ENGLISH LANGUAGE LEARNER STUDENTS' DEVELOPMENT OF THE MATHEMATICS REGISTER DURING A PROBLEM-SOLVING LESSON

Karla Matute
University of Wyoming
kmatute@uwyo.edu

Libni Catsellón
University of Wyoming
Lcastel1@uwyo.edu

Richard Kitchen
University of Wyoming
rkitchen@uwyo.edu

This paper describes how fifth-grade English Learner students (ELs) in an urban school district develop the mathematics register during a problem-solving lesson. It provides examples of students' work to illustrate how they use the mathematics register to communicate their mathematical ideas orally and in writing. The teacher implemented teaching practices such as mathematics discourse to facilitate their students' development of the mathematics register during the problem-solving lesson. Students were engaged in a problem-solving task that involved fractions. Findings provide insights into EL students' challenges when learning the mathematics register and inform instruction about the importance of incorporating teaching practices such as paraphrasing and assessing others' reasoning to support students in learning the mathematics register through problem-solving.

Keywords: Classroom discourse, Mathematics register, Problem Solving

English Learners (ELs) population is significantly growing in U.S. schools (Abedi & Gándara, 2006; Campbell et al., 2007; de Araujo et al., 2018). Almost 10% of students enrolled in public schools in the United States are classified as ELs (NCES, 2020). As the population of ELs increase, meeting their academic needs deserve an urgent national response. ELs often have limited access to a challenging education in mathematics, adequate resources, and qualified teachers (Borjian, 2008; Dong, 2016). Research highlights several issues that influence the performance gap between ELs and monolingual speakers (see, e.g., Campbell et al., 2007; Chval & Chávez, 2012; de Araujo et al., 2018). Among these issues, language is viewed as a source of difficulty in learning mathematics, particularly for ELs required to learn the content of mathematics and English language skills simultaneously (de Araujo et al., 2018; Martiniello, 2008; Moschkovich, 2007). This paper describes how two fifth-grade mathematics classes with a high percentage of ELs (30% or more) developed the mathematics register during a problem-solving lesson. It also shows how ELs use the mathematics register when engaging in a problem-solving task.

In spring 2021, the first author worked with "Ms. Ware," a 5th-grade teacher, and her students in an urban school district to conduct a case study as part of her dissertation project. One of the research goals was to examine how Ms. Ware supported her ELs in developing the mathematics register during problem-solving lessons. The research questions that we explore here are: How do ELs develop the mathematics register while engaging in a problem-solving lesson? How does Ms. Ware support ELs developing the mathematics register during a problem-solving lesson?

The Mathematics Register

The mathematics register was initially introduced into mathematics education by Halliday in 1978. He defined the mathematics register as a set of "meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes" (Halliday,1978, p.195). The literature also refers to the mathematics register as language variations constructed by

individual interactions (Schleppegrell, 2004). These variations are characterized in terms of *field* (area or subject), *tenor* (relationship between the individuals), and *mode* (organization of text or conversation) (Halliday, 1978).

The mathematics register includes semiotic resources, such as symbols, visual displays, graphs, diagrams, and language (O'Halloran, 2015). Similar to other disciplines, mathematics has its own language, and learning this discipline implies learning its language. Researchers focused on academic languages have highlighted several issues that emerge when ELs are required to use the mathematics language in informal and formal contexts (see, e.g., Barwell, 2005; Ernst-Slavit & Mason, 2011; Lucero, 2012). For example, Barwell (2005) points out that ELs often face challenges when communicating their mathematical ideas in English. He highlights that solving and writing word problems can support students in integrating mathematics content and language. Ernst-Slavit & Mason (2011) argues that ELs have limited access to mathematics classrooms in which they can consistently hear the mathematics register. Also, teachers constantly use slang, homophones, colloquialism, and idiomatic expressions that can hinder ELs' understanding (Ernst-Slavit & Mason, 2011). Lucero (2012) argues that ELs need repeated opportunities to engage in problem-solving tasks to communicate their ideas to others, define mathematics terms, and compare strategies.

Communicating mathematical ideas in oral and writing forms require students to know the knowledge of mathematics and the knowledge of the language (Wilkinson, 2018). Students need to combine everyday language, symbols, visual displays, and non-linguistic representations to communicate effectively in mathematics. Teachers should incorporate instructional strategies to help their students combine these linguistics features when communicating their mathematics ideas, and consequently, they learn the mathematics register (Moschkovich, 2014)

The mathematics register also involves linguistics features that define the communication styles (Schleppegrell, 2004; Wilkinson, 2018). For example, in an academic context, the mathematics register used can vary when the communication is oral or written (Wilkinson, 2018) Mathematics includes words borrowed from everyday English (e.g., degree, factor, relation, power, radical, product, mean, real, imaginary, rational, and natural) that can have a different meaning in other contexts than mathematics. It also involves some grammatical constructions that differ in their qualitative and quantitative meaning (e.g., *four fours*, the first *four* is an adjective while the second one illustrates a nominal status) (Pimm, 1987).

The mathematics register has two features: multiple semiotic systems and grammatical patterns (Schleppegrell, 2007). The multiple semiotic systems refer to symbol notation, oral and writing language, and visual displays to construct mathematical meaning. The grammatical patterns refer to technical vocabulary, dense noun phrases, nominalization, verbs, conjunction, and logical connectors to communicate mathematics ideas. These features work together to develop the mathematics register, and they are often used to provide mathematical arguments, justify mathematical reasoning, and build mathematical ideas and meanings (Schleppegrell, 2007).

Research Methodology

During the 2021 spring semester, I collaborated with Ms. Ware, a fifth-grade teacher, and her two 5th fifth-grade mathematics classes on three occasions. The study's goal was to examine how Ms. Ware implemented an instructional protocol called the "Discursive Mathematics Protocol" to help her ELs develop the mathematics register and how her students used the mathematics register during problem-solving lessons (see Kitchen et al., 2020, for additional information on the DMP). The DMP was built based on Polya's (1945/1986) problem-solving framework and

incorporated research-based strategies and essential teaching practices. These research-based strategies are based on theories of academic language development (Barwell, 2005; Ernst-Slavit & Mason, 2011; Moschkovich, 2015) and the essential teaching practices proposed in *Principles to Actions* (NCTM, 2014) that guide teachers in facilitating their students' development of the mathematics register ad mathematical reasoning, respectively. For the three problem-solving lessons, Ms. Ware and I worked together to plan the lesson, then Ms. Ware delivered the lesson as planned, and finally, we had a debriefing session. For the planning session, we met on Zoom to discuss how we hoped to develop the problem-solving lesson, identified questions, discussed strategies and challenges students could face, discussed the mathematics register involved in the task, and how she would support her ELs to develop it. Ms. Ware and her students met in *Google Meet* for 90 minutes each group during class. I observed the lessons and, on a few occasions, interacted with Ms. Ware and her students, particularly in group discussions. In the debriefing session, Ms. Ware and I met in Zoom for 30-minutes after the problem-solving lesson to discuss what went well in the lesson, what needed improvements for the next lesson, and what adjustments were necessary to incorporate to help ELs to develop their mathematics register.

For this study, 27 students provided consent across the two classes (15 ELs and 12 non-ELs). These classrooms had a high percentage of ELs (30% or more). Most of them speak Spanish as their first language. Ms. Ware taught one group in person and another group entirely online. Each group attended mathematics classes for 90 minutes in two blocks of 45 minutes each. Ms. Ware had Due to the COVID-19, Ms. Ware and her students used a technology tool called *Pear Deck* and *Google Meet* to participate in the class. *Pear Deck* is an application to make interactive presentations in virtual meetings, and *Google Meet* is a virtual tool that allowed Ms. Ware to teach mathematics remotely. The in-person and online groups were connected through *Google Meet* in mathematics classes.

This paper describes how students used the mathematics register while engaging in a problem-solving task. We also highlight the strategies Ms. Ware's used during the lesson to help her ELs develop the mathematics register. Data collection for this study included sources such as video recording of the planning session, problem-solving lessons in *Google Meet,* the debriefing session, ELs' work samples, and class observations. The videotapes and students' work samples were analyzed inductively using interpretive methods (Creswell & Poth, 2018). Each videotape was transcribed, and they were viewed as a whole, followed by open coding to reflect on and clarify teaching strategies to deliver the problem-solving lessons and procedures to address the mathematics register inherent in the task. The analysis process went through multiple iterations to check the consistency of the findings and look at commonalities and differences across the solutions that students provided.

Problem-Solving Task implemented

In the problem-solving lesson, we used a task called 'connecting area model to context," referred to as the multiplication story task. The multiplication story task is a performed task that is publicly accessible through the Illustrative Mathematics (IM) project. This task is considered a cognitively and linguistically demanding task. Students were required to write a multiplication story problem and give a diagram with two types of shading. They needed to establish connections among a procedure (multiplication), a diagram, and a context. Also, they needed to recognize the concept of fractions as operators (Charalambous & Pitta-Pantazi, 2007) inherent in the double shading shown in the diagram. The complexity of the language of this problem appeared when students were required to create a story with a context that related to the notion of fractions as operators represented in the diagram and translate symbols into words (3/4 of 1/5).

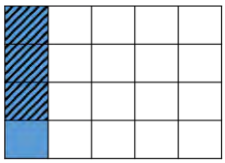
The diagram below represents one whole. 	1. Write down an equation that represents the doubled shaded area on the diagram. 2. Write a multiplication story that could be solved using the diagram with its two types of shading. Explain how your story context relates to the diagram provided.

Figure 2: Task implemented with the DMP

Findings

We now present an example of a problem-solving lesson to illustrate how students in Ms. Ware's classroom used the mathematics register when they solved the multiplication story task. Ms. Ware followed three primary teaching practices to help her students develop the mathematic register inherent in the task. These practices included: (1) implementing socio-mathematical norms and activating previous knowledge, (2) introducing the task and reviewing vocabulary, and (3) facilitating meaningful mathematical discourse as a medium to support students in developing the mathematics register.

Ms. Ware started the problem-solving lesson by establishing some mathematical norms and directions students needed to follow during the virtual meeting. She also invited her students to participate and communicate their ideas in oral and writing form through *Pear Deck* during the lesson. Students were expected to collaborate with each other, share their reasoning, engage in mathematical discussion, negotiate the meaning of mathematical terms, and reach agreements about their mathematical ideas. Prior to the problem-solving task, Ms. Ware activated her students' prior knowledge about fractions. She had students work with model diagrams and identify unit fractions (e.g., 1/3,1/5) in both columns and rows of a rectangular diagram. Students were also asked to identify expressions represented in the area model diagram that could be derived by multiplying fractions (1/4 of 1/3). The goal of the review was to emphasize the notion of fractions as operators, which would help students work on the multiplication story task.

Following the first stage, Pólya's heuristic *Understanding the Problem* (stage1 of the DMP), Ms. ware introduced the multiplication story task. She began asking questions such as: *What is the task asking for? What do you think it means to write a multiplication story? What do you guys think we're doing?* Students discussed in small groups what the problem asked them to do. Ms. Ware allowed students to use their vocabulary and familiar language to explain what they understood about the problem. She also helped her students to understand the vocabulary used in the task, such as "story," "multiplication story," and "two types of shading." Some students provided their reasoning about the meaning of these terms. For example, Marcia, an EL, explained that: a multiplication story is "like a story that has a multiplication in it," Max, a non-EL student, said, "it is like using a story, but like using math." Similarly, students defined "two types of shading" as "overlapping" of "two types of colors." Moreover, Once Ms. Ware was satisfied that most of her students understood the task and vocabulary found in the task, she continued with the activities planned and moved to stage two of Polya's heuristic, *Create a Plan* (stage 2 of the DMP)

As Ms. Ware progressed through the lesson, she engaged her students in mathematics discourse on several occasions by allowing them to share their ideas, paraphrasing others' reasoning, providing justifications, and comparing their strategies while solving the task. Throughout the second stage of the DMP, Ms. Ware asked her students to create a plan to solve

the task. She asked questions such *How could we solve this problem*? *What is our plan to solve it? What do we need to solve this problem? What two things are we looking for?* In stage 3 of Polya's heuristic, *Carry out the plan* (stage 3 of the DMP), students were required to work individually on *Pear Deck* to write an equation and a multiplication story. Then, students shared their solutions with their partners in small group discussions and justified whether the equation was related to the diagram.

During this stage, Ms. Ware engaged her students with others' reasoning. For example, Ana, an EL, wrote the numerical expression 3/4 x 1/5 to represent the area double shaded in the diagram, and she wrote the following multiplication story: "Mis. Ware ate 1/5 of a cookie; later the day, she ate ¾ of the cookie; how much did Ms. Ware eat?" Ana was able to translate "two types of shading" in the diagram into an expression (3/4 x 1/5), in which they demonstrated the ability to interpret the diagram by identifying the whole and the two factors of the multiplication. These elements are features of the mathematics register because Ana could identify the multiplication of fractions (1/5 x 3/4) shown in the diagram and use two different representations (e.g., the diagram and an equation). However, Ana's story problem demonstrated a misconception of finding fractional pieces of the original whole rather than finding a fraction of a fraction. Ana did not find a part of a part; instead, she found a part of a whole twice. Her stories did not indicate the multiplication of the two fractions represented in the diagram. Ms. Ware used Ana's story to generate a discussion and help her students reflect on their solutions. She asked her students to think and explain how the story could be fixed to match the diagram and the equation. She encouraged her students to use the mathematics register by assessing another student's strategy. Andrew, a non-EL student, gave his reasoning about Ana's story:

> I think this multiplication problem isn't correct because this was like an adding problem…if you like to write in a better way to do so, it would be like: Miss Ware split and took away 1/5 of a cookie, and later in the day, she took ¾ of the piece or something like that to represent the diagram. Also, it has a lot more sense because it shows the 1/5 that she took away and then the ¾ that you ate from the 1/5.

Ms. Ware asked another student to paraphrase what Andrew said to ensure that her students understood the issue in Ana's story. Through paraphrasing, students had the opportunity to show their understanding and use the mathematics register involved in the task. Antonio, an EL, said:

> What Aiden explained is that it [referring to Ana's problem] was an addition. So, I agree with Andrew because whenever it is multiplication, you basically have to multiply that, no matter what the denominators are, but when adding, you need to find a common denominator.

Ms. Ware revoiced Andrew's reasoning to emphasize that the notion of a part of a part did not appear in the story. She said:

> I want to point out how she [referring to Ana's story] has 1/5 of a cookie and then ¾ of a cookie. Right? And so, the ¾ is not referring to the 1/5. It is referring to a whole cookie. Whereas, as Andrew said, she [Ana] took 1/5 of something, and she ate ¾ of the 1/5 later. Do you guys hear the difference? Is it just a multiplication problem, or is this an addition problem?

Ms. Ware was aware that her students needed support in the mathematics register required to write their stories. Specifically, she helped them learn grammatical patterns (the second feature of the mathematics register) needed to write their stories. She took advantage of Ana's story and Andrews' ideas to highlight why their stories were not related to the multiplication expression ¾

x 1/5 and why the language used implied part of a whole twice in the story. Ms. Ware explained that because the unit is the same cookie used in the problem (1/5 of a cookie and then 3/4 of the cookie), the problem referred to the same "whole." Ana used the notion "part of a whole" when she wrote the phrase "1/5 of a cookie" and "¾ of the cookie," but the idea "part of a part" was not used in her story. Moreover, Ana's question, "how much did Miss Ware eat?" led students to think about adding the fractions to find the solution. Ms. Ware provided feedback on Ana's story. She suggested a sentence structure that she could use to enhance her problem "1/5 of something and then she ate ¾ of the 1/5 later." In this sentence, Ms. Ware provided her students a template they could use to write their story problem.". This sentence could help other students to use the notion of "part of a part" in their story problem. After engaging her students in the previous discussion, Ms. Ware asked them to return to Pear Deck to revise and re-write their story problems when needed.

> As another example, Ms. Ware intentionally chose another student's solution while monitoring their work on *Pear Deck*. In her multiplication story, Alejandra, an EL, demonstrated a strong understanding of fractions as operators and used the mathematics register in her answer. Ms. Ware engaged her students to assess Alejandra's problem, and Ms. Ware and I asked Alejandra to clarify the language she used in her story.
>
> *Alejandra*: My problem was Ms. Ware's daughter had a birthday party. She invited six of her friends. Ms. Ware had to get cupcakes for the party. One packet had 1/5 vanilla cupcakes, and 3/4 of those cupcakes had strawberry sprinkles on them. How many cupcakes in total had sprinkles?
>
> *Ms. Ware*: ok, what makes this work?
>
> *Andrew*: So, Alejandra's problem she explained that 1/5 of all of the cupcakes are vanilla and ¾ of the 1/5 has sprinkles on them, which is what the diagram shows because it shows 1/5 of the vanilla cupcakes, and then ¾ of the 1/5 is shaded, which shows that ¾ of the 1/5 have sprinkles on them.
>
> *Researcher:* When you said "three-fourths of those cupcakes," what do you mean? Can you say more?
>
> *Alejandra*: What I mean is that the 1/5 is the total cupcakes in the package and ¾ of those cupcakes had sprinkles on them.
>
> *Andrew*: I think Alejandra…you mean by of those cupcakes is the vanilla cupcakes, because I feel like that could be a little bit confusing to some people because like… if you are trying to be very specific in making it very clear, I'd add that ¾ of the vanilla cupcakes had sprinkles on them because somebody could ask anything that when you say of those cupcakes…they might think ¾ of the chocolate cupcakes.

The excerpt above illustrates an example of how a student combined familiar language, symbols, and diagrams to write her multiplication story. In her written solution, Alejandra also used features of the mathematics register (e.g.,1/5 of all of the cupcakes are vanilla, and ¾ of those cupcakes had sprinkles on them). For Alejandra, the phrase "of those" means 3/4 of the 1/5 of vanilla cupcakes. She used undefined references (of those cupcakes) to indicate a specific antecedent in the sentence (the vanilla cupcakes). Using this word in a sentence to translate a mathematics expression into words can lead students, especially ELs, to miss data in the problem and get misconceptions about the whole used. In this sense, I asked Alejandra to clarify what she meant when she said "of those cupcakes" to help other students understand her story. During this discussion, Andrew helped to explain Alejandra's story. He highlighted the meaning of the phrase "3/4 of those cupcakes." Both Alejandra and Andrew demonstrated a high level of

English language proficiency and understanding of the mathematics register used in the story. However, it was essential to clarify the language used in the story to help other students identify the notion of "part of a part."

During the lesson, Ms. Ware devoted significant time engaging her students in meaningful mathematical discourse to help them understand the problem, engage with their peers in mathematical reasoning, revoice and paraphrase others' ideas, and communicate their mathematical ideas. She also had her students work individually on their stories and communicate their reasoning in writing. During the last stage of Polya's heuristic, *Looking Back* (Stage 4 of the DMP), Ms. Ware asked her students to return to *Pear Deck* and revise their stories. She encouraged her students to explain how their stories related to the diagram and the equation ¾ x 1/5 = 3/20 using the mathematics vocabulary learned in the class. Some students could refine their solutions and re-write their stories in this stage.

Discussion

This paper described how fifth-grade ELs in Ms. Ware's mathematics classes used and developed the mathematics register while engaging in a problem-solving lesson. Regarding the first research question, students had consistent opportunities to develop the mathematics register during the problem-solving lesson. Ms. Ware engaged her students in meaningful mathematics discourse. Multiple linguistic recourses such as everyday language, mathematics vocabulary, and visual displays were integrated to help students learn the mathematics register. Students had multiple opportunities to communicate their ideas, assess their peers' reasoning, negotiate meanings of mathematical terms, and paraphrase others' reasoning. These strategies helped students enhance and advance their mathematics register (Moschkovich, 2015; Schleppegrell, 2007). For example, students had opportunities to learn the mathematics register inherent in the task by negotiating the mathematical meaning of terms used in the problem (e.g., multiplication story, equation), using their everyday language to explain their problems (e.g., Alejandra's story). Furthermore, students assessed Ana's problem and reflected on the language used in her story and how the phrases used (e.g., "1/5 of a cookie and ¾ of the cookie") indicated part of a whole twice instead of part of a part. Moreover, ELs had the opportunity to revise their stories and incorporate the language learned to write a multiplication story that reflected the notion of the fraction of a fraction. Alejandra's problem illustrated how students used the mathematics register to write their stories and justify the language employed in their stories (e.g., ¾ of those cupcakes").

Regarding the second research question, the multiplication story task requires students to have English language skills and abilities to translate mathematics expressions into words. Ana's story illustrates that learning the mathematics register can pose challenges for ELs, specifically when students need to use the English language to translate a mathematics expression into text (O'Halloran, 2015). In this sense, Ms. Ware implemented instructional strategies to help her students develop the mathematics register. These strategies included facilitating meaningful mathematics discourse in which students had multiple opportunities to paraphrase others' ideas, assess others' strategies, and engage with their peer's reasoning (Herbel-Eisenmann et al., 2013; Moschkovich, 2015). Ms. Ware intentionally selected specific students' stories (e.g., Ana and Alejandra's stories) to help them reflects on how they could use the mathematics register to fix their stories and represent the notion of "part of a part."

The findings in this paper provide insights into the importance of supporting ELs to develop the mathematics register through problem-solving. In Ms. Ware's mathematics classes, ELs had the opportunity to apply their knowledge about fractions and their knowledge of language to

write multiplication stories. Ms. Ware asked purposeful questions to help her students understand specific phrases that they could use to address the language complexity found in rich tasks such as the multiplication story task.

References

Abedi, J., & Gándara, P. (2006). Performance of English language learners as a subgroup in large-scale assessment: Interaction of research and policy. *Educational Measurement: Issues and Practice*, *25*(4), 36–46. https://doi.org/10.1111/j.1745-3992.2006.0007710.1111

Barwell, R. (2005). Integrating language and content: Issues from the mathematics classroom. *Linguistics and Education*, *16*(2), 205–218. https://doi.org/10.1016/j.linged.2006.01.002

Borjian, A. (2008). A new approach to meeting the needs of Latinos in American schools: Students' views on attending a redesigned small high school. *International Journal of Learning*, *15*(2), 277–285. https://doi.org/10.18848/1447-9494/CGP/v15i02/45642

Campbell, A. E., Davis, G. E., & Adams, V. M. (2007). Cognitive demands and second-language learners: A framework for analyzing mathematics instructional contexts. *Mathematical Thinking and Learning*, *9*(1), 3–30. https://doi.org/10.1080/10986060709336603

Charalambous, C., & Pitta-Pantazi, D. (2007, March). Drawing on a theoretical model to study students' understandings of fractions. *Educational Studies in Mathematics*, *64*(3), 293- 316.

Chval, K. B., & Chávez, Ó. (2012). Designing math lessons for English Language Learners. *Mathematics Teaching in the Middle School*, *17*(5), 261–265. https://doi.org/10.5951/mathteacmiddscho.17.5.0261

Creswell, J. W., & Poth, C. N. (2018). *Qualitative inquiry and research design choosing among five approaches.* (4th Edition). SAGE Publications, Inc., Thousand Oaks.

de Araujo, Z., Roberts, S. A., Willey, C., & Zahner, W. (2018). English Learners in K–12 mathematics education: A review of the literature. *Review of Educational Research*, *88*(6), 879–919. https://doi.org/10.3102/0034654318798093

Dong, Y. R. (2016). Create a responsive learning community for ELLs. *The Mathematics Teacher*, *109*(7), 534–540. JSTOR. https://doi.org/10.5951/mathteacher.109.7.0534

Ernst-Slavit, G., & Mason, M. R. (2011). "Words that hold us up:" Teacher talk and academic language in five upper elementary classrooms. *Linguistics and Education*, *22*(4), 430–440. https://doi.org/10.1016/j.linged.2011.04.004

Halliday, M (1978). *Language as a social semiotic: The social interpretation of language and meaning*. Baltimore, MD: University Press.

Herbel-Eisenmann, B., Johnson, K. R., Otten, S., Cirillo, M., & Steele, M. D. (2015). Mapping talk about the mathematics register in a secondary mathematics teacher study group. *The Journal of Mathematical Behavior*, *40*, 29–42. https://doi.org/10.1016/j.jmathb.2014.09.003

Herbel-Eisenmann, Steele, M. D., & Cirillo, M. (2013). (Developing) Teacher discourse moves A framework for professional development. Mathematics Teacher Educator, 1(2), 181–196. https://doi.org/10.5951/mathteaceduc.1.2.0181

Kitchen, R., Castellón, L. B., & Matute, K. (2020). Supporting English learner students' development of the mathematics register through an instructional intervention. In A. I. Sacristán, J. C. Cortés-Zavala, & P. M. Ruiz-Arias (Eds.), Mathematics Education Across Cultures: Proceedings of the 42nd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education Conference, México (pp. 1993-2000). Cinvestav/AMIUTEM/PME-NA. https://doi.org/10.51272/pmena.42.2020

Lucero, A. (2012). Demands and opportunities: Analyzing academic language in a first-grade dual language program. Linguistics and Education, 23(3), 277–288. https://doi.org/10.1016/j.linged.2012.05.004

Martiniello, M. (2008). Language and the performance of English-Language Learners in math word problems. *Harvard Educational Review; Cambridge*, *78*(2), 333-368,429. https://search.proquest.com/docview/212283604/abstract/A87B58C0AC984034PQ/1

Moschkovich, J. (2007). Bilingual mathematics learners. How views of language, bilingual learners' mathematics communication effect instruction. In N.S. Nasir and P. Cobb (eds.). *Improving access to mathematics: Diversity and equity in the classroom* (pp. 89 -104). New York, NY: Teachers College Press.

Moschkovich, J. (2014). Building on student language resources during classroom discussions. In. M. Civil & E. Turner, (Eds.), The common core state standards in mathematics for English language learners: Grade K-8 (pp.127-138). Alejandra, VA: TESOL.

Moschkovich, J. (2015). Academic literacy in mathematics for English Learners. *The Journal of Mathematical Behavior*, *40*, 43–62. https://doi.org/10.1016/j.jmathb.2015.01.005

National Center for Education Statistics (NCES) (2021). Preprimary, Elementary, and Secondary Education Section: Elementary and Secondary Enrollment. U.S. Department of Education. Retrieved from https://nces.ed.gov/programs/coe/indicator/cgf

National Council of Teachers of Mathematics (NCTM) (2014). *Principles to Actions: Ensuring mathematics success for all*. Reston, VA: National Council of Teachers of Mathematics.

O'Halloran, K. L. (2015). The language of learning mathematics: A multimodal perspective. *The Journal of Mathematical Behavior*, *40*, 63–74. https://doi.org/10.1016/j.jmathb.2014.09.002

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. New York, NY: Routledge.

Pólya, G. (1986). *How to solve it: A new aspect of mathematical method*. Princeton University Press. (Original work published in 1945)

Schleppegrell. (2004). *The language of schooling: A functional linguistics perspective*. Lawrence Erlbaum Associates.

Schleppegrell. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly*, *23*(2), 139–159. https://doi.org/10.1080/10573560601158461

Wilkinson, L. C. (2018). Teaching the language of mathematics: What the research tells us teachers need to know and do. The Journal of Mathematical Behavior, 51, 167–174. https://doi.org/10.1016/j.jmathb.2018.05.001

Wisconsin Center for Education Research (WCER). (2014). 2012 amplification of the English language development standards: Kindergarten – grade 12. WCER: Board of Regents of the University of Wisconsin.