"THIS ONE IS THAT": A SEMIOTIC LENS ON QUANTITATIVE REASONING

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Despite significant research exploring students' quantitative reasoning, few studies have explored the semiotic processes that mediate its development. In this report, we present a case study to show how one student constructed a semiotic chain for a quantity as he worked with a mathematical task. Importantly, we connect frameworks for quantitative reasoning and semiotics to make sense of this process. Our findings show how our case student constructed a sign for a chunk of change in a triangle to support his later construction of the quantity of amount of change of area. We also describe how the case student leveraged these signs to bolster his development of the quantity of total area. We emphasize the role of artifacts, such as physical manipulatives, a digital applet, and a diagram, in this process. Finally, we discuss the implications of this analysis for future studies that explore students' constructions of quantity.

Keywords: Algebra and Algebraic Thinking, Cognition, Mathematical Representations

Students' construction of quantities is a non-trivial process (Smith & Thompson, 2008; Thompson, 2011). According to Thompson (2011), "Too often quantities, such as area and volume, are taken as obvious, and hence there is no attention given to students' construction of quantity through the dialectic object-attribute-quantification" (p. 34). In this paper, we seek to investigate this dialectic by bridging two perspectives—namely, quantitative reasoning and semiotics. Specifically, we investigate the following research question: *What can a semiotics lens reveal about a learner's constructions of quantities?*

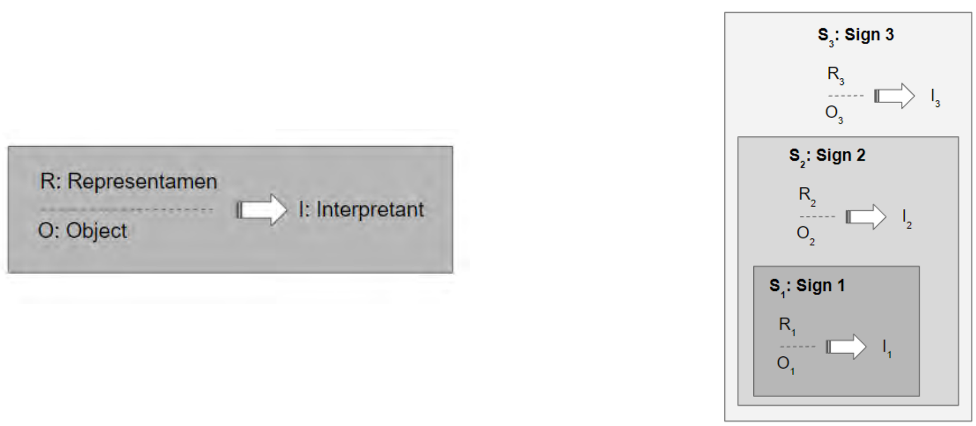
In this report, we first briefly present the frameworks of quantitative reasoning and semiotics, with attention to the ways we intend to use these perspectives concurrently. Then, we describe the method and findings from a revelatory case study (Yin, 2018) of one student's construction of a semiotic chain related to the quantity of an *amount of change* in a dynamic task. We discuss the implications of these findings for researchers and practitioners intending to support students to engage in quantitative reasoning.

Quantitative Reasoning and Semiotics

We follow Smith and Thompson (2008) in conceptualizing quantities as mental constructions of measurable attributes of objects or phenomena. Quantities establish the "conceptual content" (Smith & Thompson, 2008, p. 10) students may represent mathematically (Paoletti & Moore, 2017; Thompson, 1994). Quantitative reasoning, then, entails constructing and reasoning about relationships between conceived quantities.

Individuals can conceive of a quantity that changes its measure (or *varies*). Consider, for instance, the quantity of changing area in a growing triangle (https://ggbm.at/t6d63cun). An individual may develop a smooth image of change for the area of the triangle such that it "[takes] on values in the continuous, experiential flow of time," (Castillo-Garsow et al., 2013, p. 34). For instance, they can imagine an animation of the triangle growing continuously. An individual may also develop a chunky image of change, entailing a visualization of completed intervals without attending to variations within an interval. For example, one can imagine the triangle's growth in snapshots over time. Although both smooth and chunky images are important for students to develop, chunky images of change can also enable the conceptualization of another important quantity—an *amount of change* (hereafter AoC) between intervals (Carlson et al., 2002). Researchers have highlighted the importance of students' constructing amounts of change, for example, to differentiate between linear and non-linear growth (Paoletti & Vishnubhotla, in press), conceive of rates of change (Carlson et al., 2002), and represent quadratic (Wilkie, 2019, 2021) and trigonometric (Moore, 2014) relationships.

Given the importance of imagery in conceptualizations of quantity, we conjecture that one's mental representations necessarily meditate a student's quantity construction and corresponding meanings. Presmeg's (2006) model for semiotics in mathematics education (adapted from Peirce, 1998) provides a way to describe this process. Presmeg argued that connecting ideas in mathematics involves the negotiation of *signs*, which is the focus of semiotics. Learners construct signs as they strive to make sense of relationships between representations and their associated objects, ultimately developing interpretations for the object and representation. Presmeg follows Peirce (1998) in identifying signs as composed of these three components—*object*, *representamen* (or representation), and *interpretant* (or interpretation)—which operate together in triadic relationship as meaning is negotiated. Figure 1a shows a model for a sign.



(a) (b)

Figure 1: (a) The triadic relationship of a sign (Peirce, 1998) and (b) the process of semiotic chaining (Presmeg, 2006).

To illustrate and define these terms, consider an animation of a continuously growing triangle. One may conceive of the changing triangle (i.e., the *object*, or what a sign could stand for) and construct this object in conjunction with a mental image of the changing triangle (i.e., their *representamen* of the object, or the perceivable part of a sign). In the process of relating object and representamen, they can interpret the changing triangle in relation to a quantifiable attribute such as its area. This would be regarded as an *interpretant* for the individual, which explains the relationship they constructed between the object and the representamen. This process of arriving at an interpretant is an act of *meaning making* for the individual.

As individuals continue to engage with new ideas, they can call to mind their previous signs (i.e., triads of object-representamen-interpretant) and construct new ones. Once established, the whole of a sign can be regarded as an object onto which new representations and meanings can be built (e.g., Sfard, 1991). Presmeg (2006) refers to this process as *semiotic chaining*, whereby previous signs are nested within a new sign which "comprises everything in the entire chain to that point" (p. 169). Semiotic chains can thus explain how learners' meanings are enriched as they continuously connect the signs they make. We present a recreated visual of Presmeg's conceptualization of this process in Figure 1b.

As suggested in our example, objects, representations, and interpretations may be motivated by the artifacts with which one engages (e.g., animations, diagrams, manipulatives). As learners interact with and reconcile their interpretations of multiple artifacts, each artifact can add nuance to associated signs the learner possesses (Radford, 2010). Maffia and Maracci (2019) define this process as *semiotic interference*, which is "an enhancing of signs emerging from the contexts of use of different artifacts and referring one to the other" (p. 3-58). That is, as a learner engages a new artifact, they may translate their interpretations of "old" signs from one context in relation to new ones in another, furthering the development of meanings associated with the sign. In this way, semiotic interference describes how signs become decontextualized and generalized, and thereby more meaningful.

Method

We present a revelatory case study (Yin, 2018) to elucidate how one student's quantitative reasoning developed through a process of semiotic chaining and semiotic inference. The case study was drawn from a larger teaching experiment (Steffe & Thompson, 2000), where a pair of students engaged several tasks designed to elicit and support their covariational reasoning, detailed in Paoletti and Vishnubhotla (in press). We chose to focus on the case of a single student's activity in the beginning stages of one such task, as this account revealed how artifacts played a role in his development of meanings for a quantity via semiotic chaining.

Participant, Context, and Task

Our case student, Vicente, attended sixth grade at a public middle school in the Northeastern United States at the time of data collection. He engaged with the tasks of the teaching experiment with a partner, Lajos. One teacher-researcher (TR, second author on this paper) conducted the teaching experiment with the two students over 12 sessions.

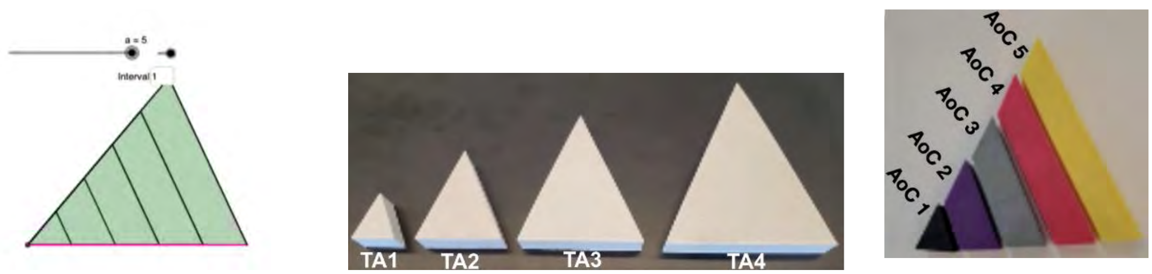
The students engaged the *Growing Triangle Task*, which asked them to consider the relationship between the length of the base segment and area of a growing triangle, presented to them through a dynamic applet (seen in Figure 2a). The ultimate goal of the *Growing Triangle Task* was for students to graph the covariational relationship between these two quantities. However, in the three 40-minute sessions that Vicente and Lajos spent working through the task, only about half of the time involved graphing activity. The TR first sought to develop students' meanings for key quantities in the scenario (e.g., base segment length, total area, AoC of area). We focus this report on Vicente's pre-graphical work as it allows us to provide evidence of his process of semiotic chaining as he constructed the AoC of area.

To support the goals of developing meanings for situational quantities, the TR designed multiple artifacts to mediate students' engagement with the task. Students had access to a digital applet that could change to display the triangle's growth as smooth or chunky (Figure 2a). They also had access to physical manipulatives of instantiations of the growing triangle (Figure 2b) as well as trapezoidal pieces intended to represent the quantity of AoC of area between instantiations (Figure 2c). We note that, in addition to these prepared artifacts, the students and TR also generated artifacts (e.g., diagrams) in the moment to support their work.

Data Collection and Analysis

We audio- and video-recorded Vicente and Lajos's activity using a free-standing camera and screen capturing software. We also collected and scanned written activity from each session. We then transcribed the recordings, including verbal utterances, gestures, and the artifacts students engaged (both prepared and actor-generated) to support our investigation of the artifacts' mediating role in Vicente's construction of quantity.

As we began to respond to our research question through our analysis of the data, we identified transcript segments of interest and the emerging meanings associated with them. First, to ensure we captured Vicente's activity and signs related to AoC of area in their context, we organized our transcript into analytical units bounded by the TR's central prompts (e.g., "How are they changing together?"). Then, through open and axial analysis (Corbin & Strauss, 2015), we examined the data for the types of meanings and related signs Vicente expressed around AoC. After this process, we analyzed each segment to consider how Vicente used or referenced artifacts in his expressions of meanings, allowing us to conjecture the occurrence of semiotic interference. We used these analyses to create models of Vicente's semiotic chain related to AoC of area in connection with his engagement with artifacts. Throughout the analysis we maintained a chain of evidence connecting our findings to data to establish trustworthiness (Yin, 2018).



(a) (b) (c)

Figure 2: Prepared artifacts from the *Growing Triangle Task*.

Results

In this section, we highlight the development of Vicente's semiotic chain involving the quantity AoC of area in the *Growing Triangle Task*. Vicente began his activity conceptualizing a chunk of change in the triangle (Sign 1). He built upon this sign to later elaborate this object as a quantity—an AoC of area (Sign 2). Finally, we show how Vicente connected these objects in his elaboration of the total area of the triangle (Sign 3). Throughout, Vicente's activity promoted semiotic interference across artifacts, which functioned to trigger, enhance, and connect signs in his semiotic chain.

Sign 1: Chunk of Change in Triangle

The activity we present in this section shows how Vicente developed a sign for *chunk of change* in the triangle. This involved his understanding that a chunk could be added after an iteration of each triangle to reach the next (e.g., move from TA1 to TA2 in Figure 2b). This interpretant emerged through Vicente's active meaning making as he negotiated the relationship between the growing triangle (an object) and the chunked compositions he came to see in that triangle (his representamen). Vicente enhanced his meanings for this sign through semiotic interference as he shifted across digital applet and physical manipulative artifacts.

To begin the *Growing Triangle Task*, the TR displayed the initial applet (smoothly and then in chunks) to Vicente and Lajos. We posit Vicente established the growing triangle as a dynamically growing object in the context of the digital applet. Furthermore, the chunky representation introduced the possibility for Vicente to conceptualize another object—namely a chunk of change.

The TR next introduced a new artifactual context to the students through physical manipulatives. He stacked two triangle manipulatives (e.g., TA1 and TA2 in Figure 2b) on top of one another (see Figure 3a) and asked Vicente, “Alright so I’m going from this one [*taps the smaller triangle*] to this one [*taps the larger triangle*]. How much has the area changed by?” Initially, Vicente responded attempting to determine a value: “Maybe by 2…like 2 of these [*taps smaller triangle twice*].” To explore how Vicente was mentally representing a change in the triangle, the TR prompted Vicente to color in “how much the area changed by from the first to the second.” Vicente proceeded to shade the darker area in Figure 3a. For Vicente, this was an instance of semiotic interference; he reinterpreted his growing triangle object and representation of a chunked composition from the digital applet to the context of the physical manipulatives.

(a) (b) (c)

Figure 3: Displays of *Growing Triangle Task* in Vicente’s construction of chunk of change.

Next, the TR prompted Vicente to consider his change in area meaning in the digital applet. The TR displayed the applet in chunks, showing the smallest triangle and then its second iteration (producing the same visual as Figure 3a, on a computer screen). He asked Vicente about “what shows the change in area,” and Vicente again pointed to the grey area in Figure 3a. Given the digital applet’s unique affordance for introducing a growing triangle object, we conjecture that this action may have supported Vicente to connect his representation of the chunked composition of the growing triangle with a notion of dynamic change.

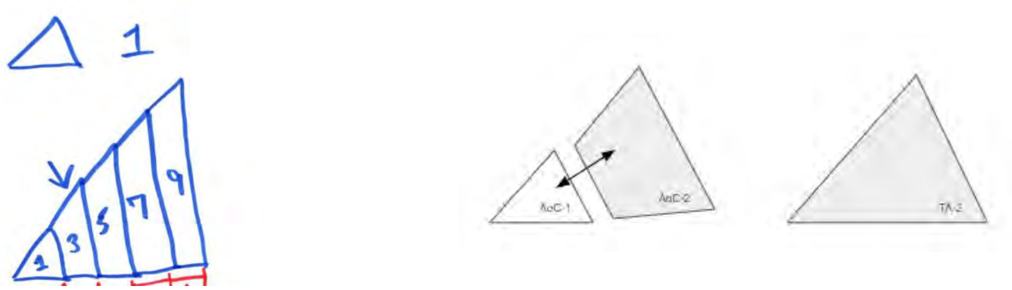
Indeed, Vicente’s subsequent activity indicated that he was beginning to develop a meaning for this change as a chunk added to previous instantiations of the triangle. Importantly, this became salient when the TR prompted him to again revisit the physical manipulatives, furthering the semiotic interference. The TR asked Vicente, “Do you see any pattern… with respect to how the changes in area are growing?” The TR arranged overlapping triangle manipulatives (e.g., TA1, TA2 and TA3 in Figure 2b) in a display like Figure 3b. Vicente’s explanation suggests that chunks of change emerged through additive operations. He stated:

> I think that it’s just, copying off each other… you can see if you counted like it, this [*points to the darkest grey area in Figure 3b*], this would be a bigger one, so you can just add it over [*gestures back toward smallest triangle in Figure 3b*].

As Vicente finished his explanation alluding to meanings for addition, Lajos picked up a new manipulative piece—a trapezoid, as shown in Figure 2c—and placed it against the longest side of the current display of stacked triangles. Vicente explained to the TR, “You can just add another one of the same size… you can kind of add that like there [in Figure 3c].” This interaction shows how the introduction of this new artifact (a trapezoidal manipulative) coincided with even greater emphasis on addition in Vicente’s language about the chunk of change. Taken together, this suggests that semiotic interference (across the digital applet, physical triangle manipulatives, and physical trapezoidal manipulatives) supported Vicente’s development of a sign for a chunk of change in the triangle: that is, a chunk of change had become a new object that could be added after an iteration of each triangle to reach the next.

Sign 2: Amount of Change (AoC) of Area

As described above, Vicente had an interest in numerically describing the magnitude of the chunk of change object he was forming once he first characterized the object (e.g., "Maybe [the area changed] by 2 [of the smallest triangles]"). At the end of the first session, the pair returned to the physical manipulatives, stacking copies of the smallest triangle on top of the trapezoidal pieces (now representing chunks of change) to measure the chunks. They recorded their findings in a diagram (Figure 4a). Thus, Vicente established two artifacts that later became central to his semiotic interference in the development of an AoC of area in his semiotic chain: the physical manipulatives (both triangular and trapezoidal) and the diagram.



(a) (b)

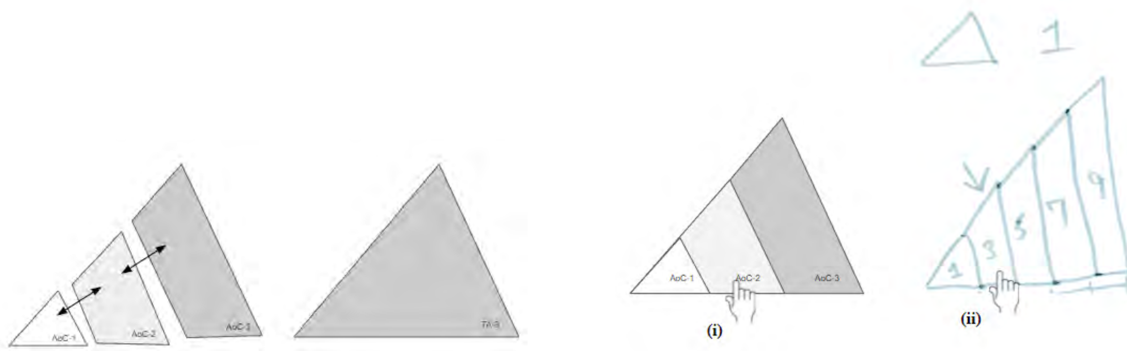
Figure 4: Displays of *Growing Triangle Task* in Vicente's construction of AoC of area.

When the pair of students returned for the second day of the *Growing Triangle Task*, the TR intended to have them to renegotiate their sign for an AoC of area, and he employed both the physical and diagrammatic artifacts to do so. First, referring to their diagram (Figure 4a) the TR asked, "What were these numbers representing, what were you talking about with these?" As Vicente swiped his finger over each corresponding trapezoidal component in the diagram, he described that "in here you could fit 3, then here you could fit 5, then here you could 7… [of] the little triangles." We interpret that Vicente understood the numeric values as representing measures of AoC of area in terms of the first triangle (e.g., AoC 1 in Figure 2c). Moreover, the diagrammatic artifact provided a recorded, concretized representation of chunks of change and their values he could draw upon in the construction of his sign for AoC of area.

The TR, however, encouraged Vicente to extend his meanings back again to the physical manipulative context of the scenario. The manipulative context was important in Vicente's sign for AoC of area due to its previous interference with the digital applet. Arranging the shapes as in Figure 4b, the TR asked Vicente, "So when you say what fits 3 of what?" Vicente at first picked up the larger triangle (i.e., TA2 in Figure 2b) and attempted to arrange copies of the smallest triangle on top. However, he quickly decided "not this one," picked up the trapezoidal piece (i.e., AoC2 in Figure 2c), and began to arrange the small triangles on top of there. He was able to fit 3 small triangles and affirmed, "Yeah, this one." We note that Vicente's activity was effortful, indicative of the semiotic interference involved in his meaning making for AoC that extended across diagrammatic and manipulative contexts. Collectively, Vicente's negotiations across artifacts indicate the development of a new sign in his semiotic chain: an AoC of area. For him, the AoC of area was an object with magnitude that could be determined using the small triangle's area as a unit, establishing a quantity constructed from his sign for a chunk of change.

Sign 3: Total Area of the Triangle

Vicente's chaining of signs for chunks of change and AoC of area supported him in developing new meanings for a related object: total area in the growing triangle. As described in the previous section, the TR had now re-introduced total area manipulatives of the growing triangle (e.g., TA3 in Figure 2b). In this interaction, the TR arranged trapezoids and a triangle piece on the table in front of the two students (see Figure 5a). He asked the pair, "This [smallest triangle] is 1; how big is that [*gesturing to large triangle displayed in Figure 5a*]?"



(a) (b)

Figure 5: Displays of *Growing Triangle Task* in Vicente's construction with total area.

Vicente explained his reasoning to Lajos in a way that first expressed a meaning for the relationship he understood to exist between the largest triangle and trapezoidal pieces in the arrangement of Figure 5a. He argued that "this [*tapping on the largest triangle in Figure 5a*] is the same as that [*gesturing to the trapezoids pushed together in Figure 5a*]." He then picked up the largest triangle piece and placed it on top of the trapezoidal arrangement to show it "fits in perfectly." We note Vicente's argument established a sign for total area as an object related to artifacts that had established his sign for AoC, suggesting a new link in his semiotic chain.

Vicente's next statements demonstrated the salience of his sign for AoC of area as he found a measurement of total area. At first, he gestured to each progressively larger trapezoid in the manipulatives, declaring that they had values of 3 and 5, respectively. When explaining to Lajos, he revealed how his own meanings for the AoC object had been supported through the semiotic chaining he had established with the diagrammatic artifact. Specifically, Vicente explained, "this one [*pointing to manipulatives as in Figure 5b(i)*] is that [*pointing to diagram as in Figure 5b(ii)*], so you can fit 3 [small triangles] in there." Here, Vicente directly related the area of the trapezoidal physical manipulatives to the areas noted in their diagram.

Finally, Vicente called upon his initial interpretations of chunks of change to determine the total area of the triangle. He concluded that "1 plus 3 is 4, and then… 4 plus 5 is 9," identifying that 9 would be the total area of the given triangle. We note that, in this explanation, Vicente does not express his conclusion for total area as simply an addition of consecutive bands in the composite triangle (that is, $1 + 3 + 5 = 9$). Instead, Vicente considers each intermediate total area in a way that might reflect an AoC of area as adding on to previous total areas in each step (i.e., $1 + 3 = 4$, then $4 + 5 = 9$). This further suggests the critical role of Vicente's previously constructed signs in his reasoning. Viewing Vicente's argument holistically, AoC of area (which was based on his sign for chunk of change) formed the basis of his exploration of magnitude with a related quantity (and sign) of total area. We summarize our hypothesis about Vicente's enacted semiotic chain in Figure 6. This figure shows the process by which his sign (1) for a chunk of change in the triangle became an object for his sign (2) AoC of area, and then these subsequently became objects for a sign (3) for total area of the triangle.

Conclusion and Discussion

In this paper, we brought together quantitative reasoning and semiotics frameworks to present a case study that highlighted the emergence of a semiotic chain (Presmeg, 2006) of a student's construction of quantity through his engagement with multiple artifacts. Specifically, we anchored our analysis in Vicente's development of signs related to the quantity of AoC of area. This included his initial constructions of a chunk of change in the triangle and his later connection to the total area of the triangle. Moreover, our evidence showed the processes of semiotic interference (Maffia & Maracci, 2019) that account for the enhancement of Vicente's signs (e.g., building a sign for the quantity of AoC of area by connecting notation in a diagram to his work with physical manipulatives). Addressing our research question, a semiotic lens on Vicente's quantitative reasoning helped us to make sense of the mediated processes that supported him as he developed associated meanings in conjunction with his representational activity through both the processes of semiotic interference and semiotic chaining.

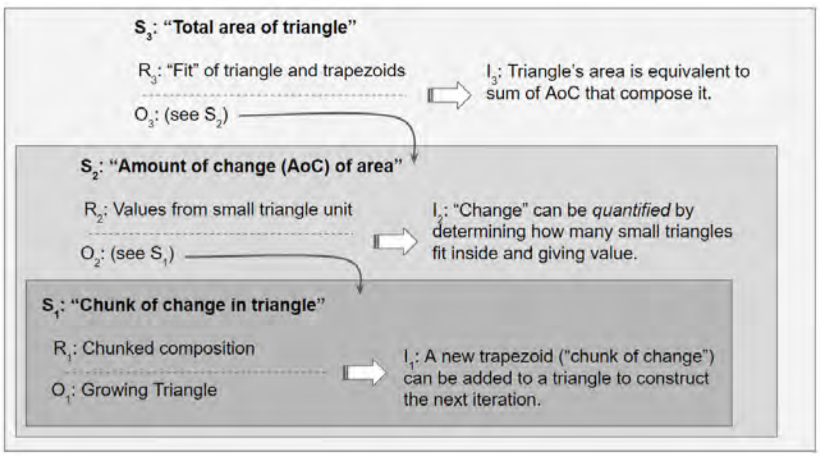


Figure 6: Model of Vicente's construction of a semiotic chain involving AoC of area

By identifying and describing the signs Vicente related to his sign for AoC of area, we charted how conceptualization of this specific quantity may develop. These findings suggest that scaffolded engagement with multiple artifacts such as applets, manipulatives, and diagrams may support learners' development of AoC in other types of tasks, particularly those involving non-linear relationships (e.g., Wilkie, 2021). They may also suggest features that could be productive to investigate beyond the scope of AoC (e.g., a quantity's relationship within and across other quantities), thus contributing to new potential lines of inquiry in the field of quantitative reasoning (e.g., Smith & Thompson, 2008).

Although not described in this paper, we also note that Vicente's signs relating to AoC of area in the *Growing Triangle Task* did not end with the semiotic chain in Figure 6. In fact, he continued to engage with these signs and construct more sophisticated meanings as he interacted with new artifacts (e.g., constructing and interpreting graphs) and as he anticipated smooth change in area (see Paoletti & Vishnubhotla, in press). By maintaining a semiotic lens on Vicente's activity, this work has offered new understandings of students' images and meanings for changing quantities (Carlson et al., 2002; Castillo-Garsow et al., 2013). Through a combined focus on quantities, artifacts, and the meaning-making process that relates them, we intend for this work to open new directions for research into how multiple artifacts can support students' conceptions of quantity.

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