EXAMINING HOW UNDERGRADUATE STUDENTS DESCRIBE THE STANDARDS FOR MATHEMATICAL PRACTICE

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While the Common Core State Standards for Mathematical Practice are a focal point of K-12 mathematics education, there is limited research examining how future teachers' (e.g., undergraduate students, teacher candidates) develop their conceptions of these standards. We investigate how opportunities within a mathematics-focused bridge course within a teacher education program provided opportunities for undergraduate students to develop their conceptions of the Standards for Mathematical Practice. Specifically, we explore how undergraduate students drew upon the Common Core provided descriptions to describe key practice ideas. This study contributes to the scholarship on mathematics teacher education and how teacher educators can support students in developing their understanding of mathematical practice.

Keywords: mathematical practices, mathematics teacher education

The Common Core State Standards for Mathematical Practice (SMPs) describe eight ways of thinking and doing mathematics that parallel the ways that mathematicians engage with mathematics (National Governors Association for Best Practices, 2010). These standards reflect a broader goal in K-12 mathematics education focused on moving beyond K-12 learners *acquiring* mathematical content knowledge, towards developing learners capable of actively *doing* something with mathematics (Kilpatrick et al., 2001; National Council of Teachers of Mathematics, 2018). Implicit in the creation and adoption of the SMPs is the assumption that if K-12 learners are to develop these skills and ways of thinking, their mathematics teachers should provide appropriate opportunities and support to develop and engage in these practices.

Initial teacher education offers a space where mathematics teacher educators can provide opportunities for undergraduate students to develop their conceptions about the SMPs including what they are (Bostic & Matney, 2014; Kruse et al., 2017), what doing an SMP looks like (Max & Welder, 2020), and how to support K-12 learners in doing them as well (Cheng, 2017; Gurl et al., 2016). While there are studies of how initial teacher education provides opportunities for undergraduate students to develop their conceptions of the SMPs, these studies are limited and there remain questions regarding these opportunities. In particular, because of the limited literature, it is unclear how different initial teacher education contexts (e.g., content courses, methods courses, bridge courses, student teaching) provide opportunities for undergraduate students to develop their conceptions of the SMPs. Understanding the differences, affordances, and limitations of the opportunities in these contexts is essential if teacher educators are to effectively support (future) teachers in providing K-12 learners with the appropriate opportunities and supports.

This paper shares how a mathematics-focused bridge course within a teacher education program provided opportunities for undergraduate students (hereto called 'students') to develop their conceptions of the SMPs. We discuss how students described the SMPs, including what ideas from the Common Core SMP descriptions students attended to in their own descriptions. We then discuss questions for further research based on these findings. We add to the scholarship on mathematics teacher education and how teacher educators can support their students in building their understanding of mathematical practices.

Framework

To understand how a bridge course provided opportunities for students to develop their conceptions of the SMPs, this study draws upon situative perspectives on learning (Greeno, 1998; Peressini et al., 2004) and sensible belief systems (Hoyles, 1992; Leatham, 2006). Specifically, these perspectives are used to examine how course learning opportunities supported the development of students' knowledge and beliefs about the SMPs. The following sections provide an overview of situative perspectives on learning, including situated knowledge and beliefs, as well as the notion of sensible belief systems.

Situative perspectives on learning

Situative perspectives conceptualize learning as both a social and individual process (Greeno, 1998; Peressini et al., 2004). Learning is social because one learns by actively participating within a context, and through this participation comes to learn the knowledge and accepted social practices of that context. Learning is also an individual process because it is changes in how an individual participates that indicate learning, as one's actions begin to reflect the accepted ways of participating in a given context. Scholars such as Putnum and Borko have argued that due to the variety of contexts within which initial teacher education occurs (e.g., content courses, methods courses, bridge courses, student teaching), applying a situative perspective to teacher education of offers a way of, "disentangling – without isolating - the complex contributions of these various contexts to novice teachers' development" (2000, p. 71).

In addition to knowledge, teachers' beliefs are also situated (Green, 1971; Hoyles, 1992; Leatham, 2006). In particular, the idea of *clustering* describes how beliefs can be connected to or isolated from one another based on the clusters within which they are held (Green, 1971), with clusters being based on the contexts within which the beliefs were formed. The idea of clustering is essential to making sense of beliefs because it describes how someone may hold beliefs that appear contradictory because they are held in different clusters. This clustering allows for the contextualization of beliefs where "a person may believe one thing in one instance and the opposite in another" (Leatham, 2006, p. 95) In other words, beliefs, like knowledge, are situated.

Sensible Systems of Beliefs

Due to the situated nature of beliefs, Leatham (2006) argues that how teachers' beliefs (and actions) are positioned in the literature needs to shift. He argues that if beliefs are situated then teachers' beliefs and actions are sensible with respect to the context within which they are working. Furthermore, any perceived 'inconsistencies' between teachers' beliefs and actions are indications that there are *other* beliefs at play in that particular context that are taking precedent over others; these beliefs may be held consciously or unconsciously and may be difficult for teachers to articulate. However, regardless of whether a belief is made explicit or not, teachers' actions are "fundamentally sensible" (Hoyles, 1992, p. 37) with respect to their beliefs.

Applying a sensible systems lens to teachers' beliefs pushes researchers to better understand the beliefs that are actually influencing teachers' actions rather than what we want to be the influence (Leatham, 2006). By understanding the actual influences on teachers' actions, teacher educators can better provide opportunities for students in teacher education courses to explore their beliefs, as well as provide learning opportunities that can shape students' beliefs so that what is sensible reflects the broader goals of K-12 mathematics education policy.

Conceptions

Finally, while knowledge and beliefs can be discussed separately, some scholars use the notion of conceptions to capture knowledge and beliefs together (Lesseig & Hine, 2021; Philipp, 2007; Thompson, 1992). This is because while knowledge and beliefs can be considered two distinct ways of knowing (beliefs are held with varying degrees of conviction while knowledge is held with certainty), distinguishing between them can be difficult. For example, the level of conviction with which one holds beliefs and knowledge can vary between people (Philipp, 2007). Moreover, what is considered knowledge to one person may be belief for another. Therefore, this study uses the notion of conception to capture students' knowledge and beliefs about the SMPs without trying to disentangle them. More specifically, this study investigates how a bridge course within a teacher education program provided opportunities for undergraduate students to develop their conceptions of the SMPs.

Context and Methods

Data for this study were collected from two sections of a mathematics-focused bridge course focused on familiarizing undergraduate students interested in becoming teachers with K-12 mathematics education policy. In five two-week modules, students in this course were introduced to the Common Core SMPs and content standards (NGA, 2010), the five strands of mathematical proficiency (Kilpatrick et al., 2001), and other key instructional ideas such as multiple representations or classroom discourse (see Table 1). To support students' developing understanding of the SMPs, the course provided opportunities for students to solve mathematical tasks, analyze their own mathematical work and the work of others for evidence of the SMPs, and reflect on their growing understanding of the SMPs and their relation to other course topics. Through activities such as these, course learning opportunities acted as a *bridge* between students thinking about *mathematics* and thinking about *teaching and learning mathematics*.

Table 1: Bridge Course Module Overview

Module	SMP	Course Topic	Strand of Proficiency
1	CCSSM & SMP Overview	Mathematical Identity	Productive Disposition
2	SMP 2: Reasoning SMP 6: Precision	Multiple Representations	Conceptual Understanding
3	SMP 7: Structure SMP 8: Repeated reasoning	Discourse	Procedural Fluency
4	SMP 3: Argumentation SMP 5: Tools	Justification	Adaptive Reasoning
5	SMP 1: Problem-solving SMP 4: Modeling	Cognitive Demand	Strategic Competence

Eight students consented to participate from the two sections of the course. Section one took place online, asynchronously during Winter 2021, with two of 12 enrolled students consenting to participate; Section two took place online, synchronously during Spring 2021 with six of 35 enrolled students consenting to participate. Of the eight participants, two expressed interest in becoming secondary mathematics teachers, five in becoming elementary teachers, and one a K-12 guidance counselor; one student was employed as an elementary classroom assistant while another was completing their elementary student teaching practicum. While limited, the consenting students capture the range of backgrounds and interests enrolled in this course.

Methodology

The primary data sources for this study are two course assignments, one completed at the end of module 3 and another completed at the end of module 5 (*n=16*; two per student); these assignments were a part of the course and would have been completed irrespective of the study. Through assignment prompts, students were asked to describe and provide evidence of their developing understanding of the SMPs including a) how they would describe key SMP ideas, b) why the SMPs are important, c) connections between the SMPs and other course topics, d) examples of how they saw themselves or others doing the SMPs when completing mathematical tasks, e) how class activities have shifted their understanding of the SMPs, and f) remaining questions they had about the SMPs.

To analyze these data, we first read through all student assignments and segmented the data by structural codes that corresponded to the assignment prompts (Saldaña, 2013). This first phase of segmenting and coding indicated that of the assignment prompts, students most frequently provided *descriptions* of key SMP ideas (65 of 231 coded segments). Based on these frequencies, second phase coding focused on the *description* text segments identified in phase one. Second phase coding used Toulmin's model of argumentation (1958) and an adaptation of Nardi et al.'s (2012) classification of warrants to capture the sources students drew upon when describing the SMPs. Due to space, we only provide a brief overview of these frameworks and how they guided analysis; more detail will be provided in our presentation.

Briefly, Toulmin's model (1958) identifies six different components of an argument: 1) a *conclusion/claim*, 2) the *data* upon which the conclusion is based, 3) the *warrant* that connects the conclusion to the data, 4) a *backing* that further supports the warrant through additional reasoning or evidence, 5) *qualifiers* to express one's confidence in the argument, and 6) *rebuttals* that address possible exceptions or refutations to the argument. While many scholars have used Toulmin's model to analyze teachers' and students' mathematical and pedagogical arguments (Krummheuer, 2015; Steele, 2005; Yackel, 2001, 2002), this model also has limitations. In particular, while Toulmin offers a way to make sense of an argument's *structure*, it does not offer a way to make sense of an argument's *quality*. To address this, scholars have proposed classifying warrants to identify the sources of influence one may draw upon when making and supporting an argument (Freeman, 2005; Nardi et al., 2012). For example, Nardi and colleagues (2012) offer seven different types of warrants mathematics educators use in their arguments: 1) *a priori epistemological*, 2) *a priori pedagogical*, 3) *institutional curricular*, 4) *institutional epistemological*, 5) *empirical personal*, 6) *empirical professiona*l, and 7) *evaluative*. These categories capture the scope of influences on mathematics teachers' arguments including their own personal or professional experiences (*empirical*), personal views or beliefs (*evaluative*), curricular resources (*institutional curricular*), shared disciplinary practices (*institutional epistemological*), and established definitions and pedagogical principles (*a priori epistemological* and *pedagogical*).

This study uses an adaptation of Nardi et al's (2012) warrant classification to which we added *policy* classifications to capture the influence of the CCSSM content standards (*policy-content*) and SMP descriptions (*policy – SMP*) (NGA, 2010), and the strands of mathematical proficiency (*policy – strand*) (Kilpatrick et al., 2001). These additional classifications were created to explicitly capture students' attention to different K-12 mathematics education policies, as well as recognize the ongoing debate about the eight SMPs in the literature. Specifically, scholars have argued the SMPs provide an inaccurate picture of mathematical practices due to the methods used to identify them (e.g., self-reported autobiographical data) or because the wide range of practices used by mathematicians cannot be captured in such a small number of standards (Moschkovich, 2013; Weber et al., 2020). Therefore, while *institutional epistemological* warrants refer to shared disciplinary practices, we determined it would be inappropriate to classify the SMPs as 'shared' due to the ongoing debate and created this new categorization. In using this adapted categorization of warrants to analyze students' SMP descriptions, our analysis allowed us to see which sources students were drawing upon as they described the SMPs, including how they related to course learning opportunities and resources.

Findings and Discussion

Within the 65 text segments coded as *description* during first-phase coding, second-phase codes were applied to 165 smaller segments, with each segment representing a different idea within students' SMP descriptions. As illustrated in Table 2, the CCSSM SMP descriptions were overwhelmingly the most frequent influence on students' SMP descriptions (*policy – SMP*; *n=114*). As almost 70% of text segments were coded in this category, further analysis focused on which components or ideas from the CCSSM students drew upon in their descriptions.

Table 2: Frequency of Second-Level Code Application

Influence	Frequency
Policy – SMP	*114*
Institutional Curricular	32
Institutional Epistemological	7
Evaluative	7
A priori Pedagogical	3
Policy – Content	1
Empirical Personal	1

Influence of the CCSSM

Overall, students attended to a range of different ideas from the CCSSM when describing the SMPs, ranging from four different ideas for SMP 5: Tools to 15 different ideas for SMP 3: Argumentation. Closer analysis indicates that how students attended to these ideas differed across SMPs. Specifically, students' descriptions of SMP 1: Problem-solving and SMP 4: Modeling focused on the overarching idea(s) of the SMP, with less attention given to specific actions related to doing the SMP. Alternatively, students' descriptions of SMP 3: Argumentation and SMP 6: Precision focused on one particular way of doing these SMPs, even though the CCSSM describes each of these SMPs as involving multiple actions. Finally, descriptions of the remaining four SMPs generally focused on a smaller range of ideas, including overarching idea(s) as well as specific actions for doing the SMPs.

Overarching idea(s)

Table 3 illustrates the different ideas from the CCSSM descriptions of SMP 1: Problem-solving and SMP 4: Modeling that were attended to in students' descriptions. For SMP 1: Problem-solving, students most frequently attended to the overarching ideas of *make sense of problems* (*n=7*) and *planning* (*n=7*). The CCSSM description also includes *how* one may do this sense making, such as *creating representations, considering analogous problems*, or *looking for entry points,* however, these actions were some of the ideas least frequently included in students' descriptions. This is similar to students' descriptions of SMP 4: Modeling which primarily focused on the overarching idea of *connecting math and the real world* (*n=9*), including *solving real world problems* (*n=7*) and *using math in real world contexts* (*n=2*). However, as with problem-solving, students attended less to the actions involved in carrying out this overarching idea, such as *making assumptions* or *identifying important quantities*. Taken together, these findings suggest students' conceptions of SMPs 1 and 4 are largely focused on the overarching idea(s) of these SMPs, with less focus on specific actions involved in doing them.

Table 3: Ideas from the CCSSM included in students' SMP 1 and SMP 4 descriptions

SMP 1: Problem-solving		SMP 4: Modeling	
Make sense of problems	7	Connecting math & the real world	9
Planning	7	Solve real world problems	7
Evaluate own work	6	Apply or use math in real-world contexts	2
Revise/change plan	3	Create representation	3
Create representation	2	Make assumptions	2
Look for entry points	1	Interpret results with respect to context	1
Analogous problems	1	Identify quantities	1
Persevere	1	Make sense of relationships to draw conclusions	1
		Example of modeling	1

Specific actions

Students' descriptions of SMP 3: Argumentation and SMP 6: Precision primarily focused on *particular ways of doing* each of these SMPs. For example, the CCSSM describes SMP 3 as the ability to "construct viable arguments and critique the reasoning of others" (NGA, 2010). In other words, argumentation in mathematics includes both "give and take" where one builds and shares an argument but also makes sense of the arguments of others. From Table 4 we can see that students primarily focused on the "give" aspect of doing SMP 3, largely describing SMP 3 as justifying one's own solution and communicating that solution or justification to others; listening to, discussing, and critiquing the arguments of others were some of the least frequently raised ideas. Overall, these findings illustrate how students generally described SMP 3: Argumentation as one-directional, focusing on students constructing and sharing their own arguments.

Students' descriptions of SMP 6: Precision also focused on a particular way of doing this SMP. Students primarily described SMP 6 as precision with respect to communication, both in general and how definitions, mathematical language, labels, and calculations can support communication (see Table 4). Precise communication, including the formulation of clear explanations and use of definitions, are explicitly included in the CCSSM SMP 6 description. Interestingly, the CCSSM also describes SMP 6 as the ability to "calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the context of the problem" (NGA, 2010). Despite this explicit attention to precise calculations, only two students referred to calculations in their descriptions, with one student including accurate calculations as a *component of* communication. Furthermore, no students touched on the notion of the appropriateness of precision for the context of the problem. These findings suggest that students' conceptions of SMP 6 are largely focused on precision with respect to communication *about* mathematics, with less attention to precision when carrying out mathematical calculations or processes.

Table 4: Ideas from the CCSSM included in students' SMP 3 and SMP 6 descriptions

SMP 3: Argumentation		SMP 6: Precision	
Communicate solution or justification to others	11	Communication	14
Grade-level appropriateness of arguments	6	Definitions	4
Justify	4	Mathematical language	3
Example of engagement	3	Explain reasoning	3
Concrete referents	2	Examine claims	1
Create diagrams	1	Correct calculations	1
Listen to other	3	Units	1
Identify mistakes/flaws	2	General	1
Critique others' ideas, solutions, or justifications	2	Label graphs	2
Compare arguments	1	Units	2
Ask clarifying questions	1	Correct calculations/answer	1
Make conjectures	1	Clear work	1
Discuss others' solutions or justifications	1	Contextual meaning	1
Use definitions and established information	1		
Analyze problem	1		
Respond to others	1		

Remaining SMPs

Students' descriptions of SMP 2: Reasoning, SMP 5: Tools, SMP 7: Structure, and SMP 8: Repeated reasoning included both overarching ideas and actions related to doing each SMP; these descriptions also generally focused on a smaller range of ideas overall. For example, while the average number of different SMP ideas raised throughout students' assignments for SMPs 1, 3, 4, and 6 was 9, students raised an average of 4.75 different ideas for SMPs 2, 5, 7, and 8.

When describing SMP 2: Reasoning, students' descriptions included multiple ideas from the CCSSM including: contextualizing, decontextualizing, reasoning about quantitative relationships, and creating representations. Similarly, students' descriptions of SMP 5: Tools included both the ability to use tools and the ability to determine which tools are available and appropriate for a given problem, with each of these ideas occurring almost equally in students' descriptions (see Table 5).

As previously discussed, students sometimes focused on a particular action in their descriptions even if multiple actions were included in the SMP title itself (e.g., SMP 3: Constructing **and** critiquing arguments). Interestingly, this was not the case for SMP 7: Structure or SMP 8: Repeated reasoning, which both include looking for *and* making use of structure/ repeated reasoning. While the "look for" aspect of SMPs 7 and 8 was the most frequently raised by students (see Table 5), students included how to "make use" almost as frequently. For example, students' descriptions of SMP 7: Structure included *identifying underlying structures* *and patterns (n=5)* and how to make use of this structure via the *(de)composition* of mathematical objects almost equally *(n=4)*. This is similar to students' descriptions of SMP 8: Repeated reasoning which focused on *identifying patterns and repetition (n=10)*, as well as how to use that repetition to *generalize (n=6)* and *determine shortcuts (n=2)*.

Table 5: Ideas from the CCSSM included in students' SMP 2, SMP 5, SMP 7, and SMP 8 descriptions

SMP 2: Reasoning	
Decontextualize	4
Contextualize	3
Make sense of quantitative relationships	3
Create representations	3
Units	1
Flexible use of operations	1

SMP 5: Tools	
Use tools	6
Identify appropriate tools	5
Examples of tools	4
Purpose of tools	2

SMP 7: Structure	
Identify underlying structures or patterns	5
Examples of using structure	5
(De)composition	4
Shift perspectives	2

SMP 8: Repeated reasoning	
Identify patterns/repetition	10
Generalize	6
Reasonableness of results	3
Examples of looking for or using repeated reasoning	2
Determining shortcuts	2

Interestingly, while students' descriptions of both SMP 7: Structure and SMP 8: Repeated reasoning included the idea of patterns, the word 'pattern' is only included in the CCSSM description for SMP 7; SMP 8 refers to noticing repeated calculations or regularity. Therefore, while students' descriptions suggest the CCSSM helped students understand what making use of structure and repeated reasoning entails, the frequent use of the word 'pattern' to describe both does raise questions about the similarity between SMPs 7 and 8.

Conclusion

This study explores how a mathematics-focused bridge course provided opportunities for undergraduate students to develop their conceptions of the SMPs. The influence of the CCSSM on students' descriptions suggests that course opportunities to read and discuss the CCSSM SMP descriptions supported students in developing their SMP conceptions. Furthermore, the ideas that students *did* include provides insight into which ideas students are holding onto and can act as a foundation upon which to build in future learning opportunities. In doing so, we hope this study will inform future research examining how different teacher education contexts provide learning opportunities for students to develop their SMP conceptions, as well as how later learning opportunities can build off conceptions established in previous courses.

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