FORMATIVELY ASSESSING NOVICES' CAPABILITIES WITH MODELING CONTENT

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This paper examines the possibilities of designing a formative assessment that gathers information about novice elementary teachers' skills with modeling content and makes sense of such information. A decomposition of the practice of modeling content was developed and used to design the assessment. Participants included ten first-year teachers who graduated from a range of different teacher education programs. The findings reveal that our formative assessment works to gather information about teachers' capabilities with modeling content and that the associated tools support making sense of the information gathered.

Keywords: Assessment; Instructional Activities and Practices; Preservice Teacher Education

Ms. Hazard is a third-grade teacher who is currently teaching her students how to name shaded parts of areas as fractions using the definition of a fraction in the Common Core State Standards (CCSSO, 2010). She knows that it is crucial that students grapple with the importance of making equal parts, naming one of the equal-sized parts, and then counting the number of shaded parts to name the fractions. In her initial lesson, she presents students with the task shown in Figure 1. There is disagreement about the fraction of the rectangle that is shaded, with students suggesting 1/3, ¼, and 1/2. She holds a rich discussion where children share their thinking and consider the thinking of others, including asking questions of their classmates. The class talks about how some students arrived at ½ because they used part of the rectangle as the whole and other students, who arrived at 1/3 or ¼, used the whole rectangle. They further talk about how some students noticed that one of the parts was bigger and that they added a line to make the parts the same sizes and that this is how some people arrived at 1/3 and others arrived at ¼. At the end of the discussion, it seems that a convincing argument for ¼ has been shared and that the class is tentatively in agreement that the shaded area is ¼ of the large rectangle. Ms. Hazard recognizes that the class needs to consolidate the core ideas shared.

What fraction of the rectangle is shaded gray?

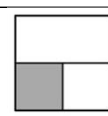

Figure 1: Third Grade Fractions Task

Ms. Hazard concludes by saying to the class, "We had such an important discussion today and I want you to listen carefully as I share how I think about naming this fraction because it connects to lots of ideas that were shared today and can help us think about other fraction problems." Ms. Hazard goes on, "The first thing I ask myself is, 'What is the whole?', and I outline the whole. Our whole is the big rectangle." As she says this, Ms. Hazard outlines the big rectangle with a whiteboard marker. "Then, I ask myself, 'Is the whole divided into equal parts?' Let's look, here the parts in the whole are different sizes. See this part [pointing to the rectangle on top] is bigger than these parts [pointing to the parts on the bottom]. So, I need to make the parts the same size, so I can divide this part [pointing to the rectangle on top] in half to make the parts the same size." Ms. Hazard uses her red marker to split the part in half. Ms. Hazard goes on, "Now I ask myself, 'how many equal parts does it take to make the whole?' I can count one, two, three, four." As Ms. Hazard counts, she points to each part and labels them one, two, three, four. Ms. Hazard goes on, "Then, I ask myself, 'What do you call one of the parts?' We call one of the parts, one out of the number of equal parts. We have four equal parts so we can call one of our equal parts, one-fourth. We have one equal part shaded in our rectangle, so one-fourth of the rectangle is shaded." Ms. Hazard labels the shaded part as one-fourth on the board. Ms. Hazard concludes, "We will continue to work with fractions and it is really important that we ask ourselves four questions. What is the whole? Trace around the whole [while tracing]. Is the whole divided into equal parts? If not, make the parts the same size [traces line with finger]. How many equal parts does it take to make the whole? What do we call one of the equal parts? We can call it one-fourth." Ms. Hazard records these questions on the board as she says them.

Ms. Hazard first engaged students in grappling with mathematical ideas related to naming a shaded part of area as a fraction and concluding that ¼ of the rectangle is shaded. She then provided access to thinking by naming, highlighting, and scaffolding important ideas surfaced by students in ways that support the whole class in doing complex mathematics. Ms. Hazard named the core ideas and provided scaffolding questions that support students in learning what to ask themselves as they approach such mathematical work.

In recent decades, there has been increased attention to several instructional practices that teachers can use to support students in constructing understanding of mathematics, including selecting rich mathematical tasks (i.e., Smith & Stein, 1998) and facilitating discussions of students' work on such tasks, either in small group or whole class (Chapin, O'Connor, & Anderson, 2013; Smith & Stein, 2011; Kazemi & Hintz, 2014). But we argue that these instructional practices are, by themselves, insufficient to enable every student to be successful with complex mathematical work and to experience of joy of mathematics as they can often leave core mathematical ideas "in the ether" rather than supporting students to synthesize and generalize the ideas they have developed individually and collectively.

Fundamentally, modeling content is about intentionally and thoughtfully providing access to content that may otherwise remain hidden to some students. Modeling, which requires the teacher to think aloud while demonstrating a skill, makes visible those practices and processes that happen internally and often remain invisible to learners if not explicitly named, explained, and shared. As we saw in the vignette from Ms. Hazard's classroom, when modeling, a teacher names, highlights and scaffolds the topics and practices in ways that support students in doing complex mathematics without taking over the work for them or compromising students' agency and their opportunity to engage in complex mathematical work.

The concept of "explicitness" in modeling is distinct from "direct instruction," which is a teacher-centered approach for delivery of content instruction (Archer & Hughes, 2011). Explicitness in modeling is not meant to reinforce or recreate the patterns of telling that are often seen in U.S. mathematics classrooms (Stein, Smith, Silver, & Henningsen, 2000), but rather it is aimed at making "access" a reality rather than a goal of collaborative development of mathematical ideas. But this sort of explicitness has often had an uncomfortable place in mathematics education. We argue both that this sort of explicitness is important and that the consideration of when to make content explicit through modeling is crucial for ensuring that all students have opportunities to engage with complex mathematical ideas. The key here is what is made explicit and what is left for students to figure out, and what "making explicit" means for the role of the teachers and students. Yet, partly due to concerns that focusing on modeling content might result in direct instruction, mathematics teacher educators have often backgrounded the work of making complex mathematics accessible through modeling. Further, this practice is complex and difficult and there is much to learn about how to support new teachers in learning to model content in productive ways (Charalambous et al., 2011).

We believe that formative assessment could make a substantial contribution to preparing new teachers to model content to support children's mathematical learning. By formative assessment, we mean assessments used to formulate subsequent learning opportunities (Cizek, 2010). Formative assessment enhances learning by revealing the current state of learners' knowledge, skills, and dispositions and ensuing action that facilitates growth (Black & Wiliam, 1998; Hattie & Timperley, 2007; Shute, 2008). We use the term skills to describe teacher candidates' (TCs) abilities to carry out specific aspects of the work of teaching at a particular moment in time, fully recognizing that TCs' capabilities will grow and change over time. Studies of the development of expertise have found that practice opportunities alone do not sufficiently support TCs to improve. Practice opportunities need to be coupled with structured directive coaching (Ericsson & Pool, 2016), which formative assessment can provide. Thus, formative assessment is a critical component in teacher preparation (AMTE, 2017; Darling-Hammond et al., 2005) and we need additional tools to assess TCs' skills with particular practices and components of teaching (Boerst et al., 2020; Shaughnessy & Boerst, 2018; Shaughnessy, Boerst, & Farmer, 2019; Shaughnessy et al., 2021). We argue that if we develop and refine formative assessments of teaching practice and tools that support teacher educators in providing timely feedback to TCs on their skills, then TCs will develop more robust skills with these teaching practices, impacting student learning. Thus, we sought to investigate whether it would be feasible to design a formative assessment focused on modeling content for use with elementary TCs. Specifically, we investigated whether such an assessment could elicit and reveal detailed aspects of TCs' skills with modeling content.

The Teaching Practice: Modeling Content

Modeling Content

Although the field of mathematics education does not yet have a shared conceptualization or common language for decomposing modeling content, a number of scholars have worked to specify and define instruction related to modeling content in ways that support our conceptualization of this instructional practice, which is distinct from behavioral modeling (demonstrating how to complete a task or behavior). The work of Collins, Brown, and Newman (1989) and Collins, Brown, and Holum (1991) in their development of the cognitive apprenticeship as a way to make thinking visible to students, name modeling as the work of performing a task while making internal processes external. Leinhardt's seminal research on instructional explanations (e.g., Leinhardt, 2010; Leinhardt et al., 1991; Leinhardt & Steele, 2005) has illustrated the complexities of explanations that are accountable to both the discipline of mathematics and pedagogical purposes. They also suggest criteria for such explanations.

Other scholars have worked to decompose the work of modeling content by examining specific types of teacher moves related to explicitness. Selling (2016) offered a decomposition of eight types of teacher moves that made different aspects of mathematical practices explicit in middle and high school mathematics discussions. This included moves like highlighting and naming when students were engaged in particular practices. Furthermore, Selling highlighted how the work of making aspects of mathematics explicit often happens at the end or in the middle of lessons, rather than at the beginning, which is also supported by other empirical research (e.g., Schwartz & Bransford, 1998; Schwartz & Martin, 2004). Further, teachers' choice and use of particular mathematical examples can support, constrain, or even obscure the mathematics that is available for students to learn within a particular domain (Ball et al., 2005; Rowland, 2008) and the choices teachers make about particular mathematical representations influence the nature of students' learning opportunities (Ball, 1993).

Overall, the mathematics education research base supports our conceptualization of modeling content and suggests that this practice is related to student learning (Cohen, 2018). The research also supports our conception of modeling as being appropriate and effective after students have had an opportunity to grapple with the mathematical ideas themselves (Schwartz & Martin, 2004). We recognize that the way that this practice is often enacted does not align with our view of this practice. When Cohen summarized her study of explicit instructional strategies (modeling & strategy instruction) in elementary ELA and mathematics lessons, she reflected, "explicitness need not be directive or prescriptive, though the instruction in this study suggests that it often is" (Cohen, 2018, p. 322). This suggests that, given the tendency for directiveness and prescriptiveness in classrooms, it is crucial that we support teachers in learning how to make context explicit in productive ways that are not directive or prescriptive.

Decomposing the Teaching Practice

To assess teachers' capabilities in modeling content, we drew upon Grossman et al.'s (2009) notion of parsing teaching practice into specific areas of work to create a "decomposition" of the practice. Because we were interested in using decomposition in teacher preparation, we attended to the importance of decomposing in ways that the practice can be taught and learned by TCs (Boerst et al., 2011). We drew upon prior research on the importance of modeling content for student learning and research on moves that teachers make when demonstrating, explaining, and modeling content to develop an unpacking of the work of modeling content, a "decomposition" of the teaching practice. We identified different aspects of what teachers have to do to model content. We acknowledge that a decomposition is a living document (Jacobs & Spangler, 2017; Shaughnessy, Ghousseini, et al., 2019; Shaughnessy et al., 2021). One challenge of decompositions is that while they are meant to provide details about the work of carrying out the practice, they cannot name the complete set of knowledge and skills required to carry out the practice or they will be unwieldy. While we recognize that teaching practice is dependent on mathematical knowledge for teaching and teachers' views of mathematics and children's ideas, the decomposition is focused squarely on teachers' enactment of the teaching practice.

Our decomposition deliberately foregrounds certain aspects of the work of modeling content with students—specifically those that we consider crucial for entry-level teaching. We further considered the aspects of modeling content that are most likely to be accessible to and learnable by novices. We began by identifying specific aspects of leading modeling content and what teachers of any level of experience and expertise do to try to accomplish those. For example, in modeling content, one important goal is to make thinking visible that might otherwise be invisible. This entails specific techniques such as annotating, which refers to adding in ideas necessary to support students' understanding, including clearly articulating what you are doing and why you are doing it. These moves are also foundational for the more complex work experienced teachers do in modeling content. We organize our decomposition for modeling content into six areas of work: (1) planning to model, (3) framing, (3) doing content area work, (5) highlighting core ideas, and (6) making thinking visible by emphasizing thinking and key elements, and (6) using language and representations carefully (see TeachingWorks, 2019 for a list of moves associated with each area of work).

Assessing Teacher Candidates' Skills with Modeling and Explaining Core Content

The design of the assessment was influenced by our view of teaching as interactive and situated in contexts (Cohen et al., 2003; Lampert, 2001). We aimed to design an assessment focused squarely on TCs' ability to model content in real time. Three considerations led us to design a standardized assessment in which TCs video recorded themselves modeling content in response to a prompt without children present. First, skillful modeling is responsive to the ideas and resources that students bring and learning goals for students. A standardized assessment enabled us to design a scenario in which we could specify the prior work that children had done and the learning goals for students. Second, modeling content can be done by the teacher, students, or co-constructed by teachers and students. These approaches differ in how much can be seen about a teacher's skills with modeling. Our standardized assessment created an instance in which we could see TCs doing all of the work rather than handing some of it off to students. Third, because the selection of content matters and we wanted a design that would enable us to see patterns in performances across TCs, a standardized assessment allowed us to standardize the content being modeled and the example and representations used. We purposefully selected content that is core to the upper elementary curriculum (comparing fractions) and focused on the use of a particular representation (the number line) to see how all TCs could model content in this situation. Specifically, we asked TCs to model comparing the fractions 5/4 and 5/6 using the number line. Figure 2 contains the "class background" and instructions. Aware that TCs might have different ideas about the practice of modeling, we included a definition of the work of modeling in the full instructions and student learning goals.

Class Background
Your fourth grade students have been working on identifying and ordering fractions. They are familiar with number lines and understand the importance of creating segments of equal length when partitioning the whole into segments of equal length. Students have worked with common fractions such as halves, thirds, fourths, sixths and eighths. They are comfortable with the relationship between halves, fourths, and eighths. Students in general know that if you divide a unit interval into *n* equal segments that *n*/*n* is one whole.
Instructions
You will model how to use a number line, or two number lines, to accurately place and compare two fractions. You can choose any number line representation(s) that you think best communicates these ideas to fourth grade students. • Begin with unmarked line(s) and model placing the necessary whole numbers. It is recommended that you place the whole numbers far enough apart that you can easily partition the intervals into fourths and sixths. • Show how you determine, with precision, where each of the fractions falls on the line. Specifically, remind students how to partition the unit interval(s) and use that information to locate the fractions on the line. • Explain how to determine which fraction is greater using the number line. You should model the process of using this technique as you would for a group of fourth graders. The goal is to make explicit the process for placing and comparing the fractions 5/4 and 5/6 on the number line.

Figure 2: Assessment Task

We used the decomposition to develop a checklist that would support us in noticing the skills with modeling content demonstrated by the TCs. We focused on five areas of work: *framing*, *doing the content area work*, *highlighting core ideas, making thinking visible by emphasizing thinking and key elements,* and *using language and representations carefully*. We excluded *planning* because we identified the content to be modeled and the example and representations for the TCs. Our checklist focuses on the teacher's use of specific techniques within our decomposition. In other words, techniques are what we look for to see if the practice happens. We could look directly for most of the techniques. However, in some cases, we had to further unpack ideas. For example, within *highlighting core ideas*, we had to identify the core ideas.

Present Study

We used the tool we had developed to investigate whether an assessment could elicit and reveal details in teachers' skills in modeling content. While the assessment was designed to be used with TCs enrolled in a teacher education program, we studied the practice of first-year teachers from a range of teacher education programs because this enabled us to study the use of the assessment in ways that were not constrained to one teacher preparation program.

Methods

The ten participants, all in their first year of teaching, spanned grades 1–5. All of the teachers were teaching in the midwestern USA. Contexts varied across urban, suburban, and rural schools. We recruited a diverse sample of teachers with respect to grade level, school district, and teacher preparation program. This sample was not intended to be representative of all first-year teachers. All teacher names used in subsequent sections are pseudonyms.

Data sources include a video record of each participant modeling in response to the provided prompt. We met with each teacher individually in their classroom after school hours or during a prep period. We gave teachers the assessment instructions and allowed 20 minutes of preparation time. Teachers were aware that we were studying their skills in modeling content. Teachers were told that they had up to 10 minutes to complete the modeling.

Data analysis proceeded in two phases. First, members of the research team independently watched each video and used the checklist to capture attributes of the performance. Second, the team discussed cases where there were differences in observations on the checklist, referencing and refining a codebook as needed to reach consensus.

Findings

Framing

Framing included both launching and closing the modeling. For launching, we found that eight of the 10 teachers made an opening statement that *named what was about to be modeled*. Two teachers *explained the purpose* of what was being modeled. Three teachers *connected the modeling to students' prior learning*, future learning, or background and/or experience. None of the teachers *closed the modeling* by summarizing or revisiting core ideas and results.

Doing Content Area Work

The content area work included four components. Six of the 10 teachers marked whole units (intervals) on the number line. To place the first fraction on the number line, six teachers marked the number line and labeled the fraction. To place the second fraction, eight teachers marked the number line and labeled the fraction. Seven teachers stated which fraction is greater.

Highlighting Core Ideas

With respect to *backgrounding* ideas, we found that only two of the 10 teachers avoided highlighting aspects of the content or task that are distracting or may lead to misconceptions.

This means that eight teachers highlighted aspects of the content or task that were confusing or might lead to misconceptions. For example, Mr. Houston highlighted that he was using 24 cm as the unit length because it was divisible by both 6 and 4.

Three of the teachers *foregrounded* mathematical ideas by using explicit verbal and non-verbal markers to draw students' attention to important aspects of the content. Ms. Wheeler used gestures to emphasize that the value of a fraction on a number line represents a distance from zero by motioning from the zero on the number line to the point representing the fractional value.

Teachers varied in *marking ideas*. We focused on five core mathematical ideas. Two of the 10 teachers elaborated the whole unit before partitioning, five teachers highlighted the meaning of the denominator (with respect to one or both of the fractions), three teachers explained the partitioning using the language of equal parts (or same-sized parts), nine teachers showed how to use the numerator and the parts that had been marked to locate the fractions (with respect to one or both fractions), and five teachers stated how to use the representation to determine the comparison. There was variation in how many ideas each teacher marked. Further, we examined the *sequencing of teachers' marking of the core ideas* and found that only three teachers compared the fractions on a number line in a logical manner.

Making Thinking Visible by Emphasizing Thinking and Key Elements

We examined whether teachers made use of three different sorts of techniques for making thinking visible by emphasizing thinking and key elements: annotating, marking metacognition, and thinking aloud. Two of the ten teachers engaged in *annotating*, adding ideas necessary to support students' understanding, including clearly articulating what you are doing and why you are doing it. For example, Ms. Wheeler emphasized why she was drawing two (stacked) number lines and the importance of lining up the number lines to later support the comparison. One teacher, again Ms. Wheeler, engaged in *thinking aloud* as a means to make thinking visible for students. After Ms. Wheeler marked both fractions on the number line, she engaged in a think aloud about how to compare the fractions. None of the teachers *marked metacognition* by using verbal, tone, or visual markers to indicate to students when thinking was being made visible.

Using Language and Representations Carefully

When looking at teachers' use of language, we found that nine of the 10 teachers consistently *used language economically*. All 10 teachers consistently *used language which was grade level appropriate*. However, only two of the teachers consistently *used language which was mathematically accurate and precise*. The mathematical language used by teachers that was not accurate and/or precise included referring to numbers as higher and/or lower, and saying fours and sixes rather than fourths and sixths. When we looked at teachers' use of representations, we found that only four teachers *drew number lines in accurate ways* (e.g., having arrows on both ends of the number line, marking 0, and making equal parts). Nine of the teachers *produced writing and representations that were legible and visible*. Only four teachers *organized the board in a way that supported understanding* how to use a number line to compare two fractions.

Discussion

Having access to detailed information about TCs' developing proficiencies with teaching practices is crucial for quality teacher education. Moreover, teacher educators must be able to use the data gathered from such assessments to focus efforts to support TCs' development. This study sought to examine the utility of using an assessment to assess teachers' skills with modeling content for formative purposes. The findings reveal details about skills with modeling content displayed by each of the teachers in the study.

We identify four key limitations of the design and use of the assessment and checklist. One limitation concerns the use of a checklist, yielding information primarily about the presence or absence of each technique rather than the use of a rubric, distinguishing the quality or quantity of teachers' enactments above the threshold. However, checklists allow for in-the-moment scoring, whereas a rubric requires that a teacher educator watch the entire performance and often rewatch it before making a judgment. Further, the checklist addressed our goal of capturing whether a TC could enact a particular technique. A second challenge is the potential interaction of mathematical knowledge for teaching (MKT) and modeling content. Even with the focus on content that matters for teaching mathematics, and supports built into the materials (e.g., student learning goals), there were instances in which MKT appeared to be a factor in the enactment. This is unsurprising as MKT is clearly intertwined throughout the work. In these cases, it was unclear whether we were accurately capturing a teacher's skill with modeling content or whether their use of MKT prevented them from demonstrating their modeling skill. This is an inherent problem in examining teaching practice. A third limitation is that we do not know whether the teachers in this study would perform differently if they modeled content in a different context. The context could be interpreted along multiple dimensions, including, but not limited to, the mathematics task used, the grade level of the students, or even engaging in the modeling work with actual children rather than through a simulation. This is an inherent problem; however, because we are not making claims about teachers' skills more broadly, we argue the assessment reveals important information about teachers' skills with modeling that can be used for formative purposes. A fourth limitation is that our study does not provide evidence of whether these teachers could decide what and when to model in their own teaching. We call out this point because, as we stated in the introduction, the goal of a teacher modeling in math class is to expand access to opportunities to engage in cognitively-demanding mathematical work. The aim is not to increase access by lowering the cognitive demand and spoon-feeding content to students. Instead, modeling is a means to consolidate ideas that students have explored. Thus, it is crucial that other sorts of information be gathered to determine (and, if necessary, intervene on) TCs' beliefs about what, when, and why to model.

Despite these limitations, our findings suggest that a standardized assessment of modeling content can reveal important information about TCs' skills. As we did not intend to make claims about this sample of teachers or a larger population of TCs, the small sample was appropriate for our goals. However, the variations in performance that are used to illustrate the capabilities of the assessment and tool cannot be interpreted as representing the skills of a larger population of TCs. This standardized assessment accomplished many of our design goals. Its ability to capture a range of skills could make the assessment and scoring tool useful in teacher education. Teacher educators and programs could use such assessments to track TCs' growth over time and to identify areas of strength and weakness with respect to the practice, which would allow for targeted support and program-level curriculum design.

We close by noting two questions that arose from this study. First, we might explore how this assessment could fit into a trajectory of assessments for assessing TCs' skills with modeling content. Second, in light of the questions that arose in this study about the role of content knowledge, we need to investigate further how we can reliably assess teaching practice in ways that account for the role of content knowledge for teaching.

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