AESTHETIC DIMENSIONS OF STUDENT MATHEMATICAL CREATIVITY

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There is a popular myth that mathematical creativity is a trait possessed by a small group of people. This is perpetuated by the lack of understanding of what influences mathematical creativity, especially in the classroom. In particular, professional mathematicians often name aesthetic experiences as a critical aspect of their mathematical creativity. Therefore, this study examines aesthetic dimensions of episodes of student creativity from the high school classroom. Several types of aesthetic experiences arose across the episodes. Students were motivated to take creative action by experiences of mystery or discomfort. Furthermore, in some instances, taking a creative action was accompanied and propelled by a sense of craziness or satisfaction.

Keywords: Affect, Emotion, Beliefs, and Attitudes; High School Education

Students must have the opportunity to be creative during mathematics class. Otherwise, they may learn that creating mathematics is for others to do; their only role is to reproduce. This is problematic because creating mathematical ideas is often necessary for practical purposes in life (e.g., crafting a personal finance plan) and can be a source of pleasure and meaning (e.g., making music or noticing patterns) (Su & Jackson, 2020). If students learn that their role with respect to mathematics is to use mathematical ideas or tools made by others, they may not be empowered to take ownership of aspects of their life with prominent mathematical components. This can be dehumanizing and could have devastating impacts (Gutiérrez, 2018).

And yet, it often seems that many students are not capable of being creative during math instruction. Some previous studies of student mathematical creativity have even been unable to detect any student creativity within the context of classroom learning (Clack, 2017; Mhlolo, 2016). I argue, though, that being mathematically creative is not an inborn trait possessed by a limited group of people, but instead a type of action that is influenced by personal, social, and cultural factors. Factors such as these have been explored in creativity in other fields (Csikszentmihalyi, 2014; Hanson, 2015), but are understudied in the case of mathematical creativity. This perpetuates the harmful myth that there are some people who can create new mathematical ideas, and others who cannot.

In mathematics, one factor that has been named by some professionals in the field as impactful in their creative process has been aesthetic experiences (Burton, 1998; Sriraman, 2004). What about students? In this paper, I examine aesthetic experiences and their role in student mathematical creativity by analyzing enacted lessons in high school mathematics classrooms. What roles can aesthetic experiences play in student mathematical creativity?

Conceptual Frameworks

In this study, I assert that mathematics is human-made and fallible (Ernest, 1998; Lakatos, 1976). Thus, human action can create new ways of doing mathematics. Creativity is the act of creating ways of doing or thinking about mathematics that were not previously possible (Riling, 2020). An action with creative mathematical potential is one that an individual takes of their own volition, rather than because of following an instruction or norm (Pickering, 1995; Sinclair, 2005). The action realizes its creative potential if it goes on to enable members of the mathematical community in which it occurs to think about or do mathematics in new ways.

Whether or not an action fulfills its creative potential is not solely based on the merits of the action, but various personal, social, and cultural forces that govern the community of mathematicians.

Because mathematics is a human endeavor, the relationship between a person and mathematics is a crucial aspect governing how mathematics develops, which I conceptualize here through *aesthetics*. Aesthetic experiences can function in multiple ways in the doing of mathematics. They may motivate a person to take some action, or they may occur as a result of a person taking an action (Brinkmann & Sriraman, 2009; Sinclair, 2004). Here I describe three forms of aesthetic experience that may play a role in student mathematical creativity.

First, aesthetic experiences can motivate the exploration of mathematics. Sinclair (2004) explains that as there are few practical reasons to pursue a great deal of the field of advanced mathematics, much of the motivation for doing so is derived from aesthetic aspects of the field. Mathematicians may become "hooked on" (p. 275) the order that can arise from completing a mathematical investigation or proof, or become intrigued by a paradox or moment of surprise. As these types of experiences have been shown to motivate mathematical activity in general, they may specially motivate mathematical action with creative potential. Thus, for the purposes of this study, a *motivating aesthetic experience* is one that prompts students to try to create new mathematical possibilities.

Sinclair (2004) also describes how an aesthetic experience can play a *generative* role in mathematics, in that people may be struck by a sense that mathematical objects or events that arise in the process of doing mathematics that they sense will be valuable or provide insight, compelling them to investigate further. It is possible that while taking an action that has the potential to create mathematics, a student might experience pleasure or excitement that generates further curiosity.

Finally, Sinclair (2004) describes the role of an aesthetic experience in considering a mathematical object that has been produced already; an *evaluative aesthetic experience*. Beauty is a particularly important form of evaluative aesthetic for many professional mathematicians (Brinkmann & Sriraman, 2009; Sinclair, 2004). This type of aesthetic experience may happen as a student regards their own work, or the work of another, and may play a part in what happens after a student has taken an action with creative potential.

Methods

To learn about the role of aesthetic experiences in student mathematical creativity, I examined enactments of secondary mathematics lessons. The lessons were designed to be aesthetically captivating for students. The lessons were analyzed by identifying instances of student action with creative potential and student aesthetic experiences, and by crafting narrative episodes of these actions and experiences, which were then examined for common themes.

Participants and Lesson Selection

The lessons in this study were enacted in high school classrooms in the northeastern region of the US, taught by teachers with 4-6 years of experience in the classroom. The teachers taught at three schools that varied in size, school type, and racial demographics (see Table 1). Each teacher taught three specially-designed lessons. These lessons were designed by small groups, including the classroom teacher, another colleague participating in the study, and university-based researchers. The topics of these lessons fell within the curricula of each course. These lessons were unique in that they were designed by teachers and researchers to provide the opportunity for aesthetic experiences. This was done by way of conceptualizing lessons as mathematical stories (Dietiker, 2015), meaning that the teachers thought of the mathematical context as a setting for the development or action of mathematical characters (e.g., numbers or geometric objects). Some teachers aimed to design stories with compelling narrative devices, such as a surprising plot twist or an air of suspense. This data set was useful for studying aesthetic dimensions of student creativity because a set of lessons designed without attention to students' aesthetic experience could fail to provide any detectable displays of student aesthetic experiences, or any variety in student aesthetic experiences.

Table 1: Participating High Schools and Students

High School	*Cooperating teachers*	*Number of participating students*	*Students' racial identification (categories >15%)*	*Mean disposition to math (1-4)*
Forte Charter Secondary (Forte)	Mr. Davis and Ms. Bacheldor	85	79% - Latinx**	2.54
Gladbury High School (GHS)	Mr. Anderson and Ms. Evans	56	58% - White**	2.73
Motion High School (MHS)	Ms. Curran and Ms. Fontaine	70	50% - Asian** 16% - Latinx**	2.81

**Based on student responses to survey items.*
***Based on demographic data reported by the state; demographics below 15% not represented in this table.*

Each specially-designed lesson was observed and audio- and video-recorded from multiple perspectives in the classroom. Some were enacted and observed for multiple classes of students, if a teacher taught multiple sections of the same course. After the lessons, all participating students completed a Lesson Experience Survey (LES). This survey included a prompt to select three descriptors to describe their experience of the lesson, out of a bank of sixteen terms. Some terms were positive (e.g., "intriguing"), some were neutral (e.g., "fine"), and others were negative (e.g., "boring"). In addition, two or three students were selected to be interviewed after each lesson. Students were typically approached about being interviewed because of some strong affective display during the lesson. For every specially-designed lesson that was filmed, the research team also observed and recorded a typical lesson in the same class of students within approximately one week, following the same observation protocols.

In order to select a group of lessons that were more likely to contain student actions with creative potential, I identified the lessons that were more frequently described on the LES with a group of target descriptors that I theorized could be selected by students to describe experiences that involved creativity (i.e., *thought-provoking*, *amazing*, *fascinating*, *fun*, *surprising*, *intriguing*, and *enjoyable*). I identified each teacher's lessons for which the highest percentage of students selected the target descriptors. Upon beginning to analyze the lessons, I began with one lesson per teacher. I iteratively added additional lessons, aiming to maintain variety in teachers and courses, until I had identified a robust set of student actions with creative potential. All lessons that were included were ones that the teachers had designed to be aesthetically engaging (see Table 2). Each lesson was transcribed, including full-group discussions and the discussions that occurred in a small group that had been filmed from an additional, close-up camera.

Table 2: Lessons Analyzed

School	*Teacher*	*Course*	*Lesson*	*High frequency descriptors*
Forte	Ms. Bacheldor	Honors Algebra 2	Extraneous solutions	Thought-provoking
Forte	Mr. Davis	AP Calculus	Derivatives of exponential functions	Intriguing, Enjoyable
Forte	Mr. Davis	AP Calculus	Solids of revolution	Fun, Enjoyable
GHS	Mr. Anderson	Algebra 2	Imaginary numbers	Thought-provoking, Surprising
GHS	Ms. Evans	Algebra 2	Introduction to inverses	Amazing, Fascinating
GHS	Ms. Evans	Algebra 2	Introduction to inverses	Thought-provoking, Enjoyable
MHS	Ms. Curran	Integrated Math 1	Geometric transformations	Amazing, Enjoyable
MHS	Ms. Curran	Integrated Math 1	Linear functions	Surprising, Fun, Enjoyable
MHS	Ms. Fontaine	Honors Integrated Math 3	Rational root theorem	Fascinating

Identifying and Categorizing Creative Actions

I next scanned the selected lessons for actions with creative potential. By creative potential, I mean actions that *could* lead to the creation of new mathematics. These are those actions that students have taken using their own agency, rather than following disciplinary or classroom norms (Pickering, 1995). In order to broaden my perspective, and thereby increase my ability to detect student creative potential, I consulted first-hand accounts of professional creators. To avoid reinforcing the narrative of mathematics' white, male history, I curated a set of professional creators who varied in medium, gender, country of origin, race and ethnicity, and age. The set of creators included individuals such as Chinese architect and conceptual artist Ai Weiwei, Black American film-maker Ava DuVernay, French-Cuban musicians Lisa-Kaindé Diaz and Naomi Diaz, of Ibeyi, and white French mathematician Sophie Germain. After learning about their creative work, I revisited the classroom data and indeed was able to identify additional student actions with creative potential.

I categorized each action with creative potential based on how the actions opened new possibility for students in the lessons (Riling, 2022). For example, *setting out* refers to instances in which students decide to embark on some kind of creative work. *Imagining* refers to the act of mentally projecting a new version of the mathematical reality. *Recognizing* involves reinterpreting some existing mathematical object in a new way. *Manifesting* is the act of taking a action that immediately changes the current mathematical context.

Identifying and Explaining Aesthetic Experiences

In order to identify any aesthetic experiences that could have had a role in students' actions with creative potential and how they developed in the classroom, I asked the following questions of the lesson data: (1) What emotions do individuals display in reference to their mathematical context? (2) Do the individuals explicitly relate their response to mathematical ideas, or do emotional displays coincide with changes in their understanding of the mathematical context? (3) What kind of aesthetic experience do the individuals appear to be having? I marked displays of emotion such as changes in vocal pitch or tone (e.g., singing or shouting), use of exclamatory language (e.g., "oh my god" or "boom"), use of explicitly aesthetic or emotional language (e.g., "mystery" or "hate"), and changes in physical behavior (e.g., smiling or dancing). I recorded my initial understanding of the type of aesthetic experience that students appeared to be having (e.g., surprise or satisfaction).

Next, I worked to establish the roles of the aesthetic factors (i.e., motivating, generative, or evaluative). I did this in part by considering when the individuals displayed evidence of having an aesthetic experience, in comparison to when the action with creative potential was taken. That is, a motivating aesthetic experience would happen before the action, a generative one would be concurrent, and an evaluative one after. In addition, in order for an aesthetic experience to be considered to have motivated an action with creative potential, there would have to be evidence that the action taken was related to the context that provoked the aesthetic response. For generative aesthetic experiences, students either would explicitly connect their affect to a mathematical object or event, or express emotion at the same time as taking an action, in the absence of any other change that could explain their expression. To be considered to be evaluative, an aesthetic expression would have to occur as the expressing student attended to a mathematical object related to the action with creative potential under consideration.

In addition to the audio and video recordings, in order to ascertain the type and role of students' aesthetic experiences, I consulted post-lesson interviews. For eight of the fourteen episodes, the student who took the action with creative potential was interviewed after the lesson. In some cases, they spoke explicitly about the action and the feelings they had related to it. For example, one student shared during their interview, "when I saw like the coefficients, with the pattern of the roots, I thought that maybe like, it's kinda crazy but maybe the coefficients are related to the polynomial." Here, the student explains how his feeling of something being "crazy" related to his actions.

I identified the episodes that seemed to contain similar dimensions within students' aesthetic experiences (e.g., all episodes that contained mysterious elements) and used memo-ing to explore the similarities in how those experiences functioned in different episodes. For example, I asked myself, "What is the role of satisfaction across these examples?" and "When does satisfaction happen in relation to creative actions?" After answering these questions for individual episodes, I constructed a model of how that type of aesthetic experience functioned with respect to actions with creative potential and to other types of aesthetic experiences. This enabled me to identify and define common dimensions in students' aesthetic experiences.

In the process of analyzing student language in relation to aesthetic experiences, I noticed that students used terms that evoked the existence of some alternative to their current accepted reality (e.g., conspiracy theories, drug use). Therefore, I examined the ways in which students described aesthetic experiences with mathematics related to the existence of a new mathematical world of some kind. This interpretation was overwhelmingly consistent with students' descriptions. Therefore, I defined the common dimensions of aesthetic experiences that I found in the episodes in terms of how they related to students' perception of the existing and/or a new mathematical world.

Findings

Within the data, when students took actions with creative potential, they sometimes seemed to be motivated by two different types of aesthetic experiences: some kind of mysterious uncertainty, or discomfort. Additionally, some actions with creative potential were accompanied by a break in mathematical reality that students referred to as "crazy," or a sense of satisfaction, which played a generative role as it spurred continued investigation, or an evaluative role as students concluded their work exploring an action with creative potential. In this section, I define *mystery*, *craziness*, *discomfort*, and *satisfaction* in terms of students' aesthetic experience with mathematics and describe how students' actions with creative potential were influenced or marked by aesthetic experiences.

Mystery

One dimension of an aesthetic experience remarked upon by students in the data was *mystery*. Mystery appeared in some instances of *imagining*, one type of action with creative potential. In several cases, students indicated feeling that their previous assumptions about the mathematical context were incorrect or incomplete in some way. This happened when students encountered something that did not make sense, such as finding that they need to take the square root of negative numbers in order to solve a problem, when they had previously been told they could not do so. After encountering such a clue, students began to doubt the general premises of the mathematical realm in which they were operating. A sense of uncertainty is inherent in mystery. For example, in the case of a group of students in a scenario that suggested that they should take the square root of negative numbers, when their teacher asked if they believed that was feasible, several students responded "I don't know." In other instances in which students sensed a mystery, they used phrases like "maybe" and "I think," conveying this lack of certainty.

Another common feature of students' references to mystery is that they referred to it broadly. For example, in one lesson, a student said "we have a situation," indicating that, in general, something about the mathematical context was amiss. She did not, however, specify exactly what the potential problem was. Another student was even less detailed, intoning "dun dun dun." This suggests that a sense of mystery might be something that students do not affix to a specific object in their mathematical context, but to the mathematical context more broadly.

Discomfort

In the lessons, students experienced various forms of discomfort as a result of the current mathematical task. This discomfort often seemed to motivate students to take actions with creative potential. Although students also can have negative emotions about other things during mathematics class, such as peer relationships or grades, I am not referring to those experiences here. One form of discomfort in the data was frustration. Frustration often occurred when students were unhappy that their attempts to solve a problem were unsuccessful. It should be noted that other students could be excited to have a problem that they could not solve quickly. To qualify as discomfort, students would have to not enjoy these experiences. Students typically demonstrated discomfort by using qualitatively negative language about mathematics. For example, "I hate *e*," or "Forty-six? Yikes." At other times, they might not verbally refer to a mathematical concept, but instead have a negative outburst (e.g., "Jesus Christ!" or "This shit's confusing") while working on a mathematical task. Another form of discomfort in the data was tedium, in which students were bored by work that they perceived as too repetitive.

In several cases, students seemed to engage in manifesting (a type of action with creative potential) in an attempt to remove discomfort. This bears something in common to students attempting to imagine their way out of a mystery. However, by manifesting, students did not return to the status quo, as the students attempted to do through imagining, but instead took an action that created a new mathematical reality. Consider that in several of these instances, the students verbally wrestled with whether or not to manifest when experiencing a state of discomfort. For example, one student who was struggling to solve an equation expressed that he was uncertain about whether he could square the equation. He asked, "Why can't we?" and then finally rejecting this felt external pressure to not square, stating, "I don't care" as he squared the equation. The evidence that students often manifested in spite of not being sure that it was allowed suggests that these students were aware that they were changing something about the mathematical context. In this way, they acted more like the students who moved toward a new mathematical reality when they experienced something crazy that they chose to explore further.

Craziness

Craziness is related to mystery in that it also indicates that students have come to doubt their current realm of known mathematics. However, it differs in that rather than only sensing that an alternate mathematical reality might exist, students are in the act of crossing over into the new realm. They are compelled to continue doing so, meaning that this aesthetic experience is generative. For example, consider a student in a lesson in which a teacher asked students to group cards that contained representations of linear functions. This student and his groupmates had begun to form pairs of cards that they believed contained representations of the same functions. When the student found out that he could make groups of more than two linear functions, he exclaimed, "more! That's crazy! There's more!" Since students who sense that something is crazy are crossing into the new reality, they can typically point to a specific mathematical object or event that provoked the feeling.

When students described a crazy aesthetic experience, they often spoke loudly in excited tones. They sometimes used language related to conspiracy theories ("it's Illuminati confirmed, bro") or drug use ("that's trippy"). These are both experiences in which an alternate version of reality is suddenly revealed, by way of learning secret information or by consuming drugs. However, the most common vocabulary that students used when expressing that they are experiencing this aesthetic is the word "crazy." It is because of the extreme frequency of this word choice that I use "crazy" to label this type of aesthetic experiences, even though it is a word I typically avoid due to its ableist connotations. To date, I have not encountered an appropriate synonym that expresses both the perceived disruption in students' understanding of the mathematical context, and the sudden onset of feelings that accompanies this shift.

Several students who took the action with creative potential of setting out seemed to be motivated by a sense that something crazy had occurred. For example, the student who found out that he could make groups of more than two linear functions then set out to learn more about individual functions in order to be able to form groups. Additionally, the student in the quadrilateral midpoint lesson set out to learn about parallelograms only after finding a parallelogram in a place that they did not expect it to appear. The feeling of craziness prompted these students to want to learn more about the possible new mathematical context. This is different from the way in which mystery functioned in the cases of *imagining*. In those cases, students responded to a prevailing sense of mystery by attempting to avoid the mystery. That is, they did not want to 'uncover the truth' and potentially create a new reality; they wanted to maintain the status quo.

Satisfaction

Evidence of the sensation of *satisfaction* also appeared in the data, accompanying several instances of recognizing. When students recognized something, be it a pattern or a familiar mathematical structure, they sometimes gasped or exclaimed a phrase such as “oh my god,” seemingly involuntarily. In some instances, the students also loudly called out to their classmates. Gaining a new perspective through which to understand the current mathematical context seemed to cause pleasure for these students. These experiences seemed to be generative, in that they compelled students to continue to investigate the implications of the action.

In some ways, satisfaction bears resemblance to the craziness that some students expressed. Both types of aesthetic experiences involved students suddenly recognizing shifts in what they knew to be the current mathematical reality. Students even used some similar language, such as “oh my god.” The difference is that, whereas the students who experienced things as being crazy were suddenly aware that a different mathematical world existed, and that the one they knew previously might even cease to exist, students who experienced satisfaction instead found evidence of being in a familiar context. For example, consider a student who recognized that an equation was in a form could be factored. Upon sensing a familiar structure, the student expressed happiness through gasps and singing. A further difference is that, whereas satisfaction is virtually always a positive feeling, the sensation of being jolted into a new reality by something that seems crazy might be enjoyable for some students, but not all, in the same way that some people enjoy the thrill of roller coasters and others do not.

Discussion

Aesthetic experiences played prominent roles in students’ creative episodes in these lesson enactments, suggesting a need for lessons to be designed with dynamic aesthetic experiences. In many cases, students seemed to be deeply impacted by their aesthetic experiences, as indicated by behaviors ranging from gasping to singing. Maybe this is why it often seems that many individuals are not capable of being creative; they have not been able to experience mathematics in a way that is conducive to mathematical creativity. I suspect that many mathematics lessons differ drastically from the dynamic enactments analyzed here, considering that mathematics is so often described as dull or boring by those who do not become professional academic mathematicians. How many more students could be moved to take action with creative potential if they were to experience suspense or mystery during their mathematics lessons?

Of course, lessons with more varied aesthetic experiences and opportunities for student creativity may make new demands of teachers, in terms of managing both behavior and mathematical ideas. The student behavior described in this study does not match the image of quiet students working diligently at their desks. It involved sudden outbursts, minor references to drugs, even the use of curse words. What new skills would teachers need to facilitate lessons in which these behaviors were not only allowed, but maybe even encouraged? This adds difficulty to the increased demands of teaching for student creativity, in which students have more control over the mathematical content of class.

Finally, these aesthetic experiences were studied within a fairly unique set of lessons, in that they were designed to be aesthetically captivating, but the lessons still aligned with standard secondary mathematics curriculum in the US. This raises the question: What other kinds of creativity might students engage in if they were to have even more diverse aesthetic experiences, such as those that integrate expressive arts or social justice topics into mathematics instruction? Future research is needed to learn what other mathematical creativity students may be capable of.

Acknowledgments

This material is based upon work supported by the National Science Foundation under Grant No. 1652513.

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