TEACHERS' MOTIVATIONS FOR AND ENACTMENTS OF AMBITIOUS MATHEMATICS TEACHING: THE CASE OF "ROUGH DRAFT MATH"

Amanda Jansen
University of Delaware
jansen@udel.edu

Crystal Collier
University of Delaware
crystalc@udel.edu

The purpose of this study was to characterize variations in how teachers enacted an approach to ambitious mathematics teaching: "rough draft math." We also examined teachers' motivations for their enactments. Thirty-two teachers from five states in the U.S.A. were recruited to participate in interviews based on recommendations from leaders of book studies focusing on this teaching practice. All participants enacted "rough draft math" by intentionally building their classroom culture to welcome students' draft thinking to achieve the goal of promoting students' productive dispositions. However, additional variations in enactments drew attention to potential tensions between multiple goals of ambitious mathematics teaching (empowering students and learning through revising). Findings suggest insights for supporting teachers' learning to teach ambitiously; findings also contribute to a knowledge base for teaching.

Keywords: Instructional Activities and Practices; Affect, Emotion, Beliefs, and Attitudes

Ambitious mathematics teaching, according to Anthony and colleagues (2015) "involves skilled ways of eliciting and responding to each and every student in the class so that they learn worthwhile mathematics and come to view themselves as competent mathematicians" (p. 46). During ambitious mathematics instruction, students engage in discourse to learn mathematics and develop positive identities. Additionally, "the motivation to do things differently is as important as knowledge and skill to creating consistently ambitious practice" (Lampert et al., 2013, p. 227). To understand how to support mathematics teachers with learning to enact ambitious mathematics instruction, investigating variations in teachers' efforts to teach ambitiously, and their motivations to do so, can provide insight.

One approach to ambitious mathematics instruction involves inviting students to publicly share their in-progress or unfinished thinking, workshopping those ideas, and explicitly inviting students to revise their thinking – engaging students in *rough draft math* (Jansen, 2020; Jansen, Cooper, Vascellaro, & Wandless, 2016). Teaching mathematics through inviting drafts and revising is a complex activity. To do this work, teachers need facility with eliciting and responding to students' thinking (e.g, Fraivillig et al., 1999). It is not easy to listen to one another (teacher-student, student-to-student, student-teacher) in ways that lead to learning for students and the teacher during a mathematics lesson (Hintz et al., 2018). Building on students' thinking (Leatham et al., 2015) so that revising takes place is demanding work. It may be challenging in-the-moment to recognize strengths (Jilk, 2016) in students' draft thinking and manage classroom interactions to position students' contributions as valuable for their peers' learning (Wood, 2013).

The purpose of this study is to document variations of enactments of an ambitious teaching practice, "*rough draft math*," to contribute to building a knowledge base for teaching (c.f., Hiebert et al., 2002). Researchers can contribute toward building a knowledge base for teaching by documenting practitioners' work in a public, sharable manner so that more educators can learn from teaching practice. Given that incorporating rough drafts and revising in math class involves a range of practices, from creating a classroom culture where drafts are welcome to

explicitly structuring revision experiences that move students' thinking forward (Jansen, 2020), there are many entry points into "rough draft math" that can be documented and understood.

Variations in Enactments and Teachers' Motivations

Ambitious teaching practices have interdependent but different goals. Consider scientific inquiry: this teaching practice has multiple goals, such as making sense of phenomena (constructing claims or explanations), articulating understandings (presenting arguments), and persuading others (critiquing, evaluating, and defending arguments) (Berland & Reiser, 2011). Similarly, "rough draft math" has multiple goals: making sense of mathematics, articulating understandings (drafting ideas), improving one's understanding (revising drafts), and promoting students' agency so that they see themselves as capable of knowing and doing mathematics. These multiple goals illustrate complexity and provide opportunities for tensions.

Berland and Reiser (2011) discuss that a complex and ambitious teaching practice, such as scientific inquiry, can vary in enactment in different ways, in part due to these multiple goals. Teachers could emphasize some goals more than others. Some goals might be adopted selectively, and other goals might be set aside. Efforts to achieve certain goals might take place more consistently. In scientific inquiry, this might look like one classroom of students learning to competitively persuade each other while another classroom of students learns to seek to understand one another's thinking (Berland, 2011). What do these variations of enactment look like in mathematics classrooms where teachers incorporate "rough draft math" and why might they occur?

Motivation can be understood through multiple perspectives, including goal-orientation theory and the role of interest. Goal orientation theory attempts to explain how and why people try to reach certain objectives (Kaplan & Maehr, 2006). Motivation can look like meaningful engagement with a practice, and such engagement develops, in part, through cultivating interest (Renninger & Hidi, 2015). Teachers with greater interest are likely to continue to enact a practice about which they have learned (Ginsberg & Wlodkowski, 2019), particularly if they see the practice as positively meeting the needs of their students (Appova & Arbaugh, 2017). However, their enactments could differ depending on the goals they seek to achieve.

In this study, we seek to understand the enactments of teachers who have expressed an interest in enacting "rough draft math," as well as the goals they sought to achieve when enacting it. This study addresses the following research questions: Among teachers with expressed interests in enacting "rough draft math," how do teachers define "rough draft math"? How do these teachers' self-reported enactments of "rough draft math" vary by their motivations?

Methods

Data Collection

We interviewed 32 mathematics teachers from five states in the U.S.A. Teachers were recruited for this study among those who had participated in book study groups to discuss *Rough Draft Math* (Jansen, 2020). Participating teachers were recommended by mathematics coaches or professional development leaders who had facilitated a book study. Additional participants were recruited through Twitter and invited to participate if they had read the book on their own and were attempting to enact relevant instructional practices in their classrooms. Teachers in this sample taught in grades K-12, from first grade through AP Calculus.

Prior to the interview, participating teachers were asked to send, via email, a digital artifact that provided an example of how they incorporated rough drafts and revising into their instruction. The artifact provided context for participants' descriptions of their enactments. Examples of artifacts included the following: web links to Desmos activities, slide decks used

during lessons, handouts of mathematics tasks, sample student work that demonstrated students' initial drafts and revisions of their thinking, videos of classroom activities, assessments with directions, and photographs of students collaborating.

Interviews lasted between 35 and 60 minutes, and all interviews were conducted via video call, one-on-one. Interview questions included the following: If you were to tell a colleague about "rough draft math," how would you explain it to them? What can you tell me about this artifact? Tell me the story of this artifact. Why did you decide to do this in your classroom? What happened in your classroom? How is this artifact an example of "rough draft math"? Participants chose their own pseudonyms. All interviews were audio recorded and transcribed.

Data Analysis

Data analysis was conducted by both authors. In our first phase of analysis, we wrote analytic memos for each participant (Saldaña, 2013). In each memo, we described our conjectures for that teachers' thinking based on our initial listening to the interview and editing of the transcript. For the next phase of analysis, we used an exploratory process to inductively create descriptive codes (Saldaña, 2013) based on teachers' talk. We then created descriptive codes for two categories: (a) motivations to enact (or functions for) "rough draft math" and (b) enactments (forms) of "rough draft math." We identified the motivations that teachers described in relation to the artifact they shared. Although teachers described other ways of enacting "rough draft math" in their practice, in addition to the artifact, we assumed that the artifact represented an important part of their teaching practice, from their perspective. The second level of analysis involved frequency coding. We assessed whether, among the motivations reported for enacting instruction aligned with the shared artifact, any of the motivations were repeated throughout the interview and how often they were repeated. Repetition of an idea signals a level of importance to the speaker or an effort to emphasize an idea in their spoken talk (Tannen, 1989).

Results

Below, we illustrate three types of motivations that teachers in this sample expressed as functions for enacting "rough draft math," along with the enactments they described as forms for achieving these motivations. However, before we describe these motivations and enactments, we share teachers' definitions for "rough draft math." These definitions illustrate that teachers could think about rough drafting and revising in mathematics in a variety of ways, which then implies that they could have different motivations to enact "rough draft math."

Rough Draft Math: Teachers' Definitions

When teachers explained "rough draft math," they described components such as, (a) eliciting initial *drafts* of students' thinking, (b) providing students with opportunities to *revise* their thinking, often after some form of (c) *collaboration* with peers, and then (d) inviting students to *reflect* on their growth in their thinking and understanding. When asked to explain what "rough draft math" meant to a colleague, Ms. Dougherty said,

> …it's simply your first draft of thinking, if you're just initially starting out your thinking. You haven't really shared those thoughts with anybody. You haven't revised any of your thinking yet. It's just your first draft… then from there we build on that, and we use our rough draft to help build that understanding to make it stronger and understand where we came from along the way.

In Ms. Dougherty's explanation, she addressed all four components, highlighting the process of first drafts in students' thinking. She alluded to revising when she spoke about needing to "build" on the first draft "to make it stronger." Her explanation included an implicit reference to

collective, collaborative work when she shifted to using a "we" pronoun. She suggested incorporating reflection on growth in thinking by saying, "understand where we came from along the way."

However, not every teacher mentioned all four components in their definitions, which suggests that, among some teachers, a subset of components of "rough draft math" resonated more than others. Ms. Briggs emphasized the process of iteratively drafting and collaborating in her explanation when she said, "I would say that it's thinking of math of less like a finished product and more like something you're continuously working on together."
Ms. Kakkar emphasized the process of revising in her explanation.

> For me, when I've heard about rough draft and when we had worked with [math coach] and talked about it, the first thought, to be honest, that was, yes, it should be in math. The rough draft particularly did not come into my mind at first, but since you know how in ELA teachers talk … this makes perfect sense in math, too… you do trials and errors and estimation and all, seeing the patterns, a couple of factors combined together to actually reach out to an answer… It has to be rough draft. It has to be multiple trials. It has to be multiple revising through your work rather than just, okay, here is my answer.

Mr. Vandelay highlighted reflecting on growth and humanizing mathematics learning in his explanation.

> I'd say it's a way of humanizing the students' mathematical experience. It gets away from a traditional feeling that math is about being good, getting right answers, and having your value either internally or affirmed from, without coming from a place of feeling like those are the things that matter. Whereas the true mathematical experience is always a work in progress. It's always about developing thinking and we learn so much from our mistakes. We learn so much from thought processes that lead the wrong direction. And so it's really a sense of honoring what all students should be having as an everyday experience learning mathematics.

In common across these teachers' descriptions, whether they expressed four components or not, was that it made sense to them to treat mathematics learning as a gradual, iterative process of drafting and revising ideas, and they expressed value for drafting and revising while learning and doing mathematics in school. These teachers' talk illustrates what drafting and revising meant to them, in their mathematics classrooms, and some teachers put relatively stronger emphases on different components of "rough draft math."

Motivations for and Enactments of "Rough Draft Math"

Teachers reported multiple motivations for enacting "rough draft math," as represented in their artifacts. The first motivation below, foster productive dispositions, was reported in some manner by every teacher. However, the other two motivations (cultivate learning through revising and reflecting; empower students) were not reported by every teacher. The table below illustrates differences in reported enactments that appeared to be aligned with specific motivations for enacting "rough draft math."

Table 1: Motivations for and Enactments of "Rough Draft Math"

Motivation	Enactment
Foster productive dispositions • Develop confidence in students	Intentionally build a classroom culture where students' ideas are valued at any stage • Labeling talk as "rough drafts"

• Increase students' participation and effort • Safety for intellectual risk taking	• Rights of Learners • Protocols for discussion • Anonymous student sharing • Multiple revision experiences in class
Cultivate learning through revising and reflecting • Normalize mistakes in math • Develop metacognition	Explicitly incorporate revising into mathematics instruction • Revision routines, such as test corrections • Reflection on revising
Empower students • Shift authority to students • Disrupt school structures that control students	Decenter teachers' authority • Student self-assessment • Peer collaboration to learn

Motivation: Foster Productive Dispositions. Every participant reported an overarching motivation to enact rough drafting in math class: to foster productive dispositions among students. The most frequently associated enactment with this motivation was teachers' efforts to intentionally build a classroom culture where students' ideas were welcome and valued at any stage of their thinking. Teachers often mentioned more than one motivation for enacting rough draft math, but this was the motivation and associated enactment shared by all participants. The function or motivation of fostering productive dispositions had a range of subdimensions: develop confidence in students, create safe experiences to encourage intellectual risk taking and collaboration, and increase students' participation and effort in math class.

One of the artifacts Ms. Dougherty shared was a set of pictures of her middle grades students collaborating – standing together at white boards on the wall working on math and sitting together on the floor working in a group, and when she described her motivations for collaborative work, Ms. Dougherty talked about building classroom culture. Ms. Dougherty described that she encouraged students to verbally label their thinking as being in a rough draft stage when they shared aloud, so that students could feel more confident to participate without being judged for imperfect or incorrect thinking.

> I definitely see the language that my students use. Um, they do a lot of, they'll use the language of rough draft math. So, very often they'll raise their hand or start their thinking saying, well, I'm not sure yet, but this is what I'm thinking right now. And they have a very comfortable space and, and I commend them. So, at the beginning of the year, I do a lot of like that [by saying], "Awesome, rough draft math talking, thank you, starting out your thinking that way. Now we all know that we're going to give you the space to share and revise if you need to." Um, I think they look for revision possibilities rather than thinking of their math as a final step. Um, so they're very aware of the idea that we can look at what others are thinking, um, and constantly revise. So I think that that's been really beneficial… I think I am making moves toward getting their confidence to be stronger in mathematics. I don't just mean confidence in ability as much as just confidence in sharing their thinking, you know, just, just get something out there. Like everybody can share something, you know?

Ms. Dougherty reported that welcoming drafts and normalizing revising in mathematics class supported students' development of their confidence so that more students would participate.

To demonstrate efforts to build culture, Ms. Briggs's artifact included a photograph of "rights of learners" (Kalinec-Craig, 2017) generated by her first-grade students, such as "I have the right to be cared for and care for others." She shared that her motivation for promoting these rights was to support students' safety and willingness to participate. According to Ms. Briggs, "…when students are talking, they have that space to work through some of their disequilibrium. … students feel psychologically safe to make those mistakes and to have confusion around a topic." She also said that it was important to her to be "really instilling that love of math and, um, making kids feel like that they were experts in math is going to have a long-term impact on them." She used "rights of learners" to build a classroom culture to support her students.

Ms. Kakkar shared an artifact of a sample of students' revised work, and she also said that she wanted to increase students' risk taking and promote participation. When describing her artifact, she said that she encouraged revising by using a protocol – learning to listen / listen to learn (Wolfe, 2021) – to teach students to hear and learn from each other. She also incorporated anonymous sharing of students' solutions during whole class discussion for workshopping and revising. Ms. Kakkar said that she enacted teaching in these ways so that students could "see their progression and how could they realize it, how much it is important to, to really build upon each other's ideas?" She also said that she wanted "to make students comfortable."

Ms. Burnett provided a short video of one of her mathematics lessons, edited into segments to illustrate an in-class, multiple revision experience, and she expressed that she wanted to encourage students to take risks, to build students' confidence, and increase their participation. She said that she wanted to share this video to illustrate what a "rough draft math" lesson looked like in her class: students worked in groups on a challenging task, she purposefully selected student strategies to be shared to the whole class while peers asked one another questions about the work, students would then revisit and revise their work in groups, and then share out again. Ms. Burnett said, "it takes the pressure off the kids" to welcome rough draft thinking. She said that, regarding a success that she has had,

> Definitely confidence building. And the students are not afraid to make mistakes… I would have [in the past] some kids just sit and do nothing. They would just hold their pencil. And I knew it was like a frozen fear of, I don't wanna make a mistake. It's [rough draft math] taken that away. Those kids are just like, let's do this.

These enactments were motivated by teachers' drives to foster productive dispositions among students and to support students with participating to learn mathematics through discourse.

Motivation: Cultivate Learning through Revising and Reflecting. Although every teacher described a motivation to promote productive dispositions, not every teacher described using rough drafting to explicitly cultivate learning through revising. Ms. Apple's artifact was an example of a revising routine and protocol that she used for students to correct their tests in AP Calculus AB, and she said that she wanted to promote growth in students' thinking and awareness of that growth.

> I like the metacognition that's involved in the protocol. So really thinking about their thinking and thinking about where their mistake, misunderstanding or, misconception or misunderstanding [was]. I think of all three of those things as different, different ways to think about the problem and where it was wrong in their thought process. Because sometimes they just make a silly mistake. Sometimes they don't understand the concept and they need to go back and revise the way they think about a problem like that.

Ms. Green also shared an activity for chapter test revisions and reflection activity and a "quicker rough draft" process for a routine for in the moment revisions.

> I do it on the test, the final draft thinking on the test revisions… the revision part is when I feel like a topic has been solidified… then the quicker rough draft would be for those moments where we're in a new learning. … I use it [rough draft math] in different ways, depending on what I'm working on. So are we working [on] solidifying an idea? Are we working on learning a new idea for the first time?

Both of Ms. Green's artifacts illustrated two revision experiences, but they had different goals: working toward solidifying thinking (test revisions) or developing new ideas (discussing and revising drafts). Teachers like Ms. Apple and Ms. Green spoke about reaching a "final draft."

Motivation: Empower Students. A subset of teachers described that they wanted to enact rough draft math in their teaching to promote equity through empowering students. Mr. Vandelay shared an artifact that documenting requirements for (and purpose of) a student portfolio for their self-assessment, which was his primary grading practice; he said that he incorporated self-assessment to transfer authority to students by dismantling traditional school grading practices. According to Mr. Vandelay,

> I personally have felt since I read *Rough Draft Math*, that one of the things that really does is honors the fact that we need to rethink the way we're evaluating students, so that what we value in terms of student thinking as a process isn't undermined by grading practices … especially with students who have low status or perceived low status or low mathematical identities, they may have come through in the high school setting case years of feeling like they're not good at math, they can't do it. They've never been successful… the greater idea is sort of building on identity. And the social sense is like, why do we believe the things about ourselves as humans, as math students as all those things, and where do those beliefs come from, whose voices have formed those beliefs and who's been neglected?

Mr. Vandelay's reflection on his assessment practices illustrate the value of empowering students to evaluate themselves and illustrate his questioning of evaluation systems in school.

In addition to photograph's Ms. Dougherty shared another artifact, which was a Desmos activity for engaging students in self-reflection of their learning and self-assessment to transfer autonomy and agency to her students.

> … I've been working toward a gradeless classroom, and I know that that's sort of a whole different topic, but, for me, the gradeless classroom is really part of rough draft math, because I really want the students to have that idea of judgment being removed from their thinking. So, each week I have the students go through a reflection... it's this constant balance between letting them know like, "Hey, yeah, I am in the room, and I am the teacher, but you have control of your learning. Own it, make it what it is."

Mr. Vandelay and Ms. Dougherty's enactments and motivations illustrate that assessments are a type of teaching practice that can be enacted in different ways for different reasons. If we contrast their assessment practice of student self-assessment with the activity of test corrections shared by Ms. Green and Ms. Apple, both sets of teachers enacted rough draft mathematics, but Ms. Green and Ms. Apple wanted students to reach perhaps an externally determined learning destination, while Mr. Vandelay and Ms. Dougherty wanted to empower their students.

Discussion

As Berland and Reiser (2011) discussed, variations in teachers' enactments of an ambitious teaching practice can be understood as variations in relative emphasis of different components of a practice, perhaps emphasizing different goals. One difference between enactments of "rough draft math" was indicated by participants' talk about assessments. Some teachers reported that they engaged their students in test revisions to help their students achieve a "final draft" understanding through revising errors. In contrast, other teachers said that they engaged their students in self-assessment to promote students' agency and autonomy. Tensions between goals are possible, such as a goal of empowering students and a goal of promoting learning through revising. Some teachers may intend for students to revise to achieve a final draft, or a pre-determined understanding, which could reflect either the discipline of mathematics or the teacher as an authority. Other teachers may want students to have a voice in not only whether they learn, but what and how much they learn, as a part of their efforts to empower students.

Future research could investigate how teachers can cultivate interest in and facility with enacting "rough draft math." We focused this study on teachers who reported a high level of interest in enacting "rough draft math." An analysis of the goals that teachers reported for enacting "rough draft math" can provide promise in generating greater uptake of the practices, as teachers could be encouraged to enact "rough draft math" if they gain awareness of the range of goals they can achieve. A contribution of this study is that understanding how these teachers enacted "rough draft math" provided insight for starting points teachers might take to learn to enact ambitious teaching; teachers could consider the goals they want to achieve and learn about possible enactments that could help them achieve these goals.

Additionally, future research could observe enactments of "rough draft math," in contrast to this study, which focused on teachers' self-reports. Observations of enactments would allow for characterizing collective practices of drafting and revising, which could provide insight into collective understandings generated and ways that students engage and participate. However, studying teachers' self-reports provides insights into teachers' goals through their voices as well as which enactments they value in pursuit of these goals.

A contribution of this study is the illustration of how ambitious mathematics teaching can vary in its enactments. Although all participating teachers reported a goal of promoting students' dispositions and enactments by encouraging a culture of drafting, they varied in their emphasis of goals to (a) learn through revising and reflecting or (b) to empower students. Additionally, results illustrated how varying goals for enacting ambitious mathematics teaching could potentially be in tension. These results contribute to building a knowledge base for how mathematics teachers can put "rough draft math" into practice.

References

Anthony, G., Hunter, R., Hunter, J., & Duncan, S. (2015). How ambitious is "ambitious mathematics teaching". *Set: Research Information for Teachers, 2, 45*, *52*(10.18296).

Appova, A., & Arbaugh, F. (2018). Teachers' motivation to learn: Implications for supporting professional growth. *Professional Development in Education*, *44*(1), 5-21.

Berland, L. K. (2011). Explaining variation in how classroom communities adapt the practice of scientific argumentation. *Journal of the Learning Sciences*, *20*(4), 625-664.

Berland, L. K., & Reiser, B. J. (2011). Classroom communities' adaptations of the practice of scientific argumentation. *Science Education*, *95*(2), 191-216.

Fraivillig, J. L., Murphy, L. A., & Fuson, K. C. (1999). Advancing children's mathematical thinking in everyday mathematics classrooms. *Journal for Research in Mathematics Education*, *30*(2), 148-170.

Ginsberg, M. B., & Wlodkowski, R. J. (2019). Intrinsic motivation as the foundation for culturally responsive social-emotional and academic learning in teacher education. *Teacher Education Quarterly*, *46*(4), 53-66.

Hiebert, J., Gallimore, R., & Stigler, J. W. (2002). A knowledge base for the teaching profession: What would it look like and how can we get one?. *Educational Researcher*, *31*(5), 3-15.

Hintz, A., Tyson, K., & English, A. R. (2018). Actualizing the rights of the learner: The role of pedagogical listening. *Democracy and Education*, *26*(2), 8.

Jansen, A. (2020). *Rough Draft Math: Revising to Learn*. Portsmouth, NH: Stenhouse Publishers.

Jansen, A., Cooper, B., Vascellaro, S., & Wandless, P. (2016). Rough draft talk in mathematics classrooms. *Mathematics Teaching in the Middle School, 22*(5), 304-307.

Jilk, L. M. (2016). Supporting teacher noticing of students' mathematical strengths. *Mathematics Teacher Educator*, *4*(2), 188-199.

Kalinec-Craig, C. A. (2017). The rights of the learner: A framework for promoting equity through formative assessment in mathematics education. *Democracy and Education*, *25*(2), 5.

Kaplan, A., & Maehr, M. L. (2007). The contributions and prospects of goal orientation theory. *Educational Psychology Review*, *19*(2), 141-184.

Lampert, M., Franke, M. L., Kazemi, E., Ghousseini, H., Turrou, A. C., Beasley, H., Cunard, A., & Crowe, K. (2013). Keeping it complex: Using rehearsals to support novice teacher learning of ambitious teaching. *Journal of Teacher Education*, *64*(3), 226-243.

Leatham, K. R., Peterson, B. E., Stockero, S. L., & Van Zoest, L. R. (2015). Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. *Journal for Research in Mathematics Education*, *46*(1), 88-124.

Renninger, K. A., & Hidi, S. E. (2015). *The power of interest for motivation and engagement*. Routledge.

Saldaña, J. (2013). *The coding manual for qualitative researchers* (2nd ed.). Sage Publishers.

Tannen, D. (1989). *Talking voices: Repetition, dialogue, and imagery in conversational discourse* (Vol. 6). Cambridge University Press.

Wolfe, J. A. (2021). Teaching is a journey: a journey in becoming. *Mathematics Teacher: Learning and Teaching PK-12*, *114*(3), 258-261.

Wood, M. B. (2013). Mathematical micro-identities: Moment-to-moment positioning and learning in a fourth-grade classroom. *Journal for Research in Mathematics Education*, *44*(5), 775-808.