SUPPORTING THE DEVELOPMENT OF POSITIVE MATHEMATICS TEACHING IDENTITIES IN PRESERVICE TEACHER EDUCATION

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Research suggests that preservice teachers enter teacher education with predominantly negative dispositions towards mathematics. We present a case study of an approach to supporting the development of positive mathematics teaching identities among students in a middle-level math methods course. Our findings suggest that engaging in narrative reflection in tandem with collaborative problem solving in a mathematical discourse community over time helped students to transform their relationships with mathematics and forge positive mathematics identities. Providing opportunities for students to share and work through their vulnerabilities was critical to this process. Our findings illustrate a promising approach for teacher educators to support the development of positive mathematics teaching identities in preservice teacher education.

Keywords: Preservice teacher education, problem solving, affect, emotion, beliefs, and attitudes

Introduction

Preparing preservice teachers (PSTs) to teach mathematics involves attending to the development of complex knowledge, skills, and reasoning practices. It also involves supporting the identity work that novice teachers must do to position themselves as mathematics teachers (Ntow & Adler, 2019; Lutovac & Kaasila, 2014). This identity work is made especially challenging by common approaches to mathematics instruction that permeate U.S. K-12 schools. Confronted by years of memorizing facts and procedures, in settings that emphasize being first and fastest, the majority of young people entering teaching view mathematics as a body of disconnected, procedural skills that need to be memorized, leading many to conclude that they are simply "not a math person" (Relich, 1996; Machalow et al., 2020).

This paper presents findings from a small study of an effort to support the mathematical identity work of a group of middle-level PSTs through mathematics problem solving and reflections. Building on the work of Machalow and colleagues (2020), we see PSTs' identities as teachers of mathematics as grounded in their experiences as learners of mathematics and intertwined with their orientations toward the subject (Gresalfi & Cobb, 2011; Ntow & Alder, 2019). Our research was guided by the following questions:

1. What connections between mathematics learning experiences and orientations toward the subject are surfaced through PSTs mathematics identity work?
2. How does engaging in collaborative problem solving influence PSTs' relationships with and orientations towards mathematics?

The participants in this study were students in a one-year teacher preparation program, seeking middle-level (ML) certification (grades 4-8). The context of the study was a semester-long mathematics methods course for all ML candidates, the majority of whom were specializing in subjects other than mathematics. The authors were both instructors of the course.

Conceptual Framework

The conceptual framework guiding our study builds on work by Machalow et a. (2020), which conceptualizes mathematics identity for teaching as being shaped by teachers'

interpretations of their experiences and their related mathematics orientations. We understand *mathematics identity* as a learner's interpretations of themself in relation to mathematics (Darragh, 2016; Graven & Heyd-Metzuyanim, 2019). Learners develop and continuously reshape their mathematics identities through experiences learning mathematics, in which multiple social factors are implicated. When learners engage with mathematics, they are also influenced by relationships with teachers and classmates, their social identities, such as race, gender, and class, and institutional systems that often make mathematics dehumanizing (Sfard & Prusak, 2005; Gutierrez, 2002). PSTs' mathematics identities are central to their development as teachers. Teachers with negative mathematics identities are more likely to focus on rote procedures and exhibit low expectations for their students (Brady & Bowd, 2005; Relich, 1996).

Teachers' orientations toward mathematics are core to their mathematics identities. Machalow and colleagues (2020) define *mathematics orientations* as one's beliefs about the nature of mathematics knowledge. They employ Skemp's (1976) two contrasting approaches to understanding mathematics to distinguish between a view of mathematics as *instrumental,* comprised of procedural knowledge, and *relational,* characterized by understanding why rules work and the relationships between them. Often, PSTs enter teacher education having learned mathematics in highly instrumental ways and have come to think of mathematics as a series of rules and procedures. Studies suggest that when students are exposed to relationally oriented approaches to mathematics, they have more of an opportunity to develop positive relationships with mathematics as a subject (McCulloch et al., 2013; Brady & Bowd, 2005).

Literature Review

Increasingly, learning is understood as a process of identity formation or becoming a certain type of person in a particular context (Lave & Wenger, 1991). Learning to teach mathematics involves developing an identity as a mathematics teacher (Hodges & Hodge, 2017; Ntow & Adler, 2019), in which mathematics identity plays a central role. Many preservice elementary teachers, however, have had problematic relationships with mathematics and are likely to experience feelings of math anxiety or lack confidence in their own ability to effectively teach mathematics based on negative formational experiences as mathematics learners (*eg.,* Brady & Bowd, 2005, Machalow et al., 2020, McGlynn-Stewart, 2010). Research on efforts to shift PSTs' relationships to mathematics offer two distinct approaches: exploring mathematics in order to "break" from one's past experiences with the subject (Ball, 1990) and engaging them in identity work to influence their mathematics teaching identities (Ntow & Adler, 2019).

Engaging PSTs in doing mathematics in ways that emphasize relational understanding (Skemp, 1976) and are likely to differ substantially from their experience as K-12 students is a well-established approach used by teacher educators (e.g., Ball, 1990). Various studies demonstrate how such opportunities to re-learn primary mathematics in methods classes can support positive shifts in mathematics orientations and identities (Harkness et al., 2007; McGlynn-Stewart, 2010). Harkness and colleagues (2017), for example, found that by working constructively in groups to make sense of mathematics increased PSTs' self-concept and self-efficacy, as well as their understanding the role that the teacher played in facilitating this productive struggle. Researchers have also found, however, that success of these approaches is dependent upon several key components of instructional design, including emphasis on relational understanding of mathematics concepts using concrete representations, positive support from teachers and peers, and collaborative work in communities of practice (Harkness et al., 2017; Kaasila et al., 2008; Machalow, et al., 2020).

Research that examines efforts in teacher education to positively influence PSTs' mathematics identities has emerged as promising in the last decade. One widely discussed strategy is to provide PSTs with opportunities to explore their own identities through narrative accounts. In these studies, researchers (who are often teacher educators) invite students to recount their own formative mathematics learning experiences as narrative texts, hypothesizing that this type of narrative work promotes reflection and understanding of one's own mathematics learning experiences (Kaasila et al., 2008; LoPresto & Drake, 2004; McCulloch, 2013). Further, asking PSTs to reflect on how their experiences have shaped their relationship with mathematics allows them to consider how common classroom practices influence how they come to think of themselves in relation to mathematics (e.g., Machalow et al., 2020; Remillard, 1993). Zazkis (2015) further argues that personifying the object of the narrative (mathematics) can emphasize the potency of the relationship learners have with the subject. The impact of identity on teachers' future orientations towards teaching underscores the importance of identity work in preservice teacher education. When teachers are able to reframe their past learning experiences through identity work, there is an increased likelihood that they will feel a sense of agency over their future mathematics teaching (Lutovac & Kaasila, 2014) and a greater capacity to foster positive math identities to future students (Machalow et al., 2020).

Our mathematics methods course combined these two approaches to surfacing and nurturing PSTs' relationships to mathematics: we engaged them in re-learning mathematics from a relational perspective and asked them to write narratives about their relationship with a *personified* version of mathematics. These narratives provided a window into PSTs' developing orientations toward mathematics and their mathematics identities. Our research used the narratives and other artifacts and observations to examine the connections between PSTs' experiences and their orientations toward and relationships with mathematics.

Methods

The eight participants in the study comprised the 2021-22 cohort of ML teacher candidates in a one-year, K-12 program, situated in a private (PWI) university in a large east-coast city. The program aims to prepare teachers to work in poverty-impacted, urban schools. The ML certification covered all subjects for grades 4-6 and content specialization for grades 7-8. Only one of the 8 participants was seeking a specialization in mathematics; the others were specializing in ELA (n=3), social studies (n=3), and science (n=1) and enrolled in content-specific courses for grades 7-12. All were enrolled in the ML mathematics methods course, taught by the authors. The fact that the majority of students in the course would be certified to teach grades 4-6 mathematics but identified as teachers of other subjects presented an instructional challenge and a research opportunity. Our aim was to explore the potential of the approach described above for nurturing non-mathematics PSTs' identities as teachers of mathematics. Three of the participants identified as white females, one as a white male, two as black females, one as a white trans-masculine non-binary, and one as an Asian male. Both authors identify as white females.

During the semester-long mathematics methods course, students were presented with a weekly Problem of the Week, an open-strategy problem involving mathematics content appropriate for grades 5-8. In the example (Figure 1), *Squares to Stairs*, students are prompted to justify their thinking as they explain the progression of the figure, followed by subsequent questions about increasingly complex case numbers and numbers of squares.

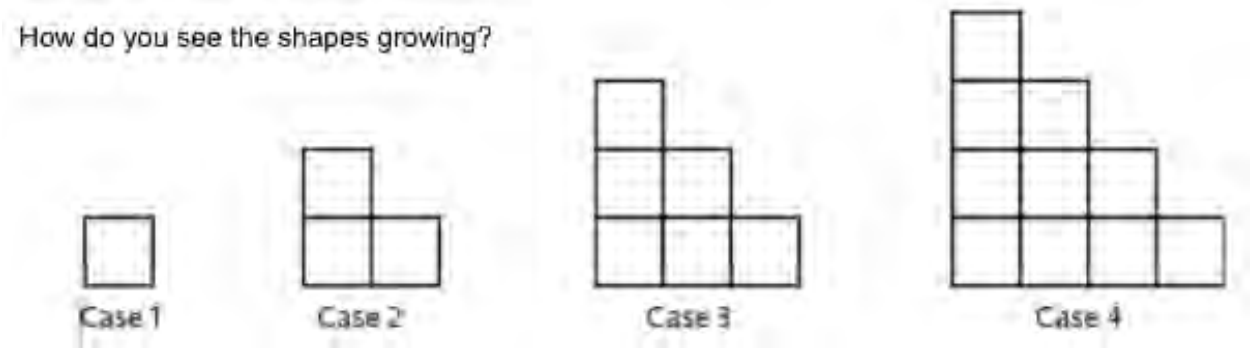

Figure 1. Squares to Stairs

Students worked on the problem outside of class and submitted a "rough draft" solution (Jansen, 2020) on an electronic document that all students had access to. The first 30 minutes of each class were devoted to paired and whole-class work on the problem, facilitated by the instructors.

The data for this study consist of two narratives written by participants, one prior to the beginning of the semester and one upon completion of the course, as well as the teacher educator/researchers' observations of students' work and interactions during in-class problem-solving sessions. The first narrative, called a "Dear Math Letter," was inspired by Bertolone-Smith and MacDonald (2020) and followed Zazkis's (2015) approach of personification. During the summer term, they were prompted to "Write a letter to Math as if Math was a person that you know." For the second narrative, students reflected on their work on the Problems of the Week in relation to the experiences and sentiments described in their initial letter.

We used inductive coding to analyze the narratives, highlighting emergent themes in the students' reflections. Themes in both narratives included salient mathematics learning experiences, shifts in views of mathematics and self-perceptions, and other influential factors such as teachers, classmates, or family members. We also applied a priori codes to indicate instrumental or relational mathematics orientations (Machalow et al., 2020). For the second narrative, we also noted specific problems mentioned by students as influential in their learning and the date during the semester when the problem was assigned. We then compared the two narratives for each participant to identify shifts and discussed themes across the trajectory of the cohort as a whole. Additionally, we used analysis of student work and observations from class sessions to supplement our understanding of the themes.

Results

Similar to previous research, we find that these PSTs entered preparation having had troubled relationships with mathematics, instrumental orientations toward the subject, and negative perceptions of themselves mathematically. PSTs described these relationships in connection to mathematics learning experiences that prioritized memorization, academic achievement, and external evaluation. These findings confirm prior accounts of the power of experiences in shaping PSTs' mathematics teaching identities (Gresalfi & Cobb, 2011; Machalow et al., 2020; Sfard & Prusak, 2005). Despite these negative entry points, students expressed a desire to improve their relationships with math for the sake of their future students.

Over the course of the semester and through participation in collaborative problem-solving sessions, we observed two parallel and related changes. As PSTs engaged with new types of mathematics learning experiences, they began to shift orientations towards mathematics and also began to form new mathematics identities, showing increased confidence and comfort with

mathematics as a subject. We hypothesize that these shifts were supported by both the nature of the mathematical problem-solving tasks and the role of a supportive mathematics discourse community within the class.

The sections that follow are structured to illustrate the trajectory of PSTs' developing mathematics teaching identities as they relate to past and current mathematics experiences. We begin by describing the findings from PSTs' "Dear Math Letters", indicating the patterns in the mathematics identities and orientations surfaced through their narrative descriptions. We then outline the changes that occurred throughout the course of the semester as described in their end-of-semester reflections and corroborated by researcher observations. These changes occurred within three discernable phases: *initial apprehensions*, as PSTs began the semester deeply influenced by their prior negative associations with mathematics; *aha moments*, which occurred several weeks into the semester, as PSTs began to experience new realizations about mathematics in their problem-solving work; and, finally, *transformed relationships with mathematics*, where PSTs were adopting new strategies and approaches to problem solving and becoming more confident in their ability as learners and teachers of mathematics. We conclude by discussing how collaboration in a mathematics discourse community served as a key mediator for the transformations described by PSTs at the end of the semester.

"Dear Math, You Are Such a Heartbreaker."

The act of personifying math in their initial *Dear Math* letters brought forth the emotional nature of students' relationships with mathematics. Students revealed predominantly negative mathematics identities, which were deeply intertwined with their social identities. The letters were moving for us as instructors to read; students shared heart wrenching accounts of tears, frustration, anxiety, pressure, and feelings of inadequacy defining their past experiences with mathematics. For example, Faith began her letter telling math: "you have always frustrated me, and I won't deny that I've cried over you more than once…the academic pressure on me felt so crippling."

These accounts of negative experiences often pointed to an instrumental understanding of mathematics, in which memorization of rote procedures and a need to get the answer "right" figured significantly. Students wrote about their experiences with "drills", "missing points", the demoralization of "never getting an A" or the devastation of "getting an F." All eight students mentioned testing as contributing to a sense of inadequacy or uncertainty in relationship to mathematics. In these and other similar comments, students described math as an individual endeavor where one fails or succeeds on their own.

Along with the individualized pressure to succeed, PSTs' descriptions of mathematics learning experiences demonstrated how institutionalized presentations of math can be seen to be highly dehumanizing, perpetuating notions that mathematics ability is linked to a person's value in society. This type of comparison often occurred in relation to other dimensions of students' social identities. In many cases, the notion of being judged in relation to mathematics was highly influential in shaping students' orientations toward and relationships with mathematics. Several students noted feeling inferior to family members or classmates who were "naturally" better in math. Some of these emotions were deeply internalized: Kevin, a student who immigrated from Korea in middle school wrote to Math: "you were the stereotype I desperately tried to run away from. I think I told myself that I was 'not good' at math because I was afraid that you would take over my whole identity".

When PSTs mentioned positive associations with math, they were fleeting; students either felt positively about math when they received good grades or as a result of a supportive

relationship with a teacher. Several students mentioned at least one teacher by name who helped them to improve their motivation to do math and their sense of ability to succeed in a particular mathematics course. Other positive orientations to math related to completing a problem, as Natalie described feeling like "the heavyweight champion of the world" after finally solving division problems. These positive experiences did not appear to have a lasting impact on their mathematics identities, perhaps in part because they were contingent upon temporary circumstances.

For many students, the conclusion of their letter to math expressed a desire to reconcile their own negative experiences and find ways to help their future students develop positive mathematics identities. They raised concerns about how their own problematic relationships with math would influence their identities as teachers, as Faith writes: "How can I teach my students to love and engage with you when I don't even do that myself?" As illustrated by this and other statements, PSTs may recognize their negative relationships with math, but pathways to rectify these relationships remain out of reach.

"I Didn't Know You Could Manipulate Numbers Like That!"

A primary component of the mathematics methods courses was collaborative work on the Problem of the Week. Problem solving discussions were structured around students' strategies, and instructors strategically paired students to share different approaches to solving the problem. During the whole class discussion, students compared different strategies and agreed on a reasonable solution. Based on our analysis, we saw three key stages that emerged over the course of the semester through students' work with these problem-solving activities.

Initial apprehensions. In the initial weeks of the semester, we found that PSTs continued to demonstrate behaviors that focused on finding algorithms and wanting to be sure to get the correct answer so as to not be judged by their classmates or teachers. Students also exhibited self-protective behaviors, such as adding self-deprecating notes or humorous memes to their submitted rough-draft thinking in an effort to deflect from their feelings of inferiority. This initial period of discomfort was evident in many students' reflections at the end of the semester. For example, Dylan explains:

> When this assignment first started, I felt very uneasy presenting my work to the class. I was constantly comparing myself to the other students… I feared my classmates would judge me as not being intelligent enough for the program. This was clearly reflected in my early Problem of the Week assignments…there is little evidence of work being completed and there is an emphasis on correctness.

This type of response was shared by the majority of the students in class, as they reported finding it difficult to engage with problems and practices that were counter to their prior mathematics learning environments. Students realized, however, that they needed to "unlearn" these deeply ingrained tendencies.

Aha moments. The structure of sharing work collectively in class pushed students to work through their vulnerabilities. Through discussing different strategies and visualizations for solving problems, students were exposed to new ways of thinking about mathematics, which led to a second phase that we refer to as a period of *aha moments*. Students gradually began to place more emphasis on strategies than arriving at a correct answer:

> When we worked on the problem in class I felt like something finally clicked and I realized that my visual model would have been useful if I had spent less time worrying about whether or not it was the "correct" way to solve the problem and instead just stuck it out and

recognized that my ideas do not have to be correct to be worthwhile, they have an inherent worth." (Brandon)

Two problems in particular, *Transparent Algorithms* and *Squares to Stairs*, helped students to see more mathematical meaning and develop deeper conceptual understanding. Both the nature of these problems as well as the timing mid-semester could have contributed to multiple students reaching *aha moments* at this point. Madison described her *aha moment,* stating: "On transparent algorithms, I did not know that you could manipulate numbers like that, straying from the basic, standard algorithms!" *Squares to Stairs* engaged students in recognizing patterns to help visualize figures in linear sequences. Unlike earlier in the semester when students focused on finding an algorithm to solve the problems, students used multiple ways to visualize and solve the pattern. Natalie's *aha moment* noted the importance of representing problems to find solutions: "Because of how we have been socialized, I would have thought that it was an elementary school tactic to visually represent a problem. I saw how my classmates drew out their patterns and I thought 'Why not?'". In these moments, students describe developing increased relational understandings as opposed to their initial instrumental orientations. As these orientations shifted, students also expressed more enthusiasm for problem solving. They began to position themselves as capable of breaking from the procedural rules of mathematics in favor of trying new strategies without a fear of being "wrong".

Transformed relationships to mathematics. During the final phase, students began taking on new strategies and approaches that they learned through their problem-solving discussions. As highlighted in their *aha moments*, working in collaboration with one another helped them to feel empowered to try out different strategies that focused on developing conceptual understanding rather than getting the correct answer. Once Natalie found that it was not indicative of a lack of understanding to attempt multiple strategies, she explained that she "looked for patterns, switched orders, crossed things out and used the process of elimination. I surprised myself and my confidence grew." Similarly, Faith reflects on how she began documenting "discoveries" as a result of her work with her classmates, which helped her "discover something new in every iteration" as she worked towards a solution.

As a result of engaging in problem solving discussions and reflective identity work, students made incredible shifts in their mathematics identities and orientations over the course of one semester. Several students continued to teach math in their field placements beyond what was required, and three students went from thinking they would never want to teach math to expressing interest in seeking certification to teach middle years math.

"I Had Eight Other Minds On the Job."

Looking across their reflections on their experiences engaging in the Problems of the Week, we found that the role of the mathematics learning community was a key mediator in helping students to shift their relationships with mathematics. Students trusted the class community in a way that enabled them to move through their vulnerabilities and in doing so, they allowed themselves to release the security of algorithms and accuracy. Brandon expressed how working with his peers supported this shift: "I felt like I was part of a community of inquiry dedicated to finding solutions instead of being evaluated on my ability to simultaneously learn, synthesize, and enact a new concept."

Further, the experience challenged their preexisting notions of math as individualized and dehumanizing. Students found support in one another and were able to move beyond their initial phase of vulnerability. While before this conceptual and identity work, Kevin purposefully disassociated from mathematics to avoid stereotypes, he described how working with others

allowed him to grow: "I always assumed that 'doing math' was an individualized process. But incorporating meaningful discourse into the process of 'inquiring' into new mathematical concepts has evidently shown me that learning math is really a journey of growth."

Discussion

Our study reaffirms prior findings that suggest PSTs benefit from opportunities to re-learn mathematics from a relational perspective (Ball, 1990) and from engaging in narrative identity work to reconcile their prior experiences with mathematics (e.g., Machalow et al., 2020; McCulloch et al., 2013; Remillard, 1993). Our data also help to illustrate that PSTs' mathematics identities are complex and intertwined with other social identities that are implicated in how they learn to teach mathematics. Addressing a personified version of mathematics allowed PSTs to unpack how their prior experiences shaped their identities and helped them to problematize their instrumental understandings of math.

Our study provides a unique window into the power of engaging in this type of narrative identity work in tandem with conceptually oriented, collaborative problem solving. We believe that these two processes were mutually supportive in our PSTs' transformation across the course of the semester, which included beginning to identify as teachers of mathematics (Ntow & Adler, 2019; Lutovac & Kaasila, 2014). The cycle of reflecting on their views of math while working with one another in community rather than individually gave students space to envision a new approach to mathematics instruction. Norms for openness, learning from mistakes, and communication throughout the problem-solving discussions helped students increase their confidence and willingness to adopt new strategies. Further, their reflections on the problem-solving experience itself helped them to see the power of community and express a desire to carry that into their own classrooms.

Importantly, we found that this process takes time and consistent, on-going work to develop a community of practice that allows students to share their vulnerabilities with one another. Although our study focuses on a small cohort of PSTs, we posit that combining opportunities for students to develop relational understandings of math with narrative identity work and community building offers opportunities for teacher educators to support the development of positive mathematics teaching identities in preservice teachers.

References

Ball, D. L. (1990). Breaking with experience in learning to teach mathematics: The role of a preservice methods course. For the Learning of Mathematics, 10(2), 10–16.

Bertolone-Smith, C.M., MacDonald, B.L. (2020, February). *Dear math…: Examining women's self-identified critical experiences in mathematics* [Paper presentation]. Annual Association of Mathematics Teacher Educators, Pheonix, AZ.

Brady, P., & Bowd, A. (2005). Mathematics anxiety, prior experience and confidence to teach mathematics among pre-service education students. *Teachers and Teaching,* 11(1), p. 37-46.

Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics*, *93*(1), 19–33.

Graven, M., & Heyd-Metzuyanim, E. (2019). Mathematics identity research: The state of the art and future directions: Review and introduction to ZDM Special Issue on Identity in Mathematics Education. *ZDM*, *51*(3), 361–377.

Gresalfi, M. S., & Cobb, P. (2011). Negotiating Identities for Mathematics Teaching in the Context of Professional Development. Journal for Research in Mathematics Education, 42(3), 270–304.

Gutiérrez, R. (2002). Enabling the practice of mathematics teachers in context: Towards a new equity research agenda. Mathematical Thinking and Learning, 4(2 & 3), 145–187.

Harkness, S. S., D'Ambrosio, B., & Morrone, A. S. (2007). Preservice elementary teachers' voices describe how their teacher motivated them to do mathematics. *Educational Studies in Mathematics*, *65*(2), 235–254.

Hodges, T. E., & Hodge, L. L. (2017). Unpacking personal identities for teaching mathematics within the context of

prospective teacher education. *Journal of Mathematics Teacher Education*, *20*(2), 101–118.
Jansen, A. (2020) *Rough draft math: Revising to learn*. Stenhouse Publishing.
Kaasila, R., Hannula, M. S., Laine, A., & Pehkonen, E. (2008). Socio-emotional orientations and teacher change. Educational Studies in Mathematics, 67(2), 111–123.
Lave, J. & Wenger, E. (1991). Situated learning: Legitimate peripheral participation. Cambridge: Cambridge University Press.
Lopresto, K. D., & Drake, C. (2004). What's Your (Mathematics) Story? Teaching Children Mathematics, 11(5), 266–271.
Lutovac, S., & Kaasila, R. (2014). Pre-service teachers' future-oriented mathematical identity work. *Educational Studies in Mathematics*, *85*(1), 129–142.
Machalow, R., Goldsmith-Markey, L. T., & Remillard, J. T. (2020). Critical moments: Pre-service mathematics teachers' narrative arcs and mathematical orientations over 20 years. *Journal of Mathematics Teacher Education.*
McCulloch, A. W., Marshall, P. L., DeCuir-Gunby, J. T., & Caldwell, T. S. (2013). Math autobiographies: A window into teachers' identities as mathematics learners. School Science & Mathematics, 113(8), 380–389
McGlynn-Stewart, M. (2010). Listening to Students, Listening to Myself: Addressing pre-service teachers' fears of mathematics and teaching mathematics. *Studying Teacher Education*, *6*(2), 175–186.
Ntow, F. D., & Adler, J. (2019). Identity resources and mathematics teaching identity: An exploratory study. ZDM, 51(3), 419–432.
Relich, J. (1996). Gender, Self-Concept and Teachers of Mathematics: Effects on Attitudes to Teaching and Learning. *Educational Studies in Mathematics*, *30*(2), 179–195.
Remillard, J.T. (1993, April). *Using experience to break from experience: An elementary mathematics methods course* [Paper presentation]. Annual Meeting of the American Educational Research Association, Atlanta, GA.
Skemp, R. R. (1976). Relational understanding and instrumental understanding. Mathematics Teaching, 77(1), 20–26.
Sfard, A., & Prusak, A. (2005). Telling Identities: In Search of an Analytic Tool for Investigating Learning as a Culturally Shaped Activity. *Educational Researcher*, *34*(4), 14–22.
Zazkis, D. (2015). Monsters, lovers, and former friends: Exploring relationships with mathematics via personification. *For the Learning of Mathematics, 35*(1), 33-38.