USE OF SIGNIFIERS IN MATHEMATICAL DISCOURSE IN KOREAN AND ENGLISH

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We consider how the existence of different signifiers for mathematical objects in different languages manifests in discourse about those objects. Based on the observation that there is a common signifier "derivative" in English used for both the derivative at a point and the derivative function and two phonetically and semantically different signifiers for those objects in Korean, we explore the differences between one Korean teacher's discourse and one American teacher's discourse about the derivative. Our analysis uncovered differences in metarules regarding the use of signifiers, as well as differences in possible connections to colloquial discourse. Additionally, we found that, after both objects are defined, the American teacher's discourse shifts in a way that precludes the simultaneous use of the common signifier for both objects whereas, in the Korean teacher's discourse, there was no similar shift.

Keywords: commognition, derivative, calculus, mathematical object, realization tree

Recently, the impact of the language in which mathematics is practiced on mathematical discourse has been emphasized. Examining students' or teachers' uses of mathematical words is an important part in this literature (e.g., Kim et al., 2012; Han & Ginsburg, 2001). Our study examines uses of terms in mathematical discourse in different languages in an effort to understand the language-dependent nature of the discourse. We focused on the discourse about the derivative in Korean and American English (English, here after), which provides a useful context for the study. The derivative is a crucial concept in Calculus and needed for multiple disciplines. Commonly, within introductory calculus, derivatives can be separated into the derivative at a point and the derivative function. By "the derivative function" we mean a function obtained by differentiating another function. Studies have addressed challenges students face distinguishing or relating these objects (Font & Contreras, 2008; Font et al., 2007; Park, 2013).

We take a commognitive perspective (Sfard, 2008) to examine discourse on the derivative at a point, the derivative function, and the narratives connecting these objects. In the commognitive approach to mathematics education, "learning mathematics is the process in which students extends their discursive repertoire by individualizing the historically established discourse called mathematics" (Sfard, 2018, p. 222). Individualizing a discourse essentially means developing one's ability to communicate with others and oneself according to the rules of the discourse. In each language, we will refer to the historically established discourse called mathematics as the canonic discourse. As Kim et al. (2012) noted, from the commognitive perspective, one should not expect mathematical discourse in different languages to be homeomorphic. Thus, one should expect differences in the canonic discourses in different languages. From this perspective, differences in canonic discourses are differences in what students are trying to individualize.

The motivation for our study is the observation that in English, the word "derivative" is included in "the derivative at a point" and "the derivative function" and the word "derivative" alone is often used for these objects, while in Korean the phonetically and semantically different terms "mi-bun-gye-su" and "do-ham-su" are used respectively for the corresponding objects and there is no common term like "derivative" that can be used for either object. Thus, this difference between English and Korean is a difference in the canonic discourses of the two languages.

While the non-homeomorphic nature of mathematical discourse in different languages has been investigated from the student perspective, to our knowledge, our study is the first to focus on differences caused by differences within the canonic discourses. In contrast to previous studies that focus on aspects of students' discourse that one would expect students to outgrow as they attain proficiency (e.g., Favilli et al., 2013; Han & Ginsburg, 2001; Kim et al., 2012; Miller & Stigler, 1987; Paik & Mix, 2003), we consider a different aspect of language-dependency in which one language has a common term for two different but related objects and the other does not, which results in a difference in the mathematics that students in each language are trying to learn. This is an important difference to investigate because several researchers have suggested that one term/notation being used for multiple objects creates challenges for teaching and learning about those objects in different mathematical discourses (e.g., inverse in algebra or discrete mathematics, Thompson & Rubenstein, 2000; tangent line in geometry and analysis, Biza & Zachariades, 2010). Especially, multiple uses of one term in the same discourse are hard to communicate with students (e.g., limit as a number and limit as a process in (Güçler, 2013)).

To investigate the differences in the canonic discourses, we examined one Korean mathematics teacher's classroom discourse and one American teacher's classroom discourse considering those teachers as participants in the canonic discourse in Korean and English, respectively, and addressed the following research question: *What are the differences between canonic discourse in English and Korean that can be observed based on an American mathematics teacher's discourse involving a common word "derivative" and a Korean mathematics teacher's discourse involving two words "mi-bun-gye-su" and "do-ham-su"?*

Based on the comparison between two teachers' discursive characteristics, we learned about the differences between canonic discourse in each language regarding metarules about word use, in relationships between canonic discourse and colloquial discourse, and in ways mathematical objects were connected. We adopted a case study approach because it allows researchers to "closely examine the data in a specific context" (Zainal, 2007, p. 1) and many recent mathematics education studies that examine teachers' discursive characteristics through in-depth analysis adopted a case study approach (e.g., González, 2015; Kontorovich, 2021). A limitation of our approach is that it may not find all of the differences between the canonic discourses or identify which differences would be most commonly observed in a large sample of classrooms. However, in-depth engagement with one teacher's discourse involving the specific terminologies of interest in each language, which a case study allows, was needed for researchers to explore mathematical discourse in each language "in action" and compare potential differences. To make our inference about differences in the canonic discourses in each language valid we followed case study design guidelines from Check and Schutt (2017), Rubin and Rubin (1995 & 2005), and Zainal (2007). We carefully chose teachers whose discourse would allow us to learn about differences in canonic discourse in each language and collected data that would enlighten us about the connections between mathematical objects made within the canonic discourse and its connection with colloquial discourse in each language. We then produced a chain of evidence about the differences in the canonic discourses in each language based on our analysis of the data from holistic point of view by examining their discourse following commognitive research guidelines from e.g., Kontorovich (2021), Nachlieli & Tabach (2012) and Sfard (2008 & 2012).

Theoretical Background

Prior Work on Differences in Mathematical Discourses in Different Languages

The commognitive framework has been used in several settings to examine the dependence of mathematical discourse on language. This conceptual and discourse-analysis framework combines cognition and communication. It asserts that thinking is communicating with oneself and views mathematics as a type of discourse. Commognition makes the language-dependent nature of mathematics almost self-evident because languages are the medium for communicating (Kim et al., 2012), which makes it a natural framework for studying such dependence.

Language studies using the commognitive framework have focused on the discourse of students. For example, Kim et al. (2012) used this framework to study differences between students' discourses about infinity in Korean and English based on phonetic and semantic disconnections between "Korean formal mathematical discourse in infinity and its informal, colloquial predecessor" and "the cohesiveness of the infinity discourse" in colloquial and formal discourse in English (p. 95). Ní Ríordáin (2013) adopts the commognitive framework, discussing how, theoretically, the syntactical structures of Irish "lend itself to easier interpretation of mathematical meaning in comparison to English" (pp. 1581-1582), which was supported empirically by the later study (Ní Ríordáin & Flanagan, 2020).

Outside of the commognitive literature, if one views the ability to correctly solve problems as a proxy for fluency in a discourse, there is a long history of investigating phenomena similar to what Ní Ríordáin and Flanagan (2020) investigated. For example, some of these studies considered the feature of Asian language terms stating mathematical concepts clearly, such as part-whole relations in fraction words, which is not a feature in other languages (e.g., English) as an explanation for Asian-language speaking students' higher performance regarding those concepts (Han & Ginsburg, 2001; Miller & Stigler, 1987; Miura, 1987; Miura et al., 1999).

Most of these studies focus on, or give insight into, phenomena within student discourse that students are intended to outgrow as they become proficient participants in the canonic discourse. In contrast, our study considers differences within the canonic discourses, which therefore will become present and persist after students become fluent in the canonic discourse.

Canonic and Teacher Discourse

From the commognitive perspective, a discourse is a "special type of communication made distinct by its repertoire of admissible actions and the way these actions are paired with re-actions" (Sfard, 2008, p. 297). Canonic discourse can be difficult to identify in general because it is historically established by practice. Thus, we grounded our identification of canonic discourse in content that has broad, objectively observable acceptance. In particular, we decided to use widely accepted curricula as the basis for identifying the canonic discourse in Korea and the United States. In Korea calculus is part of the national high school mathematics curriculum set by the Korean government (Ministry of Education, 2020), which also reviews and approves high school textbooks. In the United States there is no similar nationally controlled curriculum for calculus. Rather, there is the Advanced Placement Calculus AB curriculum set by the College Board (2020), which is a private company. Although privately set, Advanced Placement (AP) Calculus courses are taught in many high schools (public and private) and high school students may receive college credit at many colleges (including highly ranked ones like Harvard University) by scoring well on the standardized AP Calculus AB Exam at the end of each year. From this, we conclude that the AP Calculus curriculum is broadly accepted in the United States. We consider certified teachers who teach these curricula to be among the participants in the canonic discourse of their country and selected our participants from among this group.

Within the commognitive framework, teaching can be defined as "the communicational activity the motive of which is to bring the learners' discourse closer to a canonic discourse" (Tabach & Nachlieli, 2016, p.303). This view is consistent with other discursive approaches. For example, as reviewed in Schutte (2018), Pimm (1987) sees learning mathematics as similar to learning a foreign language, which is both written and spoken language which has to be used extensively within the mathematics classroom, and views the teacher as similar to a "native speaker" in a new language (p. 26). Therefore, the mathematical discourse of qualified teachers teaching courses that follow broadly accepted curricula is likely to provide examples of canonic discourse, with the understanding that teachers may occasionally include non-canonic discourse through making mistakes or adopting idiosyncratic discourse in the process of trying to bring students' discourse closer to the canonic discourse. In practice, this essentially means that, with the exception of occasional mistakes or adoptions of idiosyncratic discourse, we treat the mathematical statements endorsed as correct by qualified teachers as though they would be endorsed as correct by other qualified teachers and others who have mastered the content of the broadly accepted curriculum. We remark that the discourse of different participants in the canonic discourse may vary dramatically, see e.g. Moschkovich (2007) about the differences between academic and school discourse, but the hallmark of discourse being canonic is that it would be endorsed as correct by other participants in the canonic discourse.

Methods

This study uses teachers' discourse in each language to learn about canonic calculus discourse in English and Korean. As noted in the introduction, we adopted a case study approach to conduct an in-depth analysis. This section details the research design with our rationale to produce valid conclusions through our selection of participants, data collection, and analysis.

Participants

To ensure the validity of our choice of teachers in terms of addressing our research question, our recruiting of teachers was guided by strategies for purposive sampling in which participants are selected for a purpose (Rubin & Rubin, 1995, as cited in Check & Schutt, 2017), such as "knowledgeable about…the situation…being studied," and "willing to talk" with "a range of" view points" (p. 20). In selecting the American teacher, we discussed our study with faculty at a midsized U.S. university, who work at a center that organizes professional development (PD) for mathematics teachers. Of the teachers they recommended, William was recommended as the best "potential" participant given that William has taught Calculus multiple times, been engaged in PD for mathematics education, and participated in the design of the state-wide mathematics tests. William was also "willing to talk" about the subject of our study and to open their classroom being observed. We operationalized "the range of points of view" as using various communicational means in their teaching, especially including a range of representations and terminology in the mathematical context of our study. William described how they usually teach the topic of the study with the intention of using mathematical terms and concepts rigorously, as well as utilizing a variety of real-life contexts and mathematical representations (e.g. graphical and symbolic), which fit our operationalization of the third point. In selecting the Korean teacher, we considered Korean mathematics teachers from mathematics education master's programs at highly regarded universities in Korea and also considered Korean high school teachers who attended calculus related talks at the International Congress on Mathematical Education that happened in South Korea. Kim was an experienced high school mathematics teacher, who passed a highly competitive national test to become a secondary public school teacher, taught Calculus over several years, and was also a master's student in mathematics education. Kim also engaged in PD and taught in a mathematics program for gifted students.

Aligned with our conception of canonic discourse, the class we observed William teaching was an AP Calculus AB course in a public high school in the U.S. The Korean class we observed was a public high school course in Korea that followed the national curriculum.

Students in both classes were native speakers of the language of instruction. In general, the teachers organized their class with explanations of key words, solving examples on the board, students' individual work on problems, and then whole class discussions about the problems the they had solved. The teachers' discourse observed in their explanations and their interaction with students helped us understand how the key words of our interest were used in each language.

Data Collection

Class observations occurred at the beginning of the derivative unit before the class concentrated on computing the derivative. This part of their class was chosen to collect the relevant data for the teacher's discourse where we could potentially observe two uses of the "derivative" in English, where "mi-bun-gye-su" and "do-ham-su" were defined and connected to each other in Korean, and where other related words and real-life phenomena were discussed.

The first author video-recorded seven 50-minute lessons from Kim's class and six 90-minute lessons from William's class from the back of the classroom. There is a difference in length of the video-recorded lessons. Kim's discourse stayed related to the mathematical objects of the lesson whereas William's class included various non-mathematical discourses (e.g., information about AP exams) and reviewing of topics irrelevant to this study (e.g., computing limits), for which we did not find direct connections to our study. We included all derivative-related talk, and approximately, 400 minutes of relevant discourse were recorded in William's class.

Once the recording was completed, the first author (who is fluent in English and Korean), one Korean-speaking research assistant, and one English-speaking research assistant transcribed the videos focusing on what is said as well as what is done (non-verbal communication) following the guidelines from Sfard (2008) for transcribing the recorded data. Once the data was transcribed, the first author translated the Korean transcripts into English. A sample from the translations was checked by another mathematics education researcher who is fluent in English and Korean. Both authors reviewed the transcripts to check the match between the video data and the transcripts and added screen shots from the videos to complement "what is done".

Analysis

Our analysis of the teachers' discourse was guided by the principles of commognitive research (Kontorovich, 2021; Nachlieli& Tabach, 2012; Sfard, 2008 & 2012). In our analysis, we treated the totality of each teacher's discourse, collected over multiple days, as our unit of analysis. However, we separated lessons into several episodes to see how words were used in different contexts. Episodes were defined by each teacher's different teaching activities. Specifically, each episode was determined by completing a mathematical task, defining a new mathematical term, providing a story involving a real-life object prior to defining a mathematical term, making connections between a newly defined term and previously defined terms, or making connections between a newly defined term and a real-life object. Episodes were defined this way because changes in teaching activities correspond to changes in the context of discourse and, therefore, to potential changes in usage of words whose meaning is context-dependent (Biza & Zachariades, 2010; Thompson & Rubenstein, 2000), like "derivative" in English. Following this definition, we identified 57 episodes in Kim's class and 54 episodes in William's class.

With our interest in linguistic differences related to the signifiers "derivative" in English and "mi-bun-gye-su" and "do-ham-su" in Korean, we needed to identify the mathematical objects that these signifiers signified in each teachers' discourse. It was done based on the formalization of the derivative as a mathematical object in the commognitive framework as in (Park, 2016). After this, we examined the coded data focusing on how the derivative at a point and the derivative function were connected. Once the objects in each teacher's individualized discourse were identified, we coded each episode according to the objects and primary signifiers used to determined which objects appeared in each episode regarding how those language-specific terms were used. For example, for William's episodes, we distinguished (a) episodes in which "derivative" is used as (or as part of) a signifier for those two objects (showing the common use of the "derivative" for the two objects) from (b) episodes in which both objects signified by "derivative" appeared but in which "derivative" was only used as (or as part of) a signifier for at most one of these objects (not showing the common use of the "derivative"). Each author individually coded each of William's uses of "derivative" as described above. The coding originally matched, except for 2 episodes which were discussed until agreement was reached. The English translation of the Korean data was reviewed by another Korean and English speaking commognitive researcher, and the English-translated Korean data and English data were analyzed through discussions among the authors, one of whom is a native English speaker.

Results

Our analysis leads to three main results. First, the canonic discourse in English (AMD) contains metarules for distinguishing whether the signifier "derivative" is being used for the derivative at a point or the derivative function that has no analogue in the canonic discourse in Korean (KMD). On four occasions William provided the following rule for distinguishing the two uses of derivative: If context shows the object signified by "derivative" is a number, "derivative" is signifying the derivative at a point, and if context shows the object signified by "derivative" is a function, "derivative" is signifying the derivative function. He said, for example

> This idea of, well the derivative can be thought of as a number, which represents the slope of the tangent line at a point. But, notice that I can talk about slope of the function at every point along this curve…the slope is always changing. Now, what we get then, is a function that represents the slope of the tangent line at any point on here, as a function of x…That's called the derivative too. From the context, you understand what I'm talking about…just remember now we're talking about numbers and we're talking about functions, and we have to keep them straight. Then we get into the other half of Calc 1 and talk about integrals. There will be the same thing. There will be some integrals that are numbers, some integrals that are functions. We use the same words because the concepts are related (Day 2, 6 minutes)

One would expect that participants in AMD have some way for distinguishing between the two uses of "derivative". Thus, the existence of such metarules is not surprising, but their existence shows that the differences between AMD and KMD extend beyond simply having a different signifier structure. We concluded that this rule is part of the canonic discourse, in addition to William being a participant in the canonic discourse, because it combines two statements from the canonic discourse (that the derivative at a point is a number and that the derivative function is a function) with the generally applicable rule that one can try to use context to infer the meaning of a word that potentially has multiple meanings. We thus believe that this metarule would be endorsed by the other participants in the canonic discourse. This metarule of AMD has no counterpart in KMD because there is no signifier in KMD whose use must be distinguished.

Second, when teaching in English, one can use the fact that "derivative" is a signifier of two objects in AMD to make connections to the colloquial discourse in English for which there are no analogous connections between KMD and colloquial discourse in Korean. For example, William used "speed" in colloquial stories about reading changing numbers on a speedometer to motivate the construction of the derivative function as follows,

> As I start out on my trip, how fast am I going?...Pretty fast!…As I travel along, what is happening to the slope or my instantaneous velocity sort of thing, instantaneous rate of change? It is going down right?… I have a whole succession of changing speeds or changing rate of changes…Let's say…I have a flea, following this line and the flea has a rate of change meter on. It's the speedometer! So, he's traveling along the line, this number is changing all the time. Now, I- (Slamming hands against the board) squash the flea at one point. That number is frozen…That number I'm gonna use as the slope, and that line (drawing the tangent line at the point)…that's what we call the tangent line…that keeps visible that slope or that instantaneous rate of change at that point (Day 1, 37 minutes)

In colloquial discourse (in both Korean and English), "speed" behaves like "derivative" in AMD in the sense that it can be used as a number or as an (informal) function of time. Thus, in an English-speaking class, a teacher could explicitly recall this dual use of speed and then explain that "derivative" works in the same way students are familiar with "speed" working and, from context, one can determine if it is being used as a number or a changing quantity.

Our third result is that William's use of "derivative" shifted after he introduced the derivative function. Specifically, in episodes that only involve the derivative at a point, "derivative" was the dominant signifier used by William for this object. However, in episodes that involved both the derivative at a point and the derivative function, William dominantly used "derivative" as a signifier for the derivative function (56 out of 70 uses) and "slope of the tangent line" as a signifier for the derivative at a point. We did not observe any such shift in Kim's discourse. Rather, Kim routinely used the term "mi-bun-gye-su" to refer to the result of evaluating the "do-ham-su" at a point. Unlike our first two results, which are results about AMD and KMD, this shift depends on William's didactic choices, in particular the choice to primarily use "slope of the tangent line" to signify the result of evaluating the derivative function, and other teachers' discourse may not include such a shift. For example, one could use "derivative" assuming what it is used for would be obvious to students or always use "the derivative at a point" and "the derivative function" to avoid confusion. However, we decided that it is interesting to report because, assuming that people naturally tend to try to communicate clearly, we conjecture that not using one term for different objects in the same context is common in AMD. Similarly, we conjecture that such shifts are uncommon in KMD.

Discussion and Conclusion

In our results, we showed that the differences between the canonic mathematical discourse in English (AMD) and the canonic mathematical discourse in Korean (KMD) extend beyond the fact that AMD has a signifier "derivative" that can be used for two objects while KMD does not have common signifier for both objects. Specifically, we argued that AMD also has metarules for using context to determine which object "derivative" is signifying based on whether the object is a number or a function. Although not every instructor in AMD will necessarily include this metarule in their classroom discourse, we argued that this metarule is part of the canonic discourse, i.e. would be endorsed as correct by other qualified teachers and participants in the canonic discourse, because it combines a statement from the canonic discourse about the difference between the derivative at a point and the derivative function with the general rule that context can be used to determine the meaning of words that have multiple uses. Indeed, it has been found in other contexts that preservice mathematics teachers use context to explain the uses of other mathematical symbols that have multiple meanings (Zazkis & Kontorovich, 2016). Our result further confirms the hypothesis that Kim et al. (2012) put forward that mathematical discourse in different languages is not necessarily homeomorphic. They supported this claim by observing differences in student discourse about infinity, but we have shown differences in both the signifier structure and metarules of the canonic discourses themselves. In addition to adding to our theoretical understanding of the relationships between canonic discourses in different languages, we believe that this has some practical instructional consequences. For example, there are many students from Korea and Japan (signifiers for derivatives in Japanese and Korean are homeomorphic) who go to the U.S. for graduate school and teach calculus. These students may not be expected to know elements of AMD that do not exist in the canonic discourses of their native languages and may not be able to teach these elements to their students.

In addition to providing the metarule for distinguishing the uses of "derivative", William's first quote in Results includes a noteworthy discussion of integrals. The metarule that William provided for the use of "derivative" can be used as an analogy to explain the metarules around the use the signifier "integral" later in the course. In KMD, the terminology around integrals is similar to AMD, there is a signifier for "definite integrals" and signifier for "indefinite integrals" and a signifier like "integral" that can be used for either. However, it cannot be explained by analogy with the use of signifiers around derivatives. This further supports the conclusion of Kim et al. (2012) that "teachers need to be cognizant of those language-specific features of the discourse that may support learning and of those that may hinder successful participation" (p. 106). Because this result pertains only to the existence of certain language-specific features, whose existence we have just demonstrated, it is not limited by our case study methodology.

The discursive shift we observed in William's class, but not in Kim's leads to a theoretical hypothesis about students' learning. From commognitive theory, within higher-mathematics, which features extreme objectification and rigor that is often unfamiliar to students, new students' learning "begins with an exposure to" instructors' discursive practices (Sfard, 2014, p. 202). Then, students start "collaborating across communicational conflict" due to the differences between this new discourse and their old discourse "by observing, and then imitating, the expert's moves while also trying to figure out the reasons", which might be the only way students "come to grips with the objects" at the abstract level (Sfard, 2014, p. 202). A consequence of this is that as students start learning higher-mathematics, their discourse will include imitations of experts' discursive moves and, just as importantly, will not include moves from the canonical discourse that they have not observed (e.g, from their instructor, textbook or other resources). This leads to the hypothesis that students in courses featuring discursive shifts as in William's will initially connect evaluating the "derivative function" with obtaining "the slope of the tangent line" rather than "the derivative at a point", while students in courses like Kim's that do not feature such a shift will initially connect evaluating the "do-ham-su" with obtaining the "mi-bun-gye-su". It would be interesting to investigate whether discursive shifts as in William's class are common and to investigate the effects such shifts have on student learning.

This would be particularly interesting because the literature has shown that the connection between the derivative at a point and the derivative function can be challenging for students (Font & Contreras, 2008; Park, 2013) and implicit shifts in instructor discourse have been tied to student difficulties (Güçler, 2013). Although derivatives have been intensely studied in the literature, we have not seen shifts like the one we discuss documented before. Note, however, that the instructors in (Park, 2015) also dominantly connected "derivative function" to "slope of the of the tangent line" and this leads us to conjecture that this shift may be widespread, but not documented because previous analyses were not looking for such large-scale discursive patterns.

References

Biza, I., & Zachariades, T. (2010). First year mathematics undergraduates' settled images of tangent line. The Journal of Mathematical Behavior, 29(4), 218-229. https://doi.org/10.1016/j.jmathb.2010.11.001

Check, J., & Schutt, R. K. (2017). Qualitative Methods: Observing, Participating, Listening. In *Research methods in education*. Sage Publications. https://methods.sagepub.com/book/research-methods-in-education/i911.xml

College Board. (2020). AP Calculus AB course and exam description. Retrieved from https://apcentral.collegeboard.org/pdf/ap-calculus-ab-bc-course-and-exam-description-0.pdf

Favilli, F., Maffei, L., & Peroni, R. (2013). Teaching and learning mathematics in a non-native language: Introduction of the CLIL methodology in Italy. Online Submission, 3(6), 374-380.

Font, V., & Contreras, Á. (2008). The problem of the particular and its relation to the general in mathematics education. Educational studies in mathematics, 69(1), 33-52.

Font, V., Godino, J. D., & D'Amore, B. (2007). An onto-semiotic approach to representations in mathematics education. For the learning of mathematics, 27(2), 2-14.

González, G. (2015). The use of linguistic resources by mathematics teachers to create analogies. Linguistics and Education, 30, 81-96.

Güçler, B. (2013). Examining the discourse on the limit concept in a beginning-level calculus classroom. Educational Studies in Mathematics, 82(3), 439-453. https://doi.org/10.1007/s10649-012-9438-2

Han, Y., & Ginsburg, H. P. (2001). Chinese and English mathematics language: The relation between linguistic clarity and mathematics performance. Mathematical Thinking and Learning, 3(2-3), 201-220.

Kim, D. J., Ferrini-Mundy, J., & Sfard, A. (2012). How does language impact the learning of mathematics? Comparison of English and Korean speaking university students' discourses on infinity. International Journal of Educational Research, 51, 86-108. https://doi.org/10.1016/j.ijer.2012.01.004

Kontorovich, I. (2021) Minding mathematicians' discourses in investigations of their feedback on students' proofs: a case study. Educational Studies in Mathematics, 107, 213–234. https://doi.org/10.1007/s10649-021-10035-2

Miller, K. F., & Stigler, J. W. (1987). Counting in Chinese: Cultural variation in a basic cognitive skill. Cognitive Development, 2, 279–305.

Ministry of Education. (2020). Mathematics curriculum. Retrieved from https://down.edunet4u.net/KEDNCM/SynapConv/skin_v2021/doc.html?fn=B3E435D1-677B-A504-E5E1-578DE8B0E05E.hwp&rs=/KEDNCM/preview2021/642249b9-1e64-42e4-8953-b0ec40fd297b/&scf=/KEDNCM/NCIC/filemng/642249b9-1e64-42e4-8953-b0ec40fd297b&lfn=3.+%EB%B3%84%EC%B1%858_%EC%88%98%ED%95%99%EA%B3%BC+%EA%B5%90%EC%9C%A1%EA%B3%BC%EC%A0%95%28%EC%A0%9C2020-225%ED%98%B8%29%EC%88%98%EC%A0%95_2020.04.14_%EC%B5%9C%EC%A2%85.hwp&ttl=%EB%B3%84%EC%B1%85+8_%EC%88%98%ED%95%99%EA%B3%BC+%EA%B5%90%EC%9C%A1%EA%B3%BC%EC%A0%95%28%EC%A0%9C2020-225%ED%98%B8%29

Miura, I. T. (1987). Mathematics achievement as a function of language. Journal of Educational Psychology,79, 79–82.

Miura, I. T., Okamoto, Y., Vlahovic-Stetic, V., Kim, C. C., & Han, J. H. (1999). Language supports for children's understanding of numerical fractions: Cross-national comparisons. Journal of Experimental Child Psychology, 74(4), 356-365.

Moschkovich, J. (2007). Examining mathematical discourse practices. For the Learning of Mathematics, 27(1), 24-30.

Nachlieli, T., & Tabach, M. (2012). Growing mathematical objects in the classroom–The case of function. International Journal of Educational Research, 51, 10-27. https://doi.org/10.1016/j.ijer.2011.12.007

Ní Ríordáin, M. (2013). A comparison of Irish and English language features and the potential impact on mathematical processing. In Proceedings of the Eight Congress of the European Society for Research in Mathematics Education (pp. 1576-1585). Turkey: Middle East Technical University.

Ni Ríordáin, M. & Flanagan, E. (2020). Bilingual undergraduate students' language use and meta-level developments relating to functions. Mathematics Education Research Journal 32, 475-496. https://doi.org/10.1007/s13394-019-00268-z

Paik, J. H., & Mix, K. S. (2003). US and Korean children's comprehension of fraction names: A reexamination of cross–national differences. Child Development, 74(1), 144-154.

Pimm, D. (1987). Speaking mathematically. London: Routledge.

Park, J. (2013). Is the derivative a function? If so, how do students talk about it? *International Journal of Mathematical Education in Science and Technology 44*(5), 624–640.

Park, J. (2015). Is the derivative a function? If so, how do we teach it? *Educational Studies in Mathematics 89*(2), 233-250.
Park, J. (2016). Communicational approach to study textbook discourse on the derivative, *Educational Studies in Mathematics 91*, 395-421.
Rubin, H. J., & Rubin, I. S. (1995). Qualitative interviewing: The art of hearing data. Sage Publications.
Rubin, H. J. & Rubin, I. S. (2005) Continuing the design: Making the research credible. Qualitative interviewing. Sage Publications.
Schutte, M. (2018) Subject-specific academic language versus mathematical discourse. In J. N. Moschkovich, D. Wagner, A. Bose, J. R. Mendes, & M. Schütte (Eds.), Language and communication in mathematics education (pp. 25-36). Springer, Cham.
Sfard, A. (2008). Thinking as communication. Cambridge, England: Cambridge University Press.
Sfard, A. (2012). Introduction: Developing mathematical discourse—Some insights from communicational research. International Journal of Educational Research, 51-52, 1-9. https://doi.org/10.1016/j.ijer.2011.12.013
Sfard, A. (2014). University mathematics as a discourse–why, how, and what for?. Research in Mathematics Education, 16(2), 199-203. https://doi.org/10.1080/14794802.2014.918339
Sfard, A. (2018). On the Need for Theory of Mathematics Learning and the Promise of 'Commognition'. In P. Ernest (Ed), The Philosophy of Mathematics Education Today (pp. 219-228). Springer, Cham.
Tabach, M., & Nachlieli, T. (2016). Communicational perspectives on learning and teaching mathematics: prologue. Educational Studies in Mathematics, 91(3), 299-306. https://doi.org/10.1007/s10649-015-9638-7
Thompson, D. R., & Rubenstein, R. N. (2000). Learning mathematics vocabulary: Potential pitfalls and instructional strategies. The Mathematics Teacher, 93(7), 568-574.
Zazkis, R., & Kontorovich, I. (2016). A curious case of superscript (−1): Prospective secondary mathematics teachers explain. The Journal of Mathematical Behavior, 43, 98–110. https://doi.org/10.1016/j.jmathb.2016.07.001
Zainal, Z. (2007). Case study as a research method. Jurnal Kemanusiaan, 9 (1), 1-6.