POTENTIAL OF AN MEA TO ADVANCE BUSINESS STUDENTS' MODELING SKILLS

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The development of modeling skills in mathematics is essential for individuals to understand, describe, control, and predict phenomena around them. This article describes the results of an investigation to find out how an activity –based on a Models and Modeling Perspective– stimulates the mathematics modeling skills of undergraduate students who are in the first quarter of a business degree. As a result, it was shown that the MEA enabled students to exhibit, develop, and refine different modeling skills, such as: identification of variables, assumptions based on the real-life context, identification of patterns, and construction of mathematical representations.

Keywords: Modeling, higher education, precalculus.

Introduction

The pandemic generated by COVID 19 has highlighted the need for math education researchers and educators to reflect on the knowledge and skills that students must develop to understand, describe, control, and predict phenomena that impact humankind, and that also allows them to make decisions and take actions to relieve negative effects of these phenomena. Some educators suggest teaching strategies based on mathematics modeling that allow students to develop these skills and knowledge, as well as take active part on the solution process of these real-life problems (Lesh, 2010; Sevinc, 2021). Several authors, such as Lesh and Doerr (2003) and English et al. (2020), agree on the need to structure students' experiences in ways that allow them to develop modeling skills. According to Niss et al. (2007), learning to model implies that students develop different skills, such as "ability to identify relevant questions, variables, relations or assumptions in a given real world situation, to translate these into mathematics and to interpret and validate the solution of the resulting mathematical problem in relation to the given situation" (p. 12).

Project GAIMME indicates that "to understand how students grow in their ability to do mathematical modeling is a challenging task. Useful studies of mathematical modeling have been published, but this area has not yet been researched as deeply and systematically as other areas" (Garfunkel & Montgomery, 2019, p. 24). The National Council of Teachers of Mathematics (NCTM, 2018) mentions that mathematical modeling skills are part of modeling cycle processes that involve multiple steps and iterations. However, studies by Montero et al. (2021) and Touchstone (2014) highlight that word problems in math textbooks for higher education students in business careers have a single answer and don't usually foster modeling processes. Therefore, we need to provide learning-teaching opportunities that are purposefully designed for university students in business careers to develop modeling skills to solve real-life problems.

The purpose of this study is to understand how a model-eliciting activity (MEA) can contribute to students in business careers to develop and refine modeling skills. The research question is *What types of modeling skills do undergraduate students in business careers develop when they solve an MEA called Breaking barriers with yogurt investments?*

This research study used Model and Modeling Perspectives (MMP) as proposed by Lesh & Doerr (2003) by focusing on structuring students' experiences that foster the development of modeling skills.

Theoretical Framework

Models and Modeling Perspectives propose the design of model-eliciting activities for students to develop, amplify, and refine their thinking by enhancing math and modeling skills as they solve real-life problems (Lesh & Doerr, 2003). MEAs are situations that are purposefully designed for students to generate models using specific mathematical ideas, and are situated in real-life contexts that are meaningful to students (Aliprantis & Carmona, 2003; Doerr, 2016; Sevinc, 2021). Models are conceived as

> conceptual systems (consisting of elements, relations, operations, and rules governing interactions) that are expressed using external notation systems, and that are used to construct, describe, or explain the behaviors of other system(s)—perhaps so that the other system can be manipulated or predicted intelligently.
>
> A mathematical model focuses on structural characteristics (rather than, for example, physical or musical characteristics) of the relevant systems. (Lesh & Doerr, 2003, p. 10)

Models reside in the mind and representational media. These representational media can include written symbols, oral communication, diagrams, metaphors, tables, graphs, and algebraic symbols (Lesh & Doerr, 2003; Lesh & Harel, 2003).

Modeling activities based on real-life contexts usually have many possible solutions. They can also involve irrelevant ideas or lack information which creates a need for students to develop modeling processes such as generating assumptions and evaluating situations based on specific contexts (Sevinc & Lesh, 2021). Research studies based on MMP have provided evidence that MEAs promote students' development skills to solve problems related to their professional life in the field of business (Lesh & Yoon, 2007). According to Lesh (2010), MEAs foster students to show their thinking in ways that research can "to investigate the development of important aspects of students' mathematical thinking" (p. 29).

Methodology

We used qualitative methods to conduct this research study involving a total of 13 participants from a group of (male and female) students in their first quarter as freshmen enrolled in a business degree program. Students were registered in an in-person course on mathematics applied to business, which is the first mathematics course students take in the business undergraduate program. The topic on exponential functions had not been covered in this course previous to student participation in this study.

The MEA called Breaking barriers with yogurt investments (Yogurt MEA) was designed in the context of a colony of bacteria and yeast that produce Bulgarian yogurt (Figure 1). This MEA was designed using six principles for the construction of an MEA as proposed by Lesh et al. (2000). The mathematical ideas embedded in the design of the Yogurt MEA are: exponential function, equation, variation, growth, and rate of change. The solution requires that students identify the problem, variables and relationships, relevant assumptions, interpretation, and validation of the solution.

Business-Health Magazine

EL YOGURT ESTÁ ROMPIENDO LAS BARRERAS DE LAS INVERSIONES

LA CRECIENTE POPULARIDAD DEL YOGURT SE HA CONVERTIDO EN UN SALVADOR PARA LAS MAYORES LECHERÍAS. SUS VENTAS SUPERAN LAS VENTAS DE LECHE POR PRIMERA VÉZ

El yogurt se está convirtiendo en la "vaca en efectivo" de la industria láctea. De acuerdo con el informe de Mordor Intelligence 2018, se esperan ingresos de $106,400 millones de dólares para el año 2024 (Whitely, 2019). Muchos empresarios están emprendiendo negocios millonarios basados en el proceso de este lácteo.

La revista Forbes de negocios ha nombrado a Hamdi Ulukaya el Steve Jobs del yogurt, ya que su empresa en tan solo cinco años se puede valuar en $1,000 millones de dólares; su empresa es rentable, dominante y crece de forma importante. En estos cinco años la empresa de Ulukaya pasó de tener cinco a 2,000 empleados, convirtiéndose en el número uno del yogurt griego en Estados Unidos.

No solo las empresas de los países del continente americano han tenido estas altas utilidades, en China la creciente popularidad del yogur se ha convertido en un salvador para las mayores lecherías del país.

Las ventas de leche china crecieron sólo un 4% en 2017 en comparación con el año 2016, mientras que las ventas de yogures aumentaron un 18%.

Una de las razones por las cuales la empresa del yogurt está creciendo es debido a los grandes beneficios que tiene este lácteo para el cuerpo humano. Contiene una gran cantidad de calcio, un mineral necesario para dientes y huesos sanos. Sólo una taza proporciona el 49% de las necesidades diarias de calcio; es alto en vitaminas del grupo B, particularmente vitamina B12 y riboflavina, que pueden proteger contra enfermedades del corazón y ciertos defectos congénitos del tubo neural. Una taza también proporciona el 38% de la necesidad diaria de fósforo, 12% de magnesio y 18% de potasio. Estos minerales son esenciales para varios procesos biológicos, como la regulación de la presión arterial, el metabolismo y la salud ósea.

Un problema es que la mayoría de las variedades compradas en las tiendas pierden el grado de calidad por tres principales razones, la calidad de la leche no es buena, contienen sabores, endulzantes, colores artificiales y le adicionan azúcar.

Por esta razón, algunos negocios están emprendiendo en el sentido de hacer yogurt con características semejantes al yogurt casero que no tiene aditivos ni conservantes, esto permite al producto mantener sus nutrientes y ser un alimento sano para el cuerpo humano.

a

La empresa familiar "Kéfir healthy", después de leer el artículo pretende lanzar una línea de yogurt casero, que le permita entrar en el mercado. Uno de los principales obstáculos a los cuales se enfrenta es conocer el crecimiento de los búlgaros de leche (Kéfir). La empresa ha recabado algunos datos con un experimento en laboratorio.

Resultados de laboratorio

La empresa necesita conocer el comportamiento de crecimiento de los búlgaros, de tal manera que pueda decidir en qué momento tendrá cantidades suficientes para vender.

Tu equipo es el responsable de proponer a la empresa "Kéfir healthy" una descripción del comportamiento del crecimiento de los búlgaros y, por lo tanto, un procedimiento que le ayude a predecir en qué momento empezar a vender. Escribe una carta a la empresa donde le expliques cómo puede calcular la cantidad de búlgaros (mililitros) en cualquier momento. En tu carta debes incluir los procedimientos matemáticos, de tal manera que puedan utilizarse por cualquier otra empresa.

b

Figure 1: a) Yogurt MEA Newspaper article y b) Yogurt MEA Problem Situation

For the implementation of the Yogurt MEA, students were organized in five teams: Team 1 [T1] with two members, Team 2 [T2] with three members, Team 3 [T3] with three members, Team 4 [T4] with two members and Team 5 [T5] with three members. The Yogurt MEA was implemented in several phases:

- Phase 1. Warm-up Activity (15 min). Students read page 1 of the MEA (Figure 1a), answered the warm-up activity, and discussed the context in their teams.
- Phase 2. Team Solving the Yogurt MEA (90 min). Students worked in teams to solve the problematic situation in the MEA (Figure 1b includes the QR code with the video students watched). Students used tools such as spread sheets.
- Phase 3. Teams presented their model solutions to the whole class (30 min). Each team presented their solution model and the process they used to construct it. After each presentation, the class had the opportunity to ask questions and provide feedback about the model.

Data collection involved: video recordings of the group discussion about the model-construction processes and presentation of the final solution model; the solution letters written by each team; log notes from class observations; and the electronic files that the students submitted. Data analysis was guided by an MMP, through a qualitative lens considering the iterative modeling process, which included each team's final solution. We documented how students changed, extended, and refined their conceptual system during the solution process and during whole-class presentations and discussion. In particular, the analytical framework we used is supported in the skills as described by Niss et al. (2007), which include: the ability to identify questions, variables, relationships, relevant assumptions in a given real-life situation, interpretation, and validation of a solution.

Results and Discussion

Based on the models built by the students, we identified that the Yogurt MEA elicited modeling abilities, in particular, four: identification of variables, assumptions based on the real-life context, identification of patterns, and construction of mathematical representations (Table 1).

Table 1: Teams' Modeling Abilities

<table>
<tr><th></th><th>Identification of variables</th><th>Assumptions based on the real-life context</th><th>Identification of patterns</th><th>Representations</th></tr>
<tr><td>T1</td><td rowspan="5">They identified the relationship between the variables: amount of kefir grains, time, and temperature.</td><td rowspan="3">Assumption A. The growth of kefir grains (G) depends on the temperature (T) and time (t). The temperature and time are variables.</td><td rowspan="2">They described and predicted the growth of kefir grains. They identified a pattern of weekly cyclical growth.</td><td rowspan="2">They included verbal, tabular, and graphical representations.</td></tr>
<tr><td>T2</td></tr>
<tr><td>T3</td><td>They described the growth of kefir grains. They multiplied the amount of initial kefir grains by a constant.</td><td rowspan="2">They included verbal and tabular representations.</td></tr>
<tr><td>T4</td><td rowspan="2">Assumption B. The growth of kefir grains (G) depends on the temperature (T) and time (t). They maintained the temperature constant, and the time was variable G(t).</td><td>They described and predicted the kefir grains' growth based on the number of jars. The growth estimation was 625% in four weeks.</td></tr>
<tr><td>T5</td><td>They described and predicted the kefir grains' growth. The daily growth rate estimated was 33%.</td><td>They included verbal, tabular, and graphical representations.</td></tr>
</table>

Team's Modeling Abilities

Identification of variables. Based on the data provided and the Yogurt MEA video, all the students identified three variables related to the process of kefir grains growth. They focused their attention on the variables: growth, temperature, and time. For instance, team T3 mentioned the following in reference to their data table:

> [1] T3: Tenemos, en cuanto a los días, temperatura, mililitros y… esto es la diferencia entre un día y otro en cuanto a mililitros [We have, in relation to days, temperature, milliliters and... this is the difference in milliliters between one day and another].

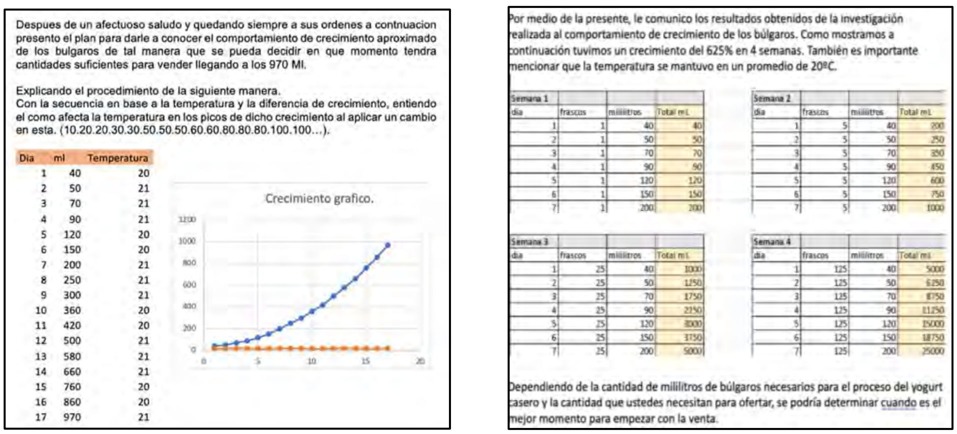
The team T1 indicated the following while they pointed to the column headings of their table (Figure 2a).

> [2] T1: Nosotros intentamos resolver esto, primero, con los datos que nos habían dado [para] los primeros siete días. Viendo así, el cambio de temperatura y [cómo] los mililitros cambiaban. [We tried to solve this, first, with the data given for the first seven days. Looking at the temperature change and how the milliliters changed].

After the team's T1 presentation, T2 mentioned that they had found a different relationship among variables.

[3] T2: Yo no estoy de acuerdo… nosotros que también nos basamos en la temperatura y en el porcentaje, nos dimos cuenta que para el día seis, siete el cambio no era de 10 10. Sino se elevaba al doble, pues, de hecho, sacamos como una tablita. [I do not agree… we also considered the temperature and the percentage, and noticed that on day sixth, seventh the change was not 10. Instead, it doubled, and we actually built a table].

[4] T1: Yo también hice esa secuencia [refiriéndose a un modelo inicial que había construido y posteriormente refinó con base en el contexto de la MEA]. [I also created this sequence [referring to an initial model that they initially build and refined later based on the MEA's context].


Despues de un afectuoso saludo y quedando siempre a sus ordenes a contnuacion presento el plan para darle a conocer el comportamiento de crecimiento aproximado de los bulgaros de tal manera que se pueda decidir en que momento tendra cantidades suficientes para vender llegando a los 970 Ml.

Explicando el procedimiento de la siguiente manera.
Con la secuencia en base a la temperatura y la diferencia de crecimiento, entiendo el como afecta la temperatura en los picos de dicho crecimiento al aplicar un cambio en esta. (10.20.20.30.30.50.50.50.60.60.80.80.80.100.100…).

Dia	ml	Temperatura
1	40	20
2	50	21
3	70	21
4	90	21
5	120	20
6	150	20
7	200	21
8	250	21
9	300	21
10	360	20
11	420	20
12	500	21
13	580	21
14	660	21
15	760	20
16	860	20
17	970	21

Por medio de la presente, le comunico los resultados obtenidos de la investigación realizada al comportamiento de crecimiento de los búlgaros. Como mostramos a continuación tuvimos un crecimiento del 625% en 4 semanas. También es importante mencionar que la temperatura se mantuvo en un promedio de 20ºC.

Semana 1			
dia	frascos	mililitros	Total mL
1	1	40	40
2	1	50	50
3	1	70	70
4	1	90	90
5	1	120	120
6	1	150	150
7	1	200	200

Semana 2			
dia	frascos	mililitros	Total mL
1	5	40	200
2	5	50	250
3	5	70	350
4	5	90	450
5	5	120	600
6	5	150	750
7	5	200	1000

Semana 3			
dia	frascos	mililitros	Total mL
1	25	40	1000
2	25	50	1250
3	25	70	1750
4	25	90	2250
5	25	120	3000
6	25	150	3750
7	25	200	5000

Semana 4			
dia	frascos	mililitros	Total mL
1	125	40	5000
2	125	50	6250
3	125	70	8750
4	125	90	11250
5	125	120	15000
6	125	150	18750
7	125	200	25000

Dependiendo de la cantidad de mililitros de búlgaros necesarios para el proceso del yogurt casero y la cantidad que ustedes necesitan para ofertar, se podría determinar cuando es el mejor momento para empezar con la venta.

a **b**

Figure 2: a) T1's Model and b) T4's Model

Discussion. The Yogurt MEA fostered an iterative discussion of the variables related to the kefir grains growth, [3] and [4]. According to Bliss et al. (2014), identification of variables is important because it allows students to determine the factors influencing the phenomenon they are analyzing and "distinguish between independent variables, dependent variables, and model parameters" (p. 7).

Assumptions based on the real-life context. The students built their models by generating assumptions based on the real-life context; they contrasted their procedures with the actual growth of kefir grains and their personal and professional experiences. We identified two types of assumptions.

Assumption A. The initial assumption of teams T1, T2, and T3 was that kefir grains growth (G) depends on the temperature (T) and time (t). They assumed that the temperature and time were independent variables. T2's personal experience allowed them to identify the possibility of additional variables that could influence the growth. The following excerpt illustrates this idea.

[5] T2: Yo, por experiencia que he cultivado muchos búlgaros, lo que platicaba con ellas, al tener siempre la misma temperatura, los búlgaros van reaccionando... lo ideal sería mantener la misma temperatura, para mayor crecimiento. Siento que depende mucho de varios factores. [I, from my experience growing kefir grains, what I was talking with them, because they always keep the same temperature, the kefir grains react… the best scenario would be to maintain the same temperature, for a higher growth. I feel that it depends of multiple factors].

T3 showed the need to investigate the kefir growth to understand the phenomenon and the influence of time and temperature. The students made the following statement:

[6] T3: Primero que nada, no sé si alguno de ustedes investigó un poco sobre este tipo de bacterias. Por medio de la investigación te das cuenta … que van creciendo solas dependiendo de la temperatura y el ambiente, y ese rollo. Se van reproduciendo, una se convierte en dos y así. [First of all, I don't know if any of you investigated about this type of bacteria. Through investigation you realize… that they grow by themselves depending on the temperature and the environment, and all that. They reproduce, one becomes two and so on].

Assumption B. Assuming an entrepreneurial role, Teams T4 and T5 considered a constant temperature based on the MEA context and the precision of industrial refrigerators [9], and they focused on modeling the kefir grains growth depending on time, $\mathrm{G}(t)$. For instance, T5 justified considering a constant temperature by contrasting their mathematical analysis with the factors that influence the growth of kefir grains in real-life.

[7] T5: noté que cuando teníamos una temperatura menor de 20º el porcentaje de crecimiento se reducía, se volvía a elevar cuando la temperatura subía porque es una bacteria. Hace rato lo comentó, el otro Bryan; que al ser una bacteria, el ambiente, la cantidad de proteínas y lo demás que hay alrededor es influyente para que se reproduzcan. Si tienen más temperatura, igual aun no conozco mucho a que temperatura se mueran y demás, pero con los datos que nos dieron, a mayor temperatura o sea 21º se reproducen más, manteniendo el sistema de las bacterias en un estándar. Entonces, mantener una temperatura, sí es posible. Yo dije mantenemos siempre la temperatura, saco un promedio en el porcentaje del crecimiento que me daba de los días de, que teníamos 21º, pues me dio un promedio del 33%. Entonces si yo mantengo siempre mi temperatura a 21º porcentualmente, más menos, voy a tener un crecimiento del 33%. [I noticed that when we had a temperature lower than 20º [C], the growth percentage decreased, it increased again when the temperature rose because it is a bacterium. If they have a higher temperature, I still don't know a lot about the temperature at which they die and so on, but with the data they gave us, the highest the temperature, i.e., 21º they reproduce more, maintaining the system of the bacteria in standard conditions. Then, maintaining the temperature is possible. I said, we always maintain the temperature, I obtain an average of the growth percentage from the days we had 21º C, and I got an average of 33%. Then if I always maintain my temperature at 21º C percentage, more or less, I will have a growth of 33%].

During the questioning phase, team T1 hesitated about the possibility of considering a constant temperature during the kefir grains growth. Based on their experience, T5 replied the following:

[8] T1: Pero está difícil, la temperatura hace un cambio de un día para el otro. [But it is hard, the temperature changes from one day to the next].
[9] T5: sí, pero no es lo mismo el sistema de refrigeración industrial que el de un salón. [yes, but the industrial refrigeration system is not the same as a classroom].

ml	temperatura	%	promedio de crecimiento
40.00	20		
50.00	21	20	33%
70.00	21	40	
90.00	21	40	
60.00	20	30	
79.80	20	25	
106.13	21	33	
141.16	21	30	
187.74	21	33	
249.69	21	33	
332.09	21	33	
441.69	21	33	
587.44	21	33	
781.30	21	33	
1,039.12	21	33	
1,382.04	21	33	
1,838.11	21	33	
2,444.68	21	33	
3,251.43	21	33	
4,324.40	21	33	

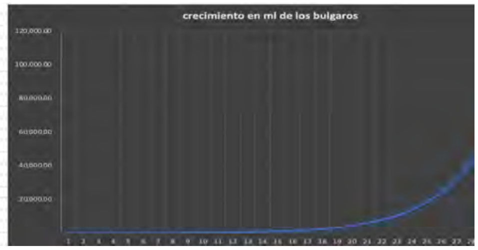


Figure 3: Mathematical Representations Included in T5's Model

Discussion. The fact that the Yogurt MEA includes three variables -time, temperature, and amount of kefir grains that influence their actual growth, allowed students to generate real-life assumptions to model the problematic situation. The whole group discussion created opportunities for students to revise and evaluate their assumptions and models. According to Garfunkel and Montgomery (2019), during the modeling process "we select 'objects' that seem important in the real-world question and identify relations between them. We decided what we will keep and what we will ignore about the objects and their interrelations" (p. 12).

Identification of patterns. The MEA allowed students to exhibit and develop their ability to identify patterns and build various strategies to solve the problem. Teams T1, T2, and T3 identified a "weekly cyclical growth" pattern.

[10] T2: Cada siete días íbamos repitiendo el ciclo, hasta terminar. [Every seventh day, we repeated the cycle until finished].

Team T3 described the kefir grains growth behavior, and teams T1 and T2 predicted the growth behavior in addition to describing it. For instance, the team T1suggested a growth pattern that established a relationship between time and temperature variation (Figure 2a). Even though the mathematical succession T1 developed requires further refinement to predict the growth in a way that is closer to the actual phenomenon, it provided a model to solve the situation.

Team T4 identified a kefir grain growth pattern that was not strongly related to the temperature. They focused on describing and predicting the growth considering the contextual factors. They suggested that the company could use jars to replicate growth whichever number of times was necessary to obtain sufficient kefir grains to produce Yogurt (Figure 2b).

[11] T4: Tomamos el crecimiento del primer día al día siete, y cerramos como semana. Volvimos a empezar en uno. Como al día siguiente teníamos 200, nosotros dividimos, 200 entre 40ml y ya fijamos los cinco frascos (segunda fila, tabla Semana 2, Figura 2b). [We considered the growth from the first to the seventh day, and we finished the week. We started again like in day one. The next day we had like 200, we divided, 200 by 40 ml and then we have five jars (second row, Week 2 table, Figure 2b)].

Team T5 identified an exponential growth pattern. They calculated the daily kefir grains growth and the corresponding percentage. Next, they obtained the average growth percentage considering only the days in which the temperature was 21ºC. The team obtained an average rate of 33% and predicted the growth (Figure 3). The students found a growth pattern that corresponds to the expression.

Discussion: The Yogurt MEA prompted students to organize and systematize information, recognize patterns, build various solution strategies and generate diverse models to estimate the growth. Sevinc and Lesh (2021) discuss that problems in realistic contexts can be solved through various solution strategies and "allow students to make mathematical and contextual inferences, an essential activity for the development of problem-solving competence" (p. 4).

Construction of mathematical representations. Given that the data included in the Yogurt MEA was included in various images embedded in a video (see QR code, Figure 1b), the students had to extract, interpret, organize and create mathematical representations to analyze the kefir grains' growth. The students used diverse representations in their models. For instance, teams T1, T2, and T5 included verbal, tabular, and graphical (Figures 2 and 3), while teams T3 and T4 included oral and tabular representations in their model. These representations allowed them to describe, explain and estimate when the kefir grain growth was sufficient to start thinking about selling yogurt (Figure 2b). The following excerpt illustrates the way they described it verbally and in their letters:

> [12] T5: En un mes ya tengo un aproximado de 3600 litros que ya los puedo mandar a producción. Porque aquí tengo los mililitros en bacterias, más no tengo la producción en yogurt. [I have approximately 3600 liters in a month that I can send to production. Because this is the milliliters in bacteria, but I do not have the production of yogurt].

Discussion: The Yogurt MEA fostered model-building using diverse representations to explain and describe the kefir grains growth phenomenon. The creation of the models reported by the students revealed their modeling abilities and the knowledge used and developed throughout the problem-solving activity. According to Lesh and Doerr (2003), "meanings associated with a given conceptual system tend to be distributed across a variety of representational media" (p. 12).

Conclusions

The analysis of the results supports the conclusion that the Yogurt MEA has the potential to support the development of diverse modeling abilities. In this study, students developed modeling abilities through model building, team interactions, and whole-group discussion. The abilities reported included: variable identification, assumptions based on the real-life context, identification of patterns, and construction of mathematical representations.

The identification of variables was fostered through the inclusion of three variables in the Yogurt MEA. The students had the opportunity to identify, select and establish relationships among variables to solve the problem. The assumptions based on the real-life context emerged because the Yogurt MEA was situated in a context close to the students. Thus, it fostered opportunities to self-evaluate and contrast models with real-life experiences growing kefir grains. This is consistent with the fact -discussed in the MMP- that the ways of thinking or models built to make sense of situations, requiring realistic decision-making, integrate with frequency ideas from more than one discipline or theory. The teams identified patterns to describe and explain the kefir grains growth and estimate the time to decide when a sufficient amount of kefir grains would be available to start selling yogurt. The teams built mathematical representations to organize data, interpret the situation, identify patterns and explain their model.

We observed students refine their modeling abilities during the MEA solution process, which was mediated by their understanding of the phenomenon, mathematical knowledge, personal and professional experience, and interaction with the environment.

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