech & Kramarsky, 1997). Conversely, graphing utilities allow students to focus on the covariation without having to worry about the graphing process itself, but students then tend to blindly trust these automatically-generated graphs without thinking more deeply about what they represent (e.g., Cavanagh & Mitchelmore, 2000).

Similarly, designers must also choose whether students will act out the covariational situation being graphed or more passively observe it. For example, designs using motion sensors paired with automatically-generated graphs allow students to literally walk their covariation and see the resulting graph appear in real time (e.g., Duijzer et al., 2019). In contrast, designs that provide animations of a covariational situation allow students to observe or even control a covariational situation but not to physically act it out themselves (e.g., Stevens et al., 2017).

We categorize the designs resulting from these two decisions by whether or not the covariational situation and the corresponding graphing are each embodied (e.g., Lakoff & Johnson, 2000; Núñez, et al., 1999; Wilson, 2002) by the students. Some designs have tried to support student embodiment of both covariation and graph creation. For example, designs aimed at teaching kinematics to college students have used ball-and-track activities paired with student-generated graphs (e.g., McDermott et al., 1987). In these activities, students both generate the motion of a physical ball in a track they have assembled and then also graph that motion. Along with instruction and practice, McDermott et al. found that this design supported improvement in college students' covariational understanding of kinematics graphs.

However, we found no design in the literature that has students simultaneously embodying both the act of graph creation and the covariational situation being graphed. We therefore conjecture that leveraging the benefits of embodying both covariation and the corresponding graph creation at the same time may help students to make these connections between graphs and the covariational situations they represent. Our research question is thus: *What kinds of graph thinking and covariational reasoning might students engage in while working with a tool designed to support collaborative instrumentation and the simultaneous embodiment of both graphing and covariation?* Accordingly, we address this question through research framed by the theories of covariational reasoning, embodied design, instrumented activity, and intersubjectivity.

Theoretical Framework

When students coordinate two varying quantities (Thompson, 1993, 2011) by attending to and making sense of the ways the quantities change in relation to each other, they engage in *covariational reasoning* (Carlson et al., 2002; Thompson & Carlson, 2017). Covariational reasoning is fundamental to students' understandings and uses of graphs precisely because *graphs* in two dimensions are representations, or traces, of the relationship between two covarying quantities. In order to support the development of students' covariational reasoning, Moore and Thompson (2015) endorse a particular way of thinking about graphs as emergent relationships called emergent shape thinking. Whereas *static shape thinking* relies on perceptual cues and global properties of graphs as static objects, *emergent shape thinking* involves understanding a graph simultaneously as what is made (a trace in progress) and how it is made (as a record of the relationship between covarying quantities). These ways of thinking about graphs will be useful in our analysis of students' graphing activity as we aim to help them make connections between graphs and the covariational situations they represent.

As we describe below, the first author embarked on a design experiment (Cobb et al., 2003) to produce a tool to mediate the formation of these connections. The tool used in this project emerged from a design cycle informed by the theory of embodied cognition. Rather than consisting solely of computational operations on symbolic propositions that occur entirely within a disconnected mind, *embodied theories of cognition* hold that cognitive processes "must be understood in the context of [the mind's] relationship to a physical body that interacts with the world. … Hence, human cognition, rather than being centralized, abstract, and sharply distinct from peripheral input and output modules, may instead have deep roots in sensorimotor processing" (Wilson, 2002, p. 625). From this perspective, even the most complex ideas are grounded in and emergent from the lived reality of bodily experiences in the world.

Empirical evidence in support of embodiment theories of cognition (e.g., Lakoff & Núñez, 2000) has impacted mathematics education research (e.g., Abrahamson & Lindgren, 2014). In that context, mathematics must be reconceptualized and accounted for not as "an objective mathematics, independent of human understanding" but rather "in terms of the human bodily-based and situated conceptual systems from which it arises" (Núñez et al., 1999, p. 47). Accordingly, such an account should be useful for devising more effective instruction through grounded learning situations that foster meaningful mathematical understanding. Abrahamson's (2014) pedagogical framework of *embodied design* provided us with a template with which to do so as it gave language and structure to the process by which the *Graph Tracer* tool (presented below) was designed. Through *phenomenalization*, we hypothesized about the embodied schema that underlie the coordination and graphical tracing of two covarying quantities. Through *concretization*, we digitally designed and 3D printed a physical artifact with which two learners could collaboratively enact their solutions to problems that would have them enact those schema. And through *dialog*, we elicited the learners' informal enactments of covariational reasoning and supported their formalization by engaging them in reflective activity.

Lastly, in order to analyze the contribution of the collaborative aspect of two students' graphing activity, we drew on the theories of instrumented activity and intersubjectivity. Vérillon and Rabardel's (1995) theory of *instrumental genesis* provides a framework for analyzing student learning in the process through which objective artifacts become tools to be used to accomplish a task. From this perspective, an artifact (e.g., a physical or digital tool) becomes an instrument (e.g., tool, sign) when a subject (e.g., actor, learner) has integrated it into a conceptual scheme for a specific implementation of the artifact in the context of their goal-directed activity. Thus, an instrument is a *psychological* construct established by the subject's instrumental relation with the artifact. In practice, an instrumented activity is characterized by individual subject-object, subject-instrument, and object-instrument interactions.

In our case, however, with two subjects working collaboratively, we will be interested in what we refer to as the subjects' *collaborative instrumented activity*. These are captured in the range of subject_1-subject_2 instrumented interactions. Then, in tandem with this analysis, we leverage Vygotsky's (1978) notion of *intersubjectivity* to examine the interpsychological process by which the two subjects negotiate and co-construct a shared conceptual scheme for their instrumented activity through the verbal and nonverbal communications in their graphing activity. Gillespie and Cornish's (2010) notion of intersubjectivity adds some texture here by considering "the mutual awareness of agreement or disagreement" (p. 3) in addition to shared understandings and even misunderstandings. Thus, for the sake of our analysis, intersubjectivity manifests itself in participants' actions and intentionalities towards each other and in their joint movement toward negotiated and developed meanings and actions.

Methodology

In a design experiment (Cobb et al., 2003), researchers iteratively design, test, and revise both an instructional design and their theories about how that design supports student learning. The designs of the tool, task, and questioning presented in this paper emerged from the third iteration of an ongoing design experiment that seeks to produce a tool that can be used to help students make meaningful connections between height/time graphs and the kinds of real covariational situations that these graphs represent. We briefly describe the second iteration design in order to provide context for the third iteration presented in this paper.

In the second iteration, the design involved a video of simultaneous covariational motion and graph creation that students could view and explore. Specifically, a video of a vertically moving object (Figure 1, left) was played on a cell phone while the phone was slid horizontally from left to right (Figure 1, right). This action was then itself recorded to create a second video in which a height/time graph of the relationship between the original object's height and the trace of its motion could be seen. Students could then use familiar video playback controls to drag the video's progress bar back and forth and observe the graph being emergently traced by the object's motion. However, even when they were provided with an animated graphical overlay depicting the graph in the final version of the video, students still struggled to connect the horizontal movement to the vertical motion in the original video. Moreover, they were unable to imagine similar sliding videos they could rely on to create or interpret other height/time graphs.

Figure 1: Vertical Video Screenshots (left) and Sliding Video Screenshots (right)

In the current iteration of this design experiment, we sought to build on that earlier iteration to create a more grounded and embodied experience for the students than the sliding video seemed to offer. While the sliding video situated the covariational motion and the graph creation as simultaneous processes, it did not directly leverage students' own simultaneous embodiment of both of these processes. The framework of embodied design thus informed the third iteration, which yielded a design that includes a tool called the Graph Tracer (Figure 2, left) along with a set of tasks using that tool. The Graph Tracer is a 3D-printed rectangular prism with two cutouts: one on the bottom so that a paper can be slid beneath it, and one down through the top so that a marker can be inserted upright and moved up and down to draw on the paper below.

Figure 2: The Graph Tracer (left) and Collaborative Graph Tracer Use (right)

The Graph Tracer is intended to be used by a pair of students, one of whom controls the speed of the paper sliding beneath it while the other controls the up and down motion of the marker (Figure 2, right). One student thus embodies the quantity of time while the other embodies the quantity of height, and we hypothesize that through their collaborative intersubjective engagement, they both embody time and height as they work together to use the Graph Tracer. The result is the creation of a visual trace of the marker's motion over time, in other words, a height/time graph representing that motion.

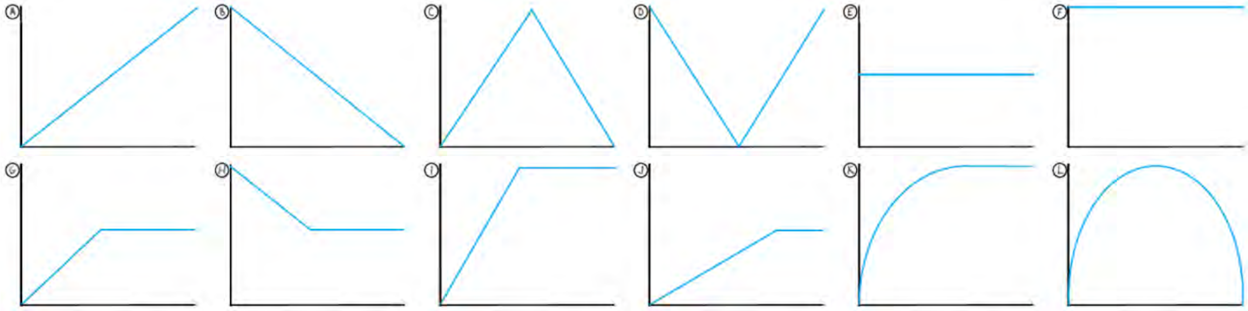

Figure 3: The Ordered Set of Traceable Graphs A-L

The task designed for use with the Graph Tracer includes a set of papers with an ordered group of preprinted graphs involving a variety of different slopes (Figure 3). A pair of students are asked to work together to use the Graph Tracer tool to trace each graph in order, reflect on their experience of doing so, and compare and contrast the different graphs. Examples of specific questions include “What did you have to do to make this graph?” and “Compare this graph to your previous graph, how is it similar or different?” The provided papers also include blank graphs, on which students can experiment with making their own traces.

As part of the design experiment, this design was tested in a task-based clinical interview (Clement, 2000; Ginsburg, 1981) with two students, “Eri” and “Robinson.” At the time of data collection, Eri was a twelve-year-old in the 7th grade who had some experience with graphing points in a coordinate plane but expressed that she was not familiar with graphing lines. Robinson, Eri’s younger brother, was a ten-year-old in the 4th grade who described having no graphing experience. The interview took place over Zoom with the designer/researcher (DR, first author) leading the interview and a partner researcher (PR, the students’ parent) in the room with the students. Collected data includes the video recording of the 45-minute interview, scanned copies of the students’ graphs, and the researchers’ field notes. Through a retrospective analysis (Cobb et al., 2003) of that data, we examined the students’ collaborative graphing activity using the Graph Tracer from both instrumented activity and covariational reasoning perspectives.

Findings

Here we present two excerpts from Eri and Robinson’s activity. The first exemplifies the intersubjective process through which the Graph Tracer became a collaborative instrument for them. The second shows how Eri leveraged her instrumented use of the Graph Tracer to revise her reasoning and apply it in a more sophisticated graphing scenario.

When they were first handed the Graph Tracer, neither Eri (Figure 4, left) nor Robinson (right) had ever seen it before or had any idea how to use it. Although we started the interview by explaining the goal of making a picture that represented height changing over time, we did not demonstrate the use of the Graph Tracer. The students thus began their explorations by enacting various ways they imagined the tool might be used, first tracing the *y*-axis of Graph A and then turning the Graph Tracer diagonally to trace the graph without moving the paper. At this point in their collaboratively instrumented activity, the students were interacting with each other and the Graph Tracer to develop a shared scheme for its use (Vérillon & Rabardel, 1995) but not yet with the object of the Graph A and its covariational meaning.

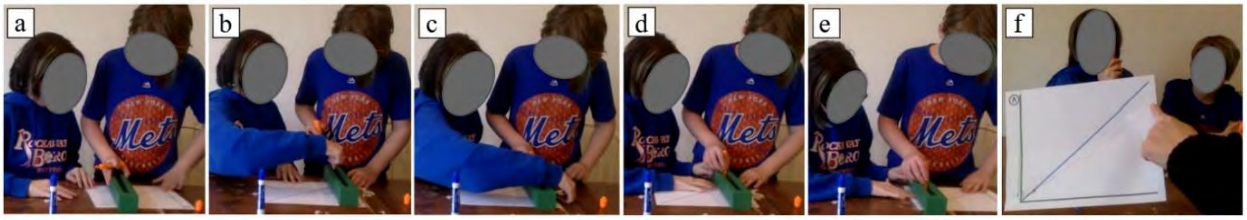

Figure 4: First Trace of Graph A

Once we clarified the constraint that the Graph Tracer must remain parallel to the *y*-axis, the students then started to explore how they could trace the graph by moving the paper and the marker simultaneously. Robinson first demonstrated how he thought the paper could be slid beneath the Graph Tracer (Figure 4a). Eri then used her finger to show the marker’s possible motion (Figure 4b) as she wondered, “What if you do [that slide of the paper] and you start moving [the marker] upward?” Figures 4c-e show how the pair attempted to implement their idea, first with Eri reaching over to pull the paper and then pushing from her side. They had reorganized themselves so that one person could hold the Graph Tracer while the other pulled the paper. Thus, the tool’s intended (Malafouris, 2013) utilization scheme (Verillon & Rabardel, 1995) and the associated task prompted the pair’s coordination of initially individual (i.e., uncoordinated movements of the pen and paper) and eventually collaborative instrumentation.

As they drew this first trace, the students began to interact with the graph as well as with each other and the Graph Tracer. In this attempt, Eri embodied the quantity of time by controlling the paper while Robinson simultaneously embodied height by controlling the marker. The pair was able to complete their trace of Graph A, but were not happy with their result (the orange line in Figure 4f; see also Figure 6). Eri had pulled the paper relatively quickly and Robinson remarked afterwards: “that was not traced.” We interpret their dissatisfaction with the result as though they struggled to negotiate a collaborative scheme to enact a smooth trace.

The pair then decided to try tracing Graph A again on the same page with a different color marker and a different tracing technique. Eri asked to switch roles so that she could control the marker as Robinson controlled the paper (Figure 5a). Robinson offered a suggestion in response: “You move up slowly while I pull the paper this way.” With this slower attempt and some help from the PR in steadying the paper (Figure 5b), they were better able to coordinate their actions to achieve the trace they wanted. In Figure 5c, the pair successfully completed a trace of Graph A using this new technique.

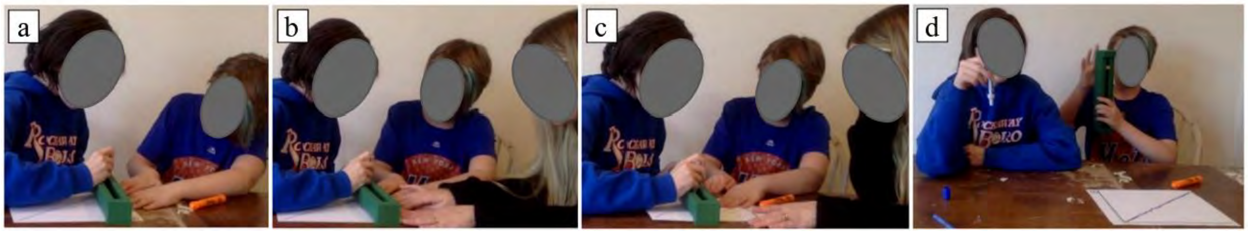

Figure 5: Second Trace of Graph A

Finally, the trace that emerged from this second attempt was placed on the table in front of the pair (Figure 5d; see also Figure 6) as they responded to the DR’s question: “How did you have to move the marker to get that picture?” Robinson enacted his answer nonverbally by holding the Graph Tracer up in the air in front of himself and moving his finger slowly upwards inside the marker track. Eri motioned similarly in the air with the marker she was still holding in her hand as she also answered out loud, “I just went like... very slowly upwards.”

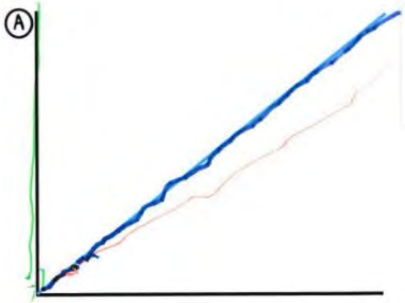

Figure 6: Eri and Robinson’s Three Graph A Traces:
The *y*-Axis (green), The First Attempt (orange), and The Second Attempt (dark blue)

The students' actions and reflections as they enacted their three traces of Graph A (Figure 6) show their process of collaborative instrumentation. As the pair negotiated how to use the Graph Tracer and attended to details of paper and marker speed in their tracing actions, the graph and Graph Tracer became an instrument with which they intersubjectively reasoned covariationally about height and time. Both their successful trace and their demonstration of the marker's motion as a change in height over time show how they had connected their enacted tracing of the graph with its covariational meaning. The completion of this trace and the students' articulations of their activity are evidence of the first major step the pair took towards enacting a shared understanding of how marker speed affects graph shape when using the Graph Tracer.

As the pair continued to enact traces of other graphs, they refined their collaborative scheme for creating smooth traces as they reflected on how their individual actions influenced their collaborative traces in different ways. For instance, when asked to compare and contrast Graphs G, I, and J by ordering them in increasing speeds of the marker motions required to trace them, Eri and Robinson initially disagreed. Robinson's intuition was that faster motion of the marker would make steeper graphs, which we interpret as an *emergent* (Moore & Thompson, 2015) view of the graph as a trace of the marker's motion. However, Eri based her reasoning on how long her subjective experience of drawing each graph had felt and the maximum height a graph reached, which we interpret as a *static* view of the graph based on its visual features.

The disagreement was resolved as they experimented by drawing three different lines at three different speeds on the same blank graph and then comparing their speeds and the steepness of those lines (Figure 7, left). This analysis convinced Eri to change her mind as she came to Robinson's way of thinking. We interpret that she did so by considering Robinson's movement of the paper in addition to the time she felt as she drew the graph. Thus, the slope of the graph seemed to have become a multiplicative object for Eri, composed of both embodied actions. As the episode continued and the pair began to develop collaborative schemes for creating and interpreting their own height/time graphs, Eri's initial static reasoning gave way to more emergent reasoning about the graph as a trace of the marker's motion over time.

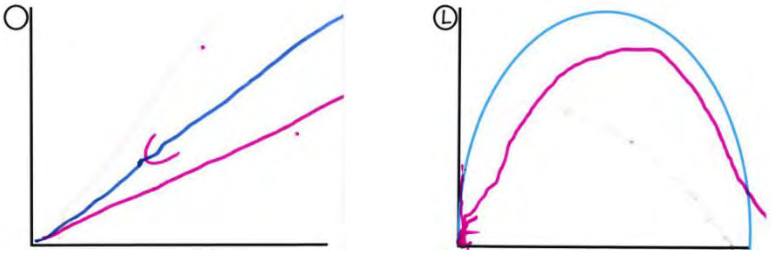

Figure 7: Eri's Marker Speed Experiment (left) and Eri's Trace of Graph L (right)

Although Robinson grew tired of the activity and eventually left the interview, Eri remained and took some time to consider Graph L. She used the Graph Tracer (with the PR's help) to attempt to trace the curve of Graph L and produced an imperfect but structurally similar result (Figure 7, right). She then expressed emergent shape thinking as she explained what she had done: She moved the marker the fastest at the ends of the curve and slowest along the top. Although Robinson's departure put an end to the pair's collaboratively instrumented activity, Eri's continued solo instrumentation via this activity seemed to support her new awareness of how the marker's speed impacted the shape of the resulting trace. This completed her process of enacting her new understanding of that relationship and showed further evidence of her emergent reasoning about the graphs she had created with the Graph Tracer.

Concluding Discussion

Using our analytic lens of collaborative instrumentation, the findings presented in this paper show how Eri and Robinson were able to move together from complete unfamiliarity with height/time graphs and the Graph Tracer tool towards a more fluent and collaborative stage in their instrumented use of the Graph Tracer and in the shared meanings they made of graphs given to or constructed by them. The Graph Tracer's requirement for simultaneous embodiment (e.g., Lakoff & Johnson, 2000; Núñez, et al., 1999; Wilson, 2002) of both the covariational situation and the creation of the corresponding graph also provided the students with a rich opportunity to reason about how these two acts are meaningfully related. Through their joint activities and their discussion of the relationship between marker speed and graph shape, both students worked together to enact their growing transformation of those artifacts into instruments they could use to reason emergently (Moore & Thompson, 2015) about height/time graphs. Eri also displayed a more sophisticated understanding of how marker speed influenced the steepness of the graph, thus beginning to interact with the idea of slope, an important graphing concept.

To continue this design experiment, we are interested in refining the design of both the Graph Tracer tool as well as the task, traceable graphs, and questioning used in this iteration. Eri and Robinson encountered physical difficulties in bracing the Graph Tracer and sliding the paper smoothly. A solution as simple as working on a smoother surface might mitigate the issue, but this solution places the burden on the user rather than the designer. Instead, the addition of a tray to brace the Graph Tracer and make the paper easier to slide could be considered. Alternately, it might be worth laminating the traceable graphs and using dry erase markers. Furthermore, modifications to the physical dimensions of the Graph Tracer like narrowing its width might improve students' ability to see their trace as they work, but this would need to be tested to determine the effects on durability and stability during use. Future iterations might also refine the questioning in order to elicit deeper and more elaborate descriptions of students' reasoning as they work. Possible directions that could benefit from deeper questioning include what students feel they are struggling with and why they choose certain actions as they use the Graph Tracer.

One limitation of the study is the messiness of the interview setting. The online setting presented barriers to conducting an effective interview, including difficulties seeing each other and the students' work as well as the challenge of two researchers conducting an interview together remotely. However, from our analysis, we conclude that the current design of the Graph Tracer has strong potential for use in the learning of covariation and graphing, thereby challenging the status quo of how these topics are traditionally taught. Future work might include refining the design of the Graph Tracer as well as gathering more data and performing further analysis to examine students' collaborative use of the Graph Tracer to reason about slope. We would also like to develop new tasks and gather new data to explore how students might use the Graph Tracer to reason about other graphing and function concepts such as rate of change, concavity, and points of inflection. Implementation of this design in a classroom setting could also support an investigation into how the Graph Tracer might be used as part of the regular middle school curriculum. Finally, it would be interesting to explore other designs that could also support the simultaneous embodiment of both covariation and covariational graphs and to further assess the analytic value of the collaborative instrumentation concept.

Acknowledgments

This material is based upon work supported by the National Science Foundation under Grant No. 1812887. We would also like to thank our colleagues Helene Leonard and Eileen Fernández for their help with this project.

References

Abrahamson, D. (2014). Building educational activities for understanding: An elaboration on the embodied-design framework and its epistemic grounds. *International Journal of Child-Computer Interaction, 2*(1), 1-16.

Abrahamson, D., & Lindgren, R. (2014). Embodiment and embodied design. In R. K. Sawyer (Ed.), *The Cambridge Handbook of the Learning Sciences* (2 ed., pp. 358-376). Cambridge University Press.

Carlson, M., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modeling dynamic events: A framework and a study. *Journal for Research in Mathematics Education, 33*(5), 352-378.

Cavanagh, M., & Mitchelmore, M. (2000). Student misconceptions in interpreting basic graphic calculator displays. In *PME CONFERENCE* (Vol. 2, pp. 2-161).

Clement, J. (2000). Analysis of Clinical Interviews: Foundations and Model Viability. In A. E. Kelly & R. Lesh (Eds.), *Handbook of Research Design in Mathematics and Science Education* (pp. 547-589). Lawrence Erlbaum Associates.

Cobb, P., Confrey, J., DiSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9-13.

Duijzer, C., Van den Heuvel-Panhuizen, M., Veldhuis, M., & Doorman, M. (2019). Supporting primary school students' reasoning about motion graphs through physical experiences. *ZDM, 51*(6), 899-913.

Gillespie, A., & Cornish, F. (2010). Intersubjectivity: Towards a dialogical analysis. *Journal for the Theory of Social Behaviour, 40*(1), 19-46.

Ginsburg, H. (1981). The Clinical Interview in Psychological Research on Mathematical Thinking: Aims, Rationales, Techniques. *For the Learning of Mathematics, v1 n3* p4-11 Mar 1981.

Lakoff, G., & Núñez, R. E. (2000). *Where Mathematics Comes From: How the embodied mind brings mathematics into being*. Basic Books.

Mevarech, Z. R., & Kramarsky, B. (1997). From verbal descriptions to graphic representations: Stability and change in students' alternative conceptions. *Educational Studies in Mathematics, 32*(3), 229-263.

McDermott, L. C., Rosenquist, M. L., & Van Zee, E. H. (1987). Student difficulties in connecting graphs and physics: Examples from kinematics. *American Journal of Physics, 55*(6), 503-513.

Moore, K. C., & Thompson, P. W. (2015). *Shape thinking and students' graphing activity*. Proceedings of the 18th Meeting of the MAA Special Interest Group on Research in Undergraduate Mathematics Education.

Núñez, R. E., Edwards, L. D., & Matos, J. F. (1999). Embodied cognition as grounding for situatedness and context in mathematics education. *Educational Studies in Mathematics*, *39*, 45-65.

Oehrtman, M., Carlson, M. P., & Thompson, P. W. (2008). Foundational reasoning abilities that promote coherence in students' function understanding. In M. P. Carlson & C. L. Rasmussen (Eds.), *Making the connection: Research and teaching in undergraduate mathematics education* (pp. 27-42). Washington, D.C.: Mathematical Association of America.

Paoletti, T., Lee, H. Y., & Hardison, H. L. (2018). *Static and emergent thinking in spatial and quantitative coordinate systems.* Proceedings of the 40th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.

Stevens, R. (2011). The Missing Bodies of Mathematical Thinking and Learning Have Been Found. *Journal of the Learning Sciences, 21*(2), 337-346.

Stevens, I. E., Paoletti, T., Moore, K. C., Liang, B., & Hardison, H. (2017). Principles for Designing Tasks that Promote Covariational Reasoning. In *Proceedings of the 20th Annual Conference on Research in Undergraduate Mathematics Education*.

Thompson, P. W. (1993). Quantitative reasoning, complexity, and additive structures. *Educational Studies in Mathematics, 25*, 165–208.

Thompson, P. W. (2011). Quantitative reasoning and mathematical modeling. In L. L. Hatfield, S. Chamberlain & S. Belbase (Eds.), *New perspectives and directions for collaborative research in mathematics education. WISDOMe Monographs* (Vol. 1, pp. 33–57). Laramie, WY: University of Wyoming.

Thompson, P. W., & Carlson, M. P. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. In J. Cai (Ed.), *Compendium for Research in Mathematics Education* (pp. 421-456).

Wilson, M. (2002). Six views of embodied cognition. *Psychonomic Bulletin & Review, 9*(4), 625-636.