MATHEMATICAL MAKING IN TEACHER PREPARATION: RESEARCH AT THE INTERSECTIONS OF KNOWLEDGE, IDENTITY, PEDAGOGY, AND DESIGN

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In this proposal, we share research that explores the potential benefits of a novel Making experience within mathematics teacher preparation that we hypothesized would inform the pedagogical and curricular thinking of prospective teachers of elementary mathematics (PMTs). That experience had PMTs exploring at the intersection of content, pedagogy, and design to digitally design, 3D print, and share an original manipulative with a child to promote their mathematical thinking. We share several vignettes of our research that aim to discern some of the potential benefits the experience might offer PMTs. These take a variety of theoretical and methodological approaches at the intersections of teacher knowledge, identity, pedagogy, and design. Implications of our findings for teacher preparation and professional learning are provided throughout the paper and in its conclusion.

Keywords: Teacher Knowledge, Technology, Problem-Based Learning, Integrated STEM/STEAM

Prospective elementary teachers (PMTs) have been characterized as coming to teacher preparation with limited conceptions of mathematics (AMTE, 2013) and a model of mathematics teaching that appeals mostly to rules and procedures (Ball, 1990; Ma, 1999). Consequently, teacher preparation must offer opportunities that challenge this model, and provide pathways to meaningful interactions and deepened understanding of both content and pedagogy. Connecting with a body of research that conceives of Making in education as the creative practice of designing, building, and innovating with analog and digital tools and materials (Halverson & Sheridan, 2014), we present one such opportunity that we centered in a novel Making experience within mathematics teacher preparation. That experience tasks PMTs with digitally designing, 3D printing, and sharing an original manipulative with a child to engage and advance their mathematical understanding. In seeking to determine what this experience might offer PMTs as they prepare to teach mathematics, our work has taken a number of theoretical and methodological approaches to address research questions at the intersections of teacher knowledge, identity, pedagogy, and design. These questions address the project's broader agenda, which is framed by the following question: *What are the potential benefits of a Making experience within mathematics teacher preparation*?

In this proposal we share some of the findings of our research along with their implications for teacher preparation and professional learning. Because the theoretical framings and methodological approaches we've taken are specific to each of these projects, this proposal is not organized in the conventional manner. Instead, we begin with the broader rationale for this project and then we present three vignettes of our research, each situated within their own literature, framings, and approaches. We conclude by looking across these and other projects to offer some reflections on the potential value of STEAM-integrated curricular experiences in teacher preparation.

Rationale

Schad and Jones's (2019) review of the research on the Maker movement in K12 education finds that students' learning through Making dominates that literature, with foci that include the improvement of STEM learning outcomes, increasing motivation and interest in STEM, and increasing equity by broadening notions of what counts as Making in STEM education. The extent to which this review mentions research on what *teachers* learn through Making is through studies of how they learn to design makerspaces and integrate maker-centered learning strategies (Clapp et al., 2016) into existing curriculum. Thus, there is almost no research on supporting teacher learning through Making. Our work (Akuom & Greenstein, 2021; Greenstein et al., 2017, 2018, 2019, 2020, 2021, 2021, under review) is situated within that gap in the research.

This work connects with a body of literature that frames *teachers as designers* (e.g., Brown, 2009; Maher, 1987) of learning experiences and of the material resources that mediate them. We conceive of design quite broadly to include the "intentional activity of transforming ideas and knowledge" (Carvalho et al., 2019, p. 79) into "tangible, meaningful artifacts" (Koehler & Mishra, 2005, p. 135). Our purpose in doing so is to introduce a pedagogically genuine, open-ended, and iterative design experience into mathematics teacher preparation that is centered on the Making of an original physical manipulative for mathematics teaching and learning. We hypothesized that the experience would afford unique pathways of diversified engagement that could promote epistemic and pedagogical shifts toward inquiry-oriented creative and participatory practices that support teaching and learning mathematics with joy and understanding. Accordingly, we view this Making experience as a *Learning by Design* (Koehler & Mishra, 2005) approach that honors the proposition that it is productive to develop teacher knowledge within a context that recognizes the interactions and connections among its constituent domains of knowledge. We also view the experience from a constructionist perspective (Harel & Papert, 1991), which argues that meaningful learning happens through the designing and sharing of digital or physical artifacts "that learners care about and have some degree of agency over" (Schad & Jones, 2019, p. 2). Indeed, when teachers take agency over the design of their own curriculum materials, they may challenge the *dissonance* that often arises at being tasked with implementing curriculum they neither chose nor endorsed. A *harmony* occurs, as a result, as these teachers come to see themselves as agents of curricular and pedagogical reform (Leander & Osborne, 2008; Priestley et al., 2012).

The Curricular Context and Experience

The study took place over three semesters in a specialized mathematics content course for PMTs at a mid-sized public university in the northeastern United States. Situated in an instructional context in which the teacher educators of those courses modeled an inquiry-oriented pedagogy, the course engaged students in a Making experience defined by the following task: "The purpose of this project is for you to 3D design and print an original physical tool (or 'manipulative') that can be used to teach a mathematical idea, along with corresponding tasks to be completed by a learner using the tool." The data corpus was comprised of video recordings of the in-class design sessions, the design of the tool, and these four written project components: 1) a "Math Autobiography," 2) an "Initial Idea Assignment," 3) a "Project Rationale," and 4) a "Final Paper/Reflection" that presents the findings from problem-solving interviews conducted by the PMTs with their tool and an elementary-age target learner.

The PMTs learned to use the Tinkercad (Autodesk, Inc., 2020; see Figure 1, left) digital modeling platform to design their manipulatives. They worked on their designs in in-class design sessions during three or four of the weekly class meetings. These sessions were held in a design lab (Figure 1, right), that we deliberately chose as we imagined that the PMTs' design activity would be more inspired in an environment intentionally configured to accommodate the kind of immersive, collaborative social space that nourishes it.

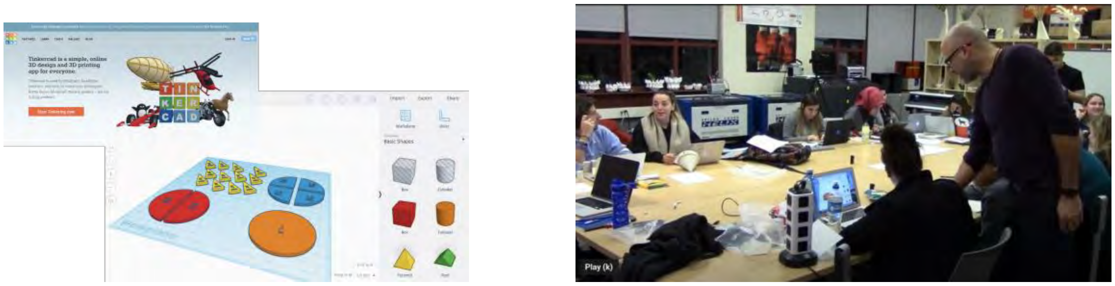

Figure 1: The Tinkercad design environment (left) and the design setting (right).

The first implementation of the Making experience occurred in the context of a pilot study (Greenstein, et al., 2019). Findings from exploratory and revelatory case studies (Yin, 2009) revealed that as the PMTs designed their manipulatives, they leveraged an appreciably rich and mature repertoire of teacher knowledge domains that we are not typically afforded opportunities to see. This finding betrays essentializing characterizations of elementary teachers as lacking in knowledge for teaching mathematics (AMTE, 2013) and suggests the promise of the Making experience. Accordingly, our pilot work became the launching point on a trajectory of further research.

Three Vignettes

In the findings that follow, we present three vignettes of research we've undertaken on that trajectory. Should this proposal be accepted, we will share the findings of these and other projects in our presentation. We propose that this body of work offers evidence of the broadly formative value of a making-oriented, learning by design experience in mathematics teacher preparation.

The Interplay of Discourses of Identity, Mathematics, Pedagogy, and Design in Mathematical Making

Here we took a commognitive perspective on learning (Sfard, 2007, 2008) in order to explore the premise that learning to teach mathematics can be seen as changes in discursive activities that include narratives about mathematics and identity (Heyd-Metzuyanim & Sfard, 2012). We adopted this perspective by foregrounding the identities (Sfard & Prusak, 2005) of teachers as learners in order to recognize what affective, interpersonal, and social matters can bring to this conversation. The following question guided the inquiry: *As prospective teachers of elementary mathematics Make new manipulatives to support the teaching and learning of mathematics, what might their discourses reveal about the epistemology of learning to teach mathematics?*

Methods. We addressed the question through a revelatory case study (Yin, 2009) of a prospective elementary teacher named "Moira" and by framing mathematics learning as the interplay between discourses about mathematical objects (*mathematizing*), participants of the discourse (*identifying*), teaching and learning (*pedagogy*), and design activity (*designing*). This framework provided us with a lens through which to study how the process of making a manipulative can provoke the four discourse activities and make visible the intertwined nature of a teacher's learning. We chose Moira because her initial design was a tool intended to simulate the "keep-change-flip" algorithm for fraction division. However, when the course's teacher educator pushed back on the idea because it did not meet the expectations for a tool that would support a students' conceptual learning, she tried to make sense of the algorithm but could not. Eventually she abandoned the idea altogether. We sought to understand this change through the lenses of the four discourses.

Findings. In this section, we present just one of the central results, which came from a follow-up interview we conducted with Moira in order to understand her rationale for the change in her design idea. It concerns our analysis of this change through the discourses of Mathematizing [*M*], Pedagogy [*P*], Designing [*D*], and Identifying [*I*]. Moira explained, "I wanted to make something that could be interpreted in many different ways [*M/P/D*] ..." As she considered her initial "keep-change-flip" tool, she explained how she realized that, "flipping the fraction upside down in my initial tool... it was just not useful [*M/P/D*] ... Then I came up with this [fraction comparison tool]" [*M/D/I*].

These reflections revealed how Moira's decision to abandon her initial design in light of the *dissonance* she wrestled with as she contemplated its purpose was not just about mathematizing, it was also about identifying. As a teacher, it was important to her that her students have the opportunity to develop their own ways of thinking about fractions with a tool that can be used in a variety of personally meaningful ways. Moira acknowledged that the pedagogy promoted in the course was also part of her decision to change her design: "Well, [the change of design] was because we were talking and you [the teacher educator] said, 'you're just teaching them how to – you're just giving them a way to solve the problem.' And I realized, you're right ..." [*M/P/D/I*]. By switching to a design for comparing fractions, Moira found *harmony* in the realization that she could participate in the discourse endorsed in the course and honor the teacher she wanted to be [*P/I*].

Implications. As in a woven tapestry, learning to teach mathematics weaves together four threads – or discourses – that are unique to a PMT's discursive experiences and particular to a learning community where an inquiry pedagogy is promoted. In this sense, to characterize Moira's learning to teach mathematics as a complex structure of discursive activities interwoven in dialectical unity is to illuminate the brilliance of a tapestry threaded by what she wants to teach (mathematizing), how she wants to teach it (pedagogy), decisions about what resources to make available (designing), and the kind of teacher she wants to be (identifying). This finding of the intertwined nature of the four discursive activities establishes that identity is as central to learning to teach mathematics as is the learning of mathematics, pedagogy, and design. And its implications speak to the formative potential of interdisciplinary project-based experiences as venues for the cultivation of prospective teachers' identities as *teachers becoming* (Greenstein et al., under review).

Designed Manipulatives as Anchors for Teacher Knowledge

This next vignette presents research that explores the 'life' of teachers as designers (e.g., Brown, 2009) of their own curricular materials, tracing their design activities from *tool-design* to *tool-use*. The aim is to explore how a PMT's designed manipulative can mediate the perennial gap (Ünver, 2014) that exists between teacher preparation and practice. The project proceeded in two phases. We began by examining the conceptual, pedagogical, social, and material resources that PMTs bring to their design decisions, their rationales for their uses of those resources, and how these intersect to mediate those design decisions. This phase is depicted in the inner ring of Figure 2. We then extended our inquiry to explore whether the designed tool could possibly be some sort of anchor for their conceptual/pedagogical visions in practice (see the outer ring of Figure 2). The following questions guided the inquiry: *As prospective teachers Make new* *manipulatives for mathematics teaching and learning, what are the resources and rationales they bring to their design decisions? Can connections be made between the resources for their design decisions and how their designs mediate the pedagogical moves they make in practice?*

Methods. To convey just one of many "images of the possible" (Shulman, 2004, p. 147), we present some of the findings of a revelatory case study (Yin, 2009) with "Anyango," a PMT whose written work explicitly expressed a wealth of design decisions and whose problem-solving interview demonstrated how her embeddings of pedagogical and conceptual knowledge in her designs served as anchors for that knowledge in practice.

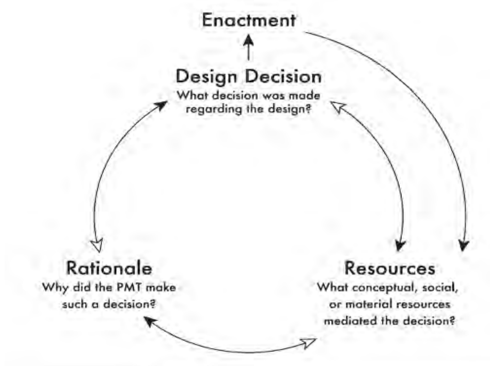

Figure 2: The 3 elements of a design decision (inner ring); Mediating resources may also be evoked in enactment (outer ring).

As the PMTs designed their manipulatives, it was their intention (Malafouris, 2013) to provide their designs with particular affordances (Gibson, 1977) for utilization schemes (Verillon & Rabardel, 1995) that they hypothesized would enable the child to abstract, through their sensorimotor manipulations, the perceptual elements that are the basis of the target concepts. Schön's (1992) notion of "knowing in action" (p. 2) directed our attention as our *Learning by Design* approach (Koehler & Mishra, 2005) enabled us to characterize the interplay between the PMTs' knowledge, experiences, intentions, and other resources that mediated their design decisions as they were invoked and made visible during the iterative design of their tool. In addition, we used the analytic concept of an *embedding* to connote a design element that embeds a PMT's pedagogical and/or conceptual knowledge in their tool. We referred to instances in which the tool served the PMT as a resource for (i.e., a reminder of) knowledge they embedded in the tool as an *anchoring* phenomenon.

Results. Anyango conceived of her design idea in response to the needs of a child she had worked with during problem-solving interviews earlier in the course. She wanted "to help… students visualize and deepen their understanding as they explore fraction relationships." Her design is "a 3D version of fraction strips (Figure 3). Each strip was made to be a rectangular/square piece that slides into individual pegs… [the] blocks stack vertically... to indicate height as value and amount." With several fractions mounted on a single "platform with the 1 (whole) always being visible… the student could begin to grasp how all the smaller parts can equate and compare to the whole." Technological knowledge, the mathematics of fractions, and a responsive pedagogy (e.g., Smith et al., 2016) mediated these and other design decisions that embed fraction values and concepts into the tool.

Anyango posed the following task in her problem-solving interview with the child: *Jack and his two friends each had the same size pizzas for lunch. Jack ate 5/8 of his pizza. Judy ate 2/3 of* *her pizza. And Sam ate 3/6 of his pizza. Who ate the most pizza? Who ate the least?* The child responded by stacking five one-eighth pieces, two one-third pieces, and three one-sixth pieces,

Figure 3: Anyango's fraction tool

each on their own pedestal with their labels facing her (Figure 3, right). Anyango's intention was for the child to compare "heights as amount" and identify the tallest as the one "who ate the most," and shortest as the one "who ate the least." When she asked the child, "Who ate the most?" the child attended exclusively to the symbolic representations engraved on each of the pegs and concluded that "It's Jack" (represented by the 5/8 piece). She went on to say that, "5 out of 8 is the biggest of all of them… 2 out of 3 is smaller and 3 out of 6 is… kind of small." Then, when Anyango asked the child to justify her answer, the child explained, "The top is two and the bottom is three." We inferred from this response that the child was basing her comparisons on interpretations of fractions as two separate whole numbers. According to this way of thinking, 5/8 is greater than 2/3.

We interpret Anyango's next move as a noticing one (Sherin et al., 2011) as she leveraged her pedagogical knowledge about the efficacy of interpreting and attending to students' thinking:

> Anyango: If I turn this [pedestal] around [Figure 2, left, such that the child's gaze can no longer be restricted to the fraction labels on the pieces], who ate the most?
> Child: <*Pointing to Judy's stack of two one-third pieces:*> This one.
> Anyango: Who has the least?
> Child: <*Pointing to Sam's stack of three sixth-pieces:*> This one.

In this excerpt, an unintentional design affordance enabled Anyango's "flipping" move and served as an anchor for her pedagogical knowledge in action. In a similar anchoring move soon thereafter, Anyango returned the tool to its initial, label-facing orientation so that the child could connect the physical representation of the amount to the symbolic one.

Implications. The diversity of design decisions made by Anyango and other PMTs, as well as the breadth of resources they leveraged and embedded within their designs, speaks to the generative power of the Making experience in terms of the agency PMTs enacted through their design activity and the wealth of conceptual resources that mediated it. In addition, the identification of anchoring phenomena in practice suggests that the Making experience yielded material epistemic scaffolding (in physical manipulative form) that supported PMTs and their commitments to the models of knowing and learning they construct in teacher preparation.

Dare to Care: A Case Study of a Caring Pedagogy on Mathematical Making, Teaching, and Learning

The mathematics and Maker cultures can be interpreted as exclusionary (Stinson, 2004; Gutiérrez, 2017; Barton et al., 2017), thereby suggesting opportunities for broadening learning opportunities to these spaces through *caring* and *Maker* pedagogies. We selected three participants for a revelatory case study (Yin, 2009), each with accompanying "outsider" traits to the project. The teacher educator (TE) and PMT ("David") brought caring pedagogies to their work but viewed themselves as interlopers to the Making culture. David's kindergarten student, "Vincent," is a student with disabilities (SWD) on the autism spectrum whose embodied acts of learning are not typically embraced in traditional mathematics classrooms. By focusing on *caring-centered relationships*, we illustrate how *together*, the participants redefined values associated with Making, traditional mathematics, and what can get celebrated as learning.

Hackenberg's (2010) *mathematical caring relations* (MCR) honor the mathematical and affective dimensions of learning. To navigate an MCR, a teacher must *decenter* "from his or her own perspectives… to help students realize and expand their ideas and worlds" (p. 239). In our project, we honor the open-ended nature of designing and Making a mathematics manipulative, the sometimes uneasy navigation through emergent mathematical "unknowns," the child's unique needs and experiences, and the tensions that are negotiated by carers (Noddings, 2012) in balancing these considerations. We therefore asked: *How does enacting a caring pedagogy during a Making-centered experience impact and broaden opportunities for meaningful mathematics learning? How does this challenge traditional notions of who can Make or participate in mathematics, and who cannot?*

Methods. We utilized purposeful sampling (Creswell, 2007) to focus this case study on our three participants. MCRs were analyzed and revealed through participants' verbal utterances and intonations, body language, actions, and mutual positionings (Simmt, 2000). The possibility of intersecting caring and Making theories called for a grounded theory approach (Glaser & Strauss, 1967) to analyzing and cross-referencing our data sources.

Results. David's Making experience began with an easy "answer" to the project task by designing an already-existing manipulative with a classmate. However, when the TE noticed the warm interactions between David and Vincent in a video recording of an earlier interview, she invited David to design a manipulative that was responsive to these interactions. When David articulated his trepidation in undertaking this more open-ended task, the TE promised to support him. We recognized this moment as the TE accepting responsibility for supporting David in caring for Vincent, and in navigating the discomfort and tensions (Noddings, 2012) that accompany this pedagogical decision. David, in turn, accepts responsibility for Vincent's care by sharing and utilizing Vincent's knowledge and love of shapes. After a few sessions with Vincent, David opts to design triangular, square, and hexagonal prisms with holes and corresponding inserts intended to create a one-to-one matching task.

During a design session, David noticed that some printed inserts did not fit into their intended holes. The TE utilized this moment of struggle to support David through his technological anxieties, and recommended including the "mis-shapes" in the matching task. David reflected on this being a "teachable moment" because his "mis-shapes" became usable for his and Vincent's learning. In another teachable moment, Vincent showed David how every shape and insert need not "match" to fill the holes (e.g., Vincent drops hexagonal inserts into the square hole). These uninhibited moments of insight suggested a transition in Vincent's attention from a shape's sides to its genus—a driving force underlying the concept of topological equivalence. These explorations culminated in Vincent aligning the hexagonal and square prisms with unlike holes to peer through them, and David receptively arranging the pieces between them to form a telescope (see Figure 4)! Together, they locked eyes and exchanged laughter and words of affirmation in an MCR where David decentered from the intended activity to *literally see his child's point of view* (Hackenberg, 2005).

Implications. Our focus on Making something *for* and *with* a specific student enabled a TE and PMT to leverage caring-centered pedagogies, and speaks to the inclusivity that caring brings to learning. As a member of the SWD community, Vincent's inclusive participation enabled him to learn with his characteristic, embodied enthusiasm in ways that defy the exclusionary notion that SWDs are not expected to participate in problem solving. The TE and David's caring-

Figure 4: Vincent sees similarities in different-shaped holes.

centered pedagogies embraced David's "mistakes" as an important part of his learning in addition to Vincent's inclination to know and learn with his body. By providing a platform "to demonstrate care for individual students and for the subject matter itself" (Bartell, 2011, p. 54), this case demonstrates how Making can create a novel opportunity to honor and invite the participation of supposed outsiders to the mathematics and Maker cultures and embrace the mathematical struggle, surprise, and discovery of all learners.

Conclusion

In addressing the question, *What are the potential benefits of a Making experience within mathematics teacher preparation?*, our research has revealed a number of positive outcomes. We shared these findings and their implications for teacher learning at the conclusion of each of the vignettes we presented above. With the conference theme of critical dissonance and resonant harmony in mind, we now provide a summary overview of the findings reported here and of other projects associated with this research.

Over and over, our findings demonstrate that immersing prospective teachers in a communal design environment and tasking them with a pedagogically genuine design experience generates opportunities for them to *harmonize emerging dissonances*. Moira's shift to a design for a conceptually promising learning tool harmonized the dissonance she faced as she confronted the rote understandings associated with her initial design idea. Anyango reconciled the dissonance that arose in her interaction with a learner by enacting a harmonizing pedagogy for the child's sound learning using a novel use of her tool. And the dissonances David felt in his interactions with a teacher and learner were harmonized as he navigated a caring pedagogy that gave rise to unforeseen and innovative perspectives on his tool for that child's learning. In addition, in projects not reported here, we observed two learners harmonize the dissonance they experienced in making sense of fraction division through embodied actions (e.g., Abrahamson, & Lindgren, 2014) with a physical "Fraction Orange" manipulative (Greenstein et al., 2021). We also observed the dissonance one PMT felt as she hesitated to leverage her cultural funds of knowledge (Moll et al., 1992) being harmonized as another PMT encouraged her to leverage it in their design of a tool for counting (Akuom & Greenstein, 2021).

These findings contribute to bodies of research on a "teacher's becoming" (Vågan, 2011; Greenstein et al., under review). They also generate new opportunities for research that moves the field forward regarding the potential value of constructionist, STEAM-integrated curricular experiences in teacher preparation. Future research could more closely explore the design of these environments in teacher preparation, the teacher educator's role in designing and facilitating these experiences, and the subsequent in-service instruction of teachers who participated in these experiences during teacher preparation.

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