SELF-EFFICACY, INSTRUCTIONAL BELIEFS AND THE USE OF THE STANDARDS FOR MATHEMATICAL PRACTICE

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Teachers' beliefs, knowledge, and decisions can affect the way teachers teach, and, consequently, what students learn. Self-efficacy beliefs may also interact with beliefs about the most appropriate and effective teaching and the selection of instructional practices to implement. We examined the relationships among teachers' mathematics self-efficacy, mathematics teaching self-efficacy, instructional beliefs, and use of the Standards for Mathematical Practices (SMP; NGACBP & CCSSO, 2010) for teachers who had been identified as effective teachers. We found that although the teachers scored similarly on beliefs surveys, there were some differences in their use of the SMP (NGACBP & CCSSO, 2010).

Keywords: Standards, Teacher Beliefs, Instructional Activities and Practices

Affective factors such as self-efficacy, motivation, anxiety, and confidence can be related to learning and teaching processes. Self-efficacy beliefs—beliefs about one's capacity to be successful in various situations—are situation-specific. Teachers have self-efficacy beliefs about mathematics, teaching, and mathematics teaching. These beliefs are important because they may interact with beliefs about the effective teaching practices (McLeod, 1987; Opera & Stonewater, 1987; Raymond, 1997) and the implementation of instructional practices (Peterson et al., 1989).

Mathematics teaching practices are often categorized as either student-centered (e.g., reform oriented, learner-centered) or teacher-centered (e.g., traditional). The use of student-centered teaching practices has been shown to relate positively to student learning in mathematics, as measured by increased post-test scores on a curriculum-based, researcher-constructed classroom test (Jong et al., 2010). Student-centered practices may also decrease negative affective factors, such as mathematics anxiety, in students (Alsup, 2004). There is evidence that teachers' beliefs concerning the use of student-centered practices and teacher self-efficacies influence teachers' decisions about what teaching practices to enact when teaching mathematics (Hadley & Dorward, 2011; Peterson et al., 1989). However, this evidence is largely based on self-reported teaching practices. Thus, it is not clear how self-efficacies may relate to the enacted teaching practices of teachers, as verified by an outside observer. Merriam and Tisdell (2016) noted that observation in the form of "firsthand encounters with the phenomenon" is more reliable than "a secondhand account" by teachers, who are sharing an interpretation of their own teaching practices (p. 137).

Further, the role that teachers' instructional beliefs play in mediating self-efficacies as part of teachers' decision-making processes about which instructional strategies to implement is not clearly understood (Allinder, 1994; Bandura & Wood, 1989; Klassen & Tze, 2014). And, there has been no exploration of the extent to which teachers who have been identified as effective may vary with respect to their beliefs and practices. Teachers, even effective teachers, are not monolithic. Knowing more about the breadth of ways to be effective would add nuance to our understanding of what it means to be effective. For these reasons, we explored this research question: How do effective teachers vary with respect to their instructional beliefs, mathematics self-efficacy, mathematics teaching self-efficacy, and classroom-observed uses of the Standards for Mathematical Practice (SMP; National Governors Association Center for Best Practices & Council of Chief State School Officers [NGACBP & CCSSO], 2010)?

Theoretical Perspectives

Ernest (1994) posited that an individual's learning cannot be separated from their social environment. In conversation with others and themselves (Ernest, 1994), students negotiate ideas in a way that expands their prior knowledge. Marchitello & Wilhelm (2014) reported that the mathematics component of the Common Core was based on cognitive principles and, in particular, the authors assumed that students learn by building on prior knowledge and working collaboratively with peers. In this way, the SMP are aligned with social constructivist perspectives on learning. Within a constructivist learning environment, teachers and students share knowledge, responsibility, and authority (Chung, 1991). A teacher's role is as a facilitator, rather than an expert, who creates and sustains a collaborative learning environment (Tam, 2000). Thus, social constructivism undergirds the student-centered teaching practices that we examined.

The theory of self-efficacy (Bandura, 1977), also aligned with social constructivism (Bandura, 2001), is a lens through which we interpreted how teachers' self-efficacies mediated their instructional practices and beliefs. A firm understanding of self-efficacy, not only as an affective factor of teachers, but also as a theory, aided in the interpretation of teachers' instructional beliefs and practices. According to Bandura (1997), an individual has the capacity to control their actions based on how confidently they exercise that control. Bandura (1997) described a phenomenon in which individuals make choices based on how successfully they believe they can perform a particular task. Thus, mathematics self-efficacy may influence the decision to use challenging tasks and mathematics teaching self-efficacy may influence a teacher's decision to engage in student-centered teaching practices that require coping with uncertainty about how students will respond when given the freedom to explore.

Literature Review

Those who have examined how the interplay between beliefs and practices can affect student learning (e.g., Allinder, 1994; Polly et al., 2013) have found that teachers' beliefs about teaching practices may be tied to other affective factors, including self-efficacy beliefs.

Self-Efficacy Beliefs of Teachers

Self-efficacy beliefs have the potential to influence teachers' choices about which instructional practices to use (Bates et al., 2013; Perera & John, 2020; Swars, 2005). In addition, self-efficacy beliefs potentially mediate teachers' instructional beliefs as they choose whether to use student- or teacher-centered practices (Wilkins, 2008). Such self-efficacy beliefs are based on a person's interpretation of past experiences (e.g., Bandura, 1986, 1997; Tschannen-Moran et al., 1998; Wilkins, 2008), are constructed from four sources (i.e., verbal persuasion, vicarious experiences, physiological arousal, and mastery experiences; see Bandura, 1986), and are situation specific (Pajares & Miller, 1994). People with high self-efficacy beliefs are more likely to: (a) attempt tasks they might find challenging; (b) expend more initial effort on successful completion of the task; and (c) persist when a task becomes difficult (Bandura, 1986).

Mathematics self-efficacy. Kahle (2008) defined mathematics self-efficacy—a situation-specific self-efficacy—as one's beliefs in their capability to successfully carry out a mathematical task. Using the Mathematics Confidence Scale (Dowling, 1978), Pajares and Miller (1984) found significant and positive correlations among gender, high school mathematics experience, college mathematics experience, and mathematics self-efficacy. Thus, they concluded, mathematics self-efficacy affects mathematics performance directly rather than through mediated variables (e.g., gender, prior experiences).

Mathematics teaching self-efficacy. Another type of self-efficacy, mathematics teaching self-efficacy, has been defined as the belief in one's capability to teach others mathematics (Enochs et al., 2000; Kahle, 2008; Swars, 2005). Teaching self-efficacy influences choices teachers make in teaching practices, curriculum delivery, and task choice (e.g., Gulistan et al., 2017). More specifically, high teaching self-efficacy has been linked to teachers' (a) openness to student responses and inquiry, including student engagement (e.g., Toropova et al., 2019); (b) positive responses to academic coaching (Ross, 1992); (c) high-quality instruction (Perera & John, 2020; Wilkins, 2008); and (d) positive effects on student attitudes toward mathematics (Nurlu, 2015).

Teaching Practices in Mathematics

Skemp (1978) distinguished between two types of student learning goals for teachers. Relational understanding is "knowing both what to do and why" (p. 9) whereas instrumental understanding refers to using "rules without reason" (p. 9). Student-centered practices align with relational thinking because student-centered practices are specifically designed to help students construct knowledge by having them grapple with both what procedures to use and why they work. Students who have experienced mathematics through student-centered practices viewed mathematics as creative and flexible, leading them to see a future in which they would use mathematics in their everyday lives (Boaler, 1997)

Teacher-centered practices align with instrumental understanding because the teacher is viewed as one who possesses knowledge and who should impart that knowledge to passive learners. These learners, in turn, are responsible for emulating the procedures that the teacher has demonstrated without necessarily reflecting on the reasons undergirding those procedures. For this study, because effectively implemented student-centered practices are aligned with social constructivist assumptions about learning and regarded as appropriate for the teaching and learning of mathematics (National Council of Teachers of Mathematics [NCTM], 2000, 2014; NGACBP & CCSSO, 2010; NRC, 2001; Smith & Stein, 2018), we equated effective student-centered practices with effective mathematical teaching practices.

Teachers' Mathematics Instructional Beliefs

Wilkins (2008) posited that instructional beliefs are mediated by teachers' attitudes about mathematics and mathematics teaching, and those instructional beliefs, in turn, influence instructional practices. O'Hanlon et al. (2015) described teachers' instructional beliefs—views teachers hold about the best teaching practices—as a complex system that integrates teachers' beliefs concerning the nature of mathematics with their beliefs about the relationship between the teaching and learning of mathematics. Woolley et al. (2004) classified teachers' instructional beliefs as aligned with either constructivist or traditionalist (i.e., behaviorist) approaches to learning. Yet researchers have found that such instructional beliefs sometimes conflict with enacted teaching practices (Peterson et al., 1989; Raymond, 1997; Yurekli et al., 2020).

We examined the interplay of these factors: teachers' self-efficacy beliefs, teachers' mathematics instructional beliefs, and teachers' implementations of the SMP.

Methods

Participants

For this study, we focused on two mathematics teachers who were identified as effective by both mathematics teacher educators and their school principals. Kathy and Frances (pseudonyms) both taught at the same suburban school in the midwestern United States. Kathy taught kindergarten and Frances taught Grade 5/6 mathematics.

Data Collection

To determine levels of self-efficacies, instructional beliefs, and use of effective teaching practices, we used the instruments and procedures described in this section.

Self-efficacy beliefs. To measure both mathematics and mathematics teaching self-efficacy, we used Kahle's (2008) Mathematics Teaching and Mathematics Self-Efficacy (MTMSE) survey. The MTMSE has a measured reliability of $\alpha = .942$ and produced positive results for both face and content validity. The MTMSE originally was intended for teachers who taught grade 3–6, but Kahle (personal communication, June 8, 2020) verified that the survey is suitable for elementary and middle school teachers.

We assessed mathematics teaching self-efficacy beliefs with Part 2 and Part 4 of the MTMSE survey. In Part 2 of the MTMSE teachers rated, using a Likert scale of 1 (strongly disagree) to 6 (strongly agree), their agreement with statements such as "I will generally teach mathematics ineffectively" or "I will typically be able to answer students' questions." In Part 4 of the survey, teachers rated (on a scale of 1–6, from low to high) how confident they felt about teaching specific content (e.g., fractions, decimals, shapes).

We measured mathematics self-efficacy using Part 1 and Part 3 of the MTMSE survey. These components focus on teachers' beliefs about their capabilities regarding a variety of mathematical tasks. In the survey, teachers are asked to rate their level of confidence, on a scale from 1–6, in their own ability to complete certain tasks, though they are not required to complete the tasks. One of the tasks was: "On a map, 7/8 inch represents 200 miles. How far apart are two towns whose distance apart on the map is 3 1/2 inches?"

During pre-lesson interviews, the first author asked teachers to rate their confidence about the content and the teaching of the content.

Instructional beliefs. We used an instructional beliefs survey that incorporated items from O'Hanlon et al.'s (2015) Teaching and Learning Mathematics Beliefs survey to determine whether teachers' instructional beliefs were primarily aligned with student-centered or teacher-centered instructional practices. The beliefs survey is applicable to all the teachers of Grades K–12 as the items were designed to elicit teachers' instructional beliefs about the content they teach at their own grade level. In developing the survey, O'Hanlon et al. designed items to reflect NCTM's (2000) process standards recommendations, which are aligned with the values described in the CCSS-M standards (NGA & CCSSO, 2010). O'Hanlon et al.'s items represented three perspectives: personal learning, student learning, and teaching. For this study, we used items that focused on teachers' perspectives about student learning and teaching.

Use of practice standards. To assess teachers' use of practice standards, the first author observed 5 or 6 lessons for each teacher and collected all lesson plans and materials used in those lessons. The first author also conducted short pre- and post-observation interviews to discuss the lesson plan.

To obtain a detailed description of each teacher, the first author conducted five in-person observations with the kindergarten teacher and six with the 5th and 6th grade mathematics teacher during the spring of 2021. The teacher selected the lessons for each observation. We video and audio recorded each observation. To assess the frequency with which teachers engaged students in the Standards of Mathematical Practice (SMP; NGACBP & CCSSO, 2010), we used the Mathematics Classroom Observation Protocol for Practices (MCOP^2; Gleason et al., 2015) during the observation and subsequent viewing of video data. The MCOP^2 was developed to examine aspects of teaching facilitation (TF) and student engagement (SE) for the purpose of teaching mathematics for conceptual understanding and was grounded in the Instruction as Interaction framework (Cohen et al., 2003). Gleason et al. (2017) found the MCOP^2 to have an interrater reliability for both TF ($IRR = 0.616$) and SE ($IRR = 0.669$) subscales. Teacher facilitation refers to the role of the teacher to provide lesson structure and guidance through problem solving and mathematical discourse. Student engagement refers to students fulfilling their roles as active learners within the classroom environment (Gleason et al., 2015). The MCOP^2 includes both TF and SE because an observer cannot assess whether teacher actions are effective without observing whether those teacher actions produce meaningful engagement from students. The MCOP^2 observation guide aligns with teacher implementation of the SMP. In Table 1, from Gleason et al.'s (2017) publication, we note the alignment between the MCOP^2 and the SMP. Gleason et al. (2015) recommended that multiple observations occur over a period of time so that a comprehensive view of teachers' practices can be established.

	Standards for Mathematical Practice							
MCOP^2 item	Make sense of problems and persevere in solving them	Reason abstractly and quantitatively	Construct viable arguments and critique the reasoning of others	Model with mathematics	Use appropriate tools strategically	Attend to precision	Look for and make use of structure	Look for and express regularity in repeated reasoning
1	X						X	X
2	X				X			
3	X							
4	X		X					X
5	X	X	X		X			
6							X	X
7		X		X				
8							X	X
9	X							
10			X			X		
11	X							
12			X					
13			X					
14	X							
15			X					
16	X							

Table 1. Relationship between the MCOP^2 and the SMP (NGACBP & CCSSO, 2010)

Data Analysis

We did not perform statistical tests on survey data because of the small sample size. We used the quantitative tools only to estimate levels for each teacher's self-efficacies as part of a description of that teacher's beliefs and practices.

Self-efficacy beliefs. We used analysis results from the MTMSE survey for the purpose of establishing whether teachers' mathematics and mathematics teaching self-efficacy beliefs were high or low. Survey questions were worded both positively and negatively. We reordered ratings so that all statements were on a consistent scale with high scores representing more self-efficacious views. Unlike Kahle (2008), who based analysis on the sums of survey ratings, we tallied the responses for each question so that we could consider each type of self-efficacy separately. For mathematics self-efficacy, we classified teachers as having a high mathematics self-efficacy if they provided at least 17 responses (out of 31 questions) that were fours, fives, or sixes on a Likert scale. For mathematics teaching self-efficacy, we classified teachers as having a high mathematics teaching self-efficacy if they recorded at least 15 responses (out of 26 questions) that were fours, fives, or sixes.

Instructional beliefs. The survey questions were constructed both as positively and negatively worded statements. For the analysis, we again reordered ratings so that all statements were on a consistent scale with high scores representing more student-centered views. Rather than looking at the mean rating of survey items, we examined the frequencies of each level, which was more appropriate for our purpose. Following O'Hanlon et al. (2015), we classified teachers whose responses were primarily in the top half of the Likert scale (scores of 4, 5, or 6) as having instructional beliefs that aligned with more student-centered practices. In contrast, teachers for whom a majority of their responses were in the bottom half of the Likert scale were classified as having primarily teacher-centered instructional beliefs (O'Hanlon et al., 2015).

Use of practice standards. We classified teachers' use of the SMP by their scores in the relevant section of the MCOP^2. The MCOP^2 rates teachers' performance on a scale of 0–3 for each of the 16 items, with a 0 indicating the non-use or incorrect use of the item description and a 3 indicating the use of the item description. For each teacher, we reported the median scores across all observations for individual component of the scale (see Table 1). We used a median because the data were ordinal. We focused our analysis on the frequency of attaining medians at the top two rating levels (i.e., 2 and 3) within each of the SMP because those levels indicate at least an intentional effort to used student-centered practice.

Results

The purpose of this research was to determine the relationships among effective teachers' instructional beliefs, mathematics and mathematics teaching self-efficacies, and their use of the SMP. Using the collapsed rating method described above, both teachers scored similarly with respect to their instructional and self-efficacy beliefs. However, there were differences in their MCOP^2 results. In the following sections we discuss the specific results to each of the surveys and the MCOP^2 observations.

Self-Efficacy and Instructional Beliefs

We found that both teachers scored identically on the sections measuring mathematics teaching self-efficacy, meaning that both teachers felt as though they were capable of teaching mathematics in a student-centered approach. The teachers' mathematics self-efficacy survey results showed a slight difference between the two participants, Kathy scored higher than Frances (see Figure 1). However, both teachers achieved scores that we interpret as indicating a high mathematics self-efficacy, because both scores were above the cutoff score of 17.

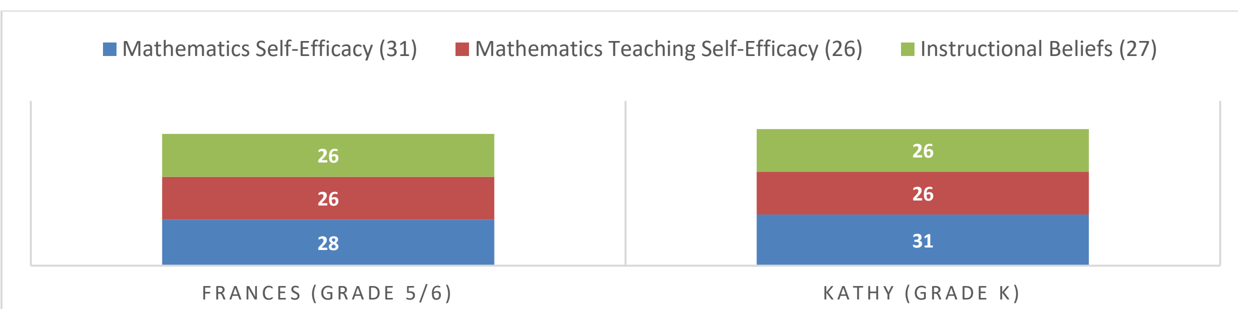

Figure 1. Beliefs Survey Results

Both teachers seemed to score similarly on instructional beliefs indicating that they both believed in a student-centered approach to teaching. Of the 26 items, each teacher selected a single 3-level rating, and their remaining 25 ratings were within the 4–6 range. However, when we examined the distribution of ratings, Frances' rated all 25 statements with a 4, whereas Kathy's ratings were all 5s and 6s (see Figure 2). Although both teachers had student-centered instructional beliefs, Kathy's beliefs may be more robustly student-centered than Frances' beliefs. We found further evidence of a possible disparity during the pre- and post-observation interviews, as Kathy spoke more frequently—without prompting—about her student-centered instructional beliefs. Frances often required prompting to discuss her instructional beliefs.

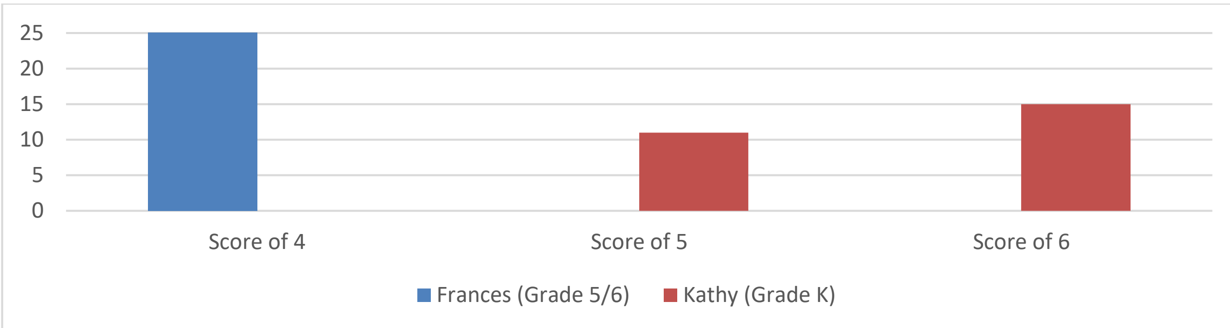

Figure 2. Specific Score Categories for Instructional Beliefs

Use of Standards of Mathematical Practice

Using the MCOP^2 observation tool, Frances received more medians of 3 than Kathy, and Frances' medians were always equal to or greater than Kathy's (see Table 2). For teacher facilitation items across all observations, Frances' median score was 3 and Kathy's was 2. For student facilitation items, Frances' median score was 2.5, whereas Kathy's median score was 2.

Table 2. Medians of Scores on the MCOP2 Across All Items and Observations

	Make sense of problems and persevere in solving them		Reason abstractly and quantitatively		Construct viable arguments and critique the reasoning of others		Model with mathematics		Use appropriate tools strategically		Attend to precision		Look for and make use of structure		Look for and express regularity in repeated reasoning	
MCOP^2 Items	K	5/6	K	5/6	K	5/6	K	5/6	K	5/6	K	5/6	K	5/6	K	5/6
1	3	3											3	3	3	3
2	1	2							1	2						
3	3	3														
4	2	3			2	3									2	3
5	2	2	2	2	2	2			2	2						
6													2	3	2	3
7			2	2			2	2								
8													3	3	3	3
9	1	3														
10					2	3					2	3				
11	2	2														
12					2	2										
13					2	3										
14	2	3														
15					2	3										
16	1	3														

Each teacher consistently facilitated the SMP in their classroom. Although Kathy received a median of 2 more often than a 3 on the MCOP^2, she regularly provided her students with opportunities to look for and make use of structure and look for and express regularity in repeated reasoning. Frances—having a median of 3 more often—provided opportunities for her students to make sense of problems and persevere in solving them, attend to precision, look for and make sense of structure, and look for and express regularity in repeated reasoning.

Relationships Among Beliefs and Practices

Although teachers had only minor variation in the robustness of their beliefs, based on their survey responses, their MCOP[2] showed somewhat greater variation in their use of the SMP in their lessons. For example, Kathy had more robust student-centered beliefs, as measured by the beliefs survey (see Figure 2); but then Kathy employed the SMP less consistently throughout the observations. Figure 3, showing the medians of level 2 and level 3 implementations of the SMP, shows that Kathy had a greater median of level 2 implementations, although her beliefs were more robustly student centered. In contrast, although Frances' student-centered beliefs were less robust, she scored a 3 across more categories.

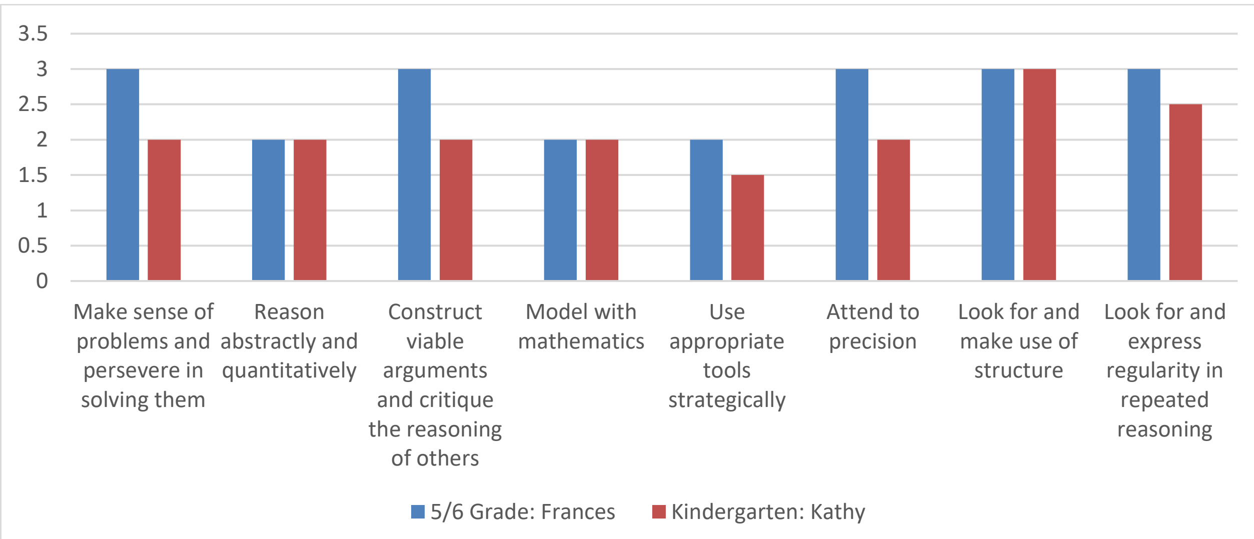

Figure 3. Median SMP Scores Across All Observations

Discussion

Echoing prior research, we noted a connection between the teachers' beliefs and their use of instructional practices (Hadley & Dorward, 2011; Peterson et al., 1989). However, there may be some differences with respect to which practices teachers use and how often they use those practices. We conjecture that our observed differences between the teachers were not simply due to grade-level differences but instead were due to the complexity of teaching and teacher beliefs. Building on prior work (e.g., Yurekli et al., 2020) by using classroom observations, we conclude that teachers with more-robust student-centered beliefs may not enact those beliefs in practice.

This study was limited by the number of participants and the pandemic context in which we observed them[1]. More research, using in-depth interviews, is needed to explore the extent to which beliefs, experiences, and other factors influence teachers' use of the SMP. Even so, this study adds to the growing body of research on the beliefs of teachers.

1 Because of the pandemic, we had limited access teachers who were teaching in person and there were many constraints to the classroom environment. Thus, the practices we observed may not have reflected teachers' typical or desired approaches.

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