LEARNING TRAJECTORIES RESEARCH NEEDS A HARD RE-SET: USING PCTM TO CENTER COGNITION, CONTEXT, AND CULTURE

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Research on LTs remains a central topic in mathematics education. In this plenary paper, I argue that LT-based research needs a hard re-set if it is to play a role in creating more equitable and anti-oppressive experiences for historically marginalized students. I begin with an overview of LT-based research presented during PME-NA plenary sessions, which I examine through a lens of cognition, context, and culture. I assert that a continued focus on cognition reproduces the status quo and causes dissonance for many learners. I then discuss equity in LT research and how it has evolved throughout the years. Next, I offer Political Conocimiento in Teaching Mathematics (PCTM) as a framework that can support us in asking the complex sociopolitical questions needed to create liberatory spaces in mathematics teaching and learning. I end by inviting the field to commit to centering equity in their LT-based research as a political act.

Keywords: Equity, Inclusion, and Diversity; Social Justice; Teacher Educators; Learning Trajectories and Progressions

Introduction

For many of us, the past two years have been challenging mentally, emotionally, physically, and spiritually. There are multiple pandemics affecting our students. And while news of a looming recession and midterm elections may be dominating media outlets, the pandemics of racism, cisheteropatriarchy, redlining, xenophobia, ableism, wealth inequality, food insecurity, and climate change are still alive and well. Right-leaning states, politicians, and media are openly attacking supports aimed at remedying these pandemics. Capitalism continues to thrive at the expense of the very Black and Brown people whose ancestors built this country. Wars and threats of wars are happening around us. And while the United States stands ready to send money overseas to maintain its interests, it fails to protect its own citizens from police brutality or ensure clean drinking water as a human right.

Teachers and schools are also facing some of the greatest attacks we have seen in decades. Texas, Georgia, and Florida continue to compete in a "race for the bottom" as they seek to define and ban "divisive topics," create anti-woke laws, ban books, whitewash this nation's history, and further marginalize students who identify as LGBTQIA+ by developing policies intended to destroy their safety. Teachers are left exhausted by the pandemics, ongoing attacks, and fears of being sued or otherwise humiliated in any attempt to support students specifically harmed by these pandemics. Education remains a political pawn. Schools are underfunded. And politicians would rather spend tax dollars enforcing racist laws instead of paying teachers what they deserve. These conditions further exacerbate teacher burnout leading many school districts to start the year with an unprecedented number of vacancies.

Instead of critical mathematics education scholars being consistently asked to defend the relevance of their work as if the context of concurrent pandemics doesn't impact the teaching and learning of mathematics, we need to shift the conversation to examining how each of these cells of mathematics education research serves to maintain or liberate us from the multiple

pandemics plaguing our nation. As we meet on these stolen lands, I implore our community to take up Aguirre et al.'s (2017) call for engaging in equity-oriented mathematics education research as a political act as well as what Ladson-Billings (2021) termed a "hard re-set." Ladson-Billings (2021) used the term hard re-set as a mantra and called for us to center students and culture in an effort to build a more humane future. Specifically, she stated:

> We must re-think the purposes of education in a society that is straining from the problems of anti-Black racism, police brutality, mass incarceration, and economic inequality. The point of the hard re-set is to reconsider what kind of human beings/citizens we are seeking to produce. (Ladson-Billings, 2021, p. 72)

LT-based research needs a hard re-set that can only be achieved if our community adopts a critical stance that centers equity. Doing so requires holding cognition, culture, and context together while using a critical lens. To date, LT research has privileged cognition at the expense of culture and context, and that disregard has led to inequitable uses. To make this point, I first discuss the history of LTs in plenary sessions at PME-NA in relation to cognition, culture, and context. Next, I establish my positionality and history with LT-based research. I then discuss the current state of equity-based approaches in LT research, and offer Political Conocimiento in Teaching Mathematics (PCTM) as a framework for our field to consider in order to enact the hard re-set needed (Gutiérrez, 2017). I end with questions and implications for the field.

Learning Trajectories at PME-NA

The North American Chapter of the Psychology of Mathematics Education (PME-NA) has a detailed history of centering LTs in its conference. In his 2010 plenary session and paper, Mike Battista spoke about the similarities and differences between Learning Progressions (LPs) and LTs, and what it meant for students to move through LPs/LTs (e.g., milestones, levels of sophistication). After highlighting differences in theoretical framings, the nature of levels, and the inclusion of instruction as a way to differentiate LTs and LPs, Battista turned to his work with Cognitive Based Assessments (CBAs). He noted that a CBA LP outlines students' conceptions, obstacles, plateaus, and mental processes needed to advance for a given topic (2010, p. 66). While Battista notes that movement through progressions is not unilateral because "students' learning backgrounds and mental processing differ[s]," there was no specific mention or acknowledgment of the sociocultural and political environment this instruction, assessment, and research validation occurred in. We are also left to wonder how Battista defines learning backgrounds and to what extent, if any, that definition captures the rich knowledge students bring from their homes and communities. Battista ended his paper by calling on researchers to exercise caution when using quantitative techniques to develop and validate LPs so as to not misapply such techniques or ignore how this thread of work interacts with research on learning (Battista, 2010, p. 69). It is noteworthy that Battista's caution to the field centered cognition and upholding principles of research methodology while tangentially addressing context and ignoring culture.

Susan Empson (2010) questioned the novelty and usefulness of LTs in Battista's plenary in an invited critique and reaction. After offering a summary of LTs and drawing on Simon's (1995) discussion of a hypothetical learning trajectory, Empson pondered LTs' place in teaching and research. In addition to posing questions for consideration, Empson acknowledged the importance of context when she cautioned the field not to underestimate the role of tasks, teachers, and teaching in LT research. She also reminded us to acknowledge disciplinary practices in the same way we focus on content, which is critical since LTs are tools used for

teaching and ultimately derive meaning from classroom contexts. Finally, this paper noted that LTs ultimately need to be useful for teachers and that the creation and use of LTs must be an interactive process that involves careful study of how teachers use them. Empson's paper reminded us of the importance of context in cognition-based research. This paper did not explicitly address culture.

In 2012, Confrey offered a plenary paper summarizing how LTs were used in the development of the Common Core State Standards for Mathematics (CCSM). A major premise of this work was to support state leaders in distilling elements of the new standards into smaller pieces of information supported by research on student learning over time. Another key element of her paper was presenting components of LTs in ways that were useful for teachers, resulting in the creation of a hexagon map of K-8 mathematics standards. As Confrey described the hexagon map, she provided a rationale for its design, noting how different big ideas were connected (e.g., counting, addition, and subtraction), how some content supported the learning of other ideas (e.g., equipartitioning supporting the development of division and multiplication), and why some topics were visually clustered between others (e.g., length area and volume are nestled between equipartitioning on one side and shapes/angles on the other). This plenary offered a detailed analysis of a multiplication and division LT coupled with figures, strategies, and multiple representations. Confrey and team unpacked each trajectory by articulating conceptual principles, strategies, representations, misconceptions, meaningful distinctions in language, coherent structure, and bridging standards, which centered cognition.

Confrey explicitly stated that the hexagon map did not address the standards for mathematical practice (which could have inserted relevant connections to culture and context) but noted that students would surely use various practices as they progressed through the trajectory (2012, p. 8). She then posited that when LTs are properly unpacked and coordinated with standards, teachers can be better supported to connect underlying mathematical principles. She ended this paper by inviting the field to consider the usefulness of the hexagon maps as one example of what coherence across standards could look like, with the ultimate goal of supporting teachers as they transitioned to using the CCSS (Confrey, 2012). While the hexagon maps and associated unpacking offered valuable information related to cognition, a sociopolitical lens was not evident. As such culture was not discussed and context was not addressed. Given the social and political nature of teaching and how the CCSS were created, such connections would have been valuable as they could have contextualized this national movement and addressed teachers' concerns around new mandated curricula and tests that accompanied these standards.

The next plenary talk on LTs featured Julie Sarama in 2018. In a response to the conference theme, *Looking back, looking ahead: Celebrating 40 years*, Sarama discussed how mathematical knowledge developed in young children and shared brief highlights from her work. Her definition of LTs, which she noted is rooted in constructivism, acknowledged the importance of instruction and mathematical tasks. As Sarama unpacked the tenets of *Hierarchic Interactionalism*, she noted several points that were central to young childrens' innate skills and environment. She ended this chapter by offering an example of a student named Justin progressing through the LT-based *Building Blocks* curriculum. Sarama highlighted this student's growth in counting, addition, and subtraction as one example of how student thinking across multiple LTs is interactive and can grow concurrently (Sarama, 2018). Again, this plenary paper, and the original paper from which the example was drawn, centered cognition (and cognitive science) and did not provide insight on the context of Justin's "growth," his culture, who the teacher was, and the broader sociopolitical context this study occurred in.

In reflecting on these plenaries, Battista's took a cognitive approach. Empson pushed back and acknowledged context but left much unsaid about culture. Using a constructivist paradigm, Confrey and Sarama attend to context in limited ways (e.g., standards, curriculum) but did not include culture or other sociopolitical factors. My goal in providing this brief historical overview through a lens of cognition, context, and culture was not to freeze any of these scholars in time as working toward equity is a journey and not a destination. Rather, I sought to highlight how each of the plenary papers privileged cognition (when written) at the expense of context and culture, thus casting equity to the sidelines. While these scholars moved the field forward in monumental ways by disrupting understandings of how mathematical content is organized and reframing students as capable of rich mathematical thinking, they did not go far enough to disrupt other problematic strongholds in mathematics teaching and learning (e.g., tracking, low expectations). This review of previous LT-focused plenaries at PME-NA highlights the timeliness and necessity of the current conference theme, as scholars were specifically invited to consider their work through a sociopolitical lens. I hope this paper and resulting discussion contribute to a collective examination of why a hard re-set is needed in LT-based research if it is to play *any* role in leading toward a more "antioppressive and equitable human experience" in mathematics teaching and learning (Aguirre et al., 2017, p. 127).

Positionality

Before moving forward, it is important that I provide context on who I am and how I came to this work. I am not an outsider to LT-based research. In fact, I have an intimate history with LTs, and most of my time as a graduate research assistant for my master's and doctoral programs was spent on large-scale, NSF-funded, LT-focused grants. Earlier in my program, I worked on a research team to develop LTs for equipartitioning and rational numbers. As a graduate assistant, I conducted numerous clinical interviews and worked with my teammates to construct and validate LTs. Our team regularly met and engaged with other LT experts in the field, many of whom I cite in this paper, and worked to support the development of our state's mathematics standards. During that time, several concerns began to arise in the field around the construction and validation of LTs. Many of these questions were aimed at the diversity, or lack thereof, of the student population upon whom LTs were constructed and validated. Our team considered that feedback and began to intentionally recruit research participants in various settings to diversify our student sample. I recall being curious about this critique and wondering if students from different racial, cultural, and linguistic backgrounds would demonstrate different pathways through our sets of tasks. At that time, my equity lens was not sophisticated enough to recognize that engaging diverse students in a fairly "rigid" set of tasks was unlikely to produce different outcomes. In fact, as I reflect on this approach to diversifying the student sample in our research, I now see that we worked to accumulate more in the sample, rather than pause and fundamentally reorganize the research design. I was also only beginning to understand the impact of context and interlocking systems of oppression. And as such, I was not yet able to a) question the rationale for centering cognition, b) form arguments about how the clinical interview structure excluded context, c) understand how students intersectional identities influenced their work on our tasks, or d) understand how the social and political context of standards, funding, and other external forces impacted our research work.

After transitioning to another project, my focus shifted from developing and validating LTs in rational numbers to designing LT-based professional development. While on this project, our team worked on translating and coordinating early grades LTs focused on number, counting, addition, and subtraction, into useful tools for teachers across grades K-5. This research project,

titled *Learning Trajectories Based Instruction* (LTBI), served as the basis for my dissertation. As we sought to understand and develop a model for *how* teachers learned to use LTs, I, along with other team members, became interested in how teachers' implementation of LTBI looked different across various subsets of their student population. I had grown as a person and a scholar and was much more aware of equity, justice, and how the systemic nature of marginalization in schools maintained opportunity gaps and systematically excluded minoritized learners.

After ending my high school teaching career to complete my doctorate full-time, I was much more attentive to culture and context. I had also grown increasingly frustrated with the ways "new trends" and "innovative curricular materials" in mathematics teaching and learning yielded the same results year after year (e.g., opportunity gaps, tracking). Because of the national attention LTs were receiving at the time, and how they were being used to develop curriculum and assessment, I wanted to be proactive in considering how they could be used equitably. Therefore, I worked with my team to articulate a theory of Equitable Learning Trajectory Based Instruction (E-LTBI), which I then investigated in a case study of four teachers for my dissertation (Myers, 2014). This E-LTBI framework resulted from simultaneously considering Gutiérrez's (2007) four dimensions of equity and our existing LTBI framework (Myers et al., 2015). I share more about this work in my review of LTs and equity after providing a brief overview of LTs/LPs and critiques.

Learning Trajectories

Several definitions of LTs have been offered in the field. Clements and Sarama (2004) define LTs as

> descriptions of children's thinking and learning in a specific mathematical domain and a related, conjectured route through a set of instructional tasks designed to engender those mental processes or actions hypothesized to move children through a developmental progression of levels of thinking, created with the intent of supporting children's achievement of specific goals in that mathematical domain. (p. 83)

Confrey and Maloney's (2010) definition of LTs features similar language but notes that trajectories a) are empirically supported, b) include activities, tools, and assessments in addition to tasks, and c) highlight the iterative nature movement, reflection, and refinement as students move from informal understandings to formal ideas (p. 2). Research around LTs exists in three primary areas: development and validation (constructing LTs in different domains and content strands) (Battista, 2004; Blanton et al., 2015; Confrey et al., 2009; Maloney & Confrey, 2010; Gravemeijer et al., 2003; Fonger et al., 2020), informing instructional tools (e.g., standards, curriculum, and assessment) (Clements, 2002; Clements & Sarama, 1998; Confrey, 2012; Daro, Mosher, & Corcoran, 2011; Mosher, 2011), and, more recently, professional learning for teachers (Bargagliotti & Anderson, 2017; Clements & Sarama, 2009; Edgington, 2012; Sarama et al., 2016; Suh & Seshaiyer, 2015; Sztajn et al., 2012; Wickstrom, 2014; Wilson et al., 2013; Wilson et al., 2015; Wilson et al., 2017). Lobato and Walters (2017) conducted a detailed review of research on LTs and LPs in mathematics and science education. They produced a taxonomy of approaches to learning trajectories and progressions, which they refer to as LT/Ps. At each level, they described the approach, offered an example, highlighted the features, outlined the methods used, and discussed the purpose, benefits, and tradeoffs. I invite readers to study the full paper to learn more about the breadth and depth of research around developing and validating various LT/Ps.

It is important to note that I do not seek to offer a distinction between LTs and LPs nor advocate for one over the other. I encourage readers interested in the LT vs. LP discussion to read Battista (2010) and Ellis, Weber, & Lockwood (2014), as both papers offer a detailed account. The relevant similarity from my perspective is that while LTs and LPs center cognition and narrowly reference context (in noting the importance of carefully selected tasks and pedagogical moves), neither body of research explicitly addresses the social, cultural, or political context in which the research, validation, creation, and intended uses occurred. Moreover, I argue that both LTs and LPs offer a narrow definition of what mathematics is, whose mathematics is privileged, and why we engage with it, thus missing an opportunity to expand the view of what counts as mathematics (Aguirre et al., 2017).

Critiques of LTs (and LPs)

Critiques of LTs and LPs in mathematics and science education are not new. In the National Research Council (NRC) report *Taking Science to School*, the authors provide an overview of teaching and learning science in grades K-8. In their chapter on learning progressions, much of which aligns with LTs mathematics education, the committee highlighted how LPs can be used to map students' understandings and unify science topics that have previously been disconnected. The committee ended this chapter by discussing the design challenges of LPs and stated,

> No single learning progression will be ideal for all children, since they have different instructional histories, bring different personal and cultural resources to the process of learning science, and learn in different social and material environments. The best learning progressions are those that make effective use of the resources available to different children and in different environments. This is the challenge that we are farthest from responding to effectively with the current research base. (NRC, 2007, p. 222)

The committee later noted that although they recognized inequities in science education and the dire need to address them, they were unsure about what recommendations to make related to modifying instruction for diverse learners. Their suggested agenda for future action included focusing on the effectiveness of different instructional strategies, the unpacking of systemic inequities across schools, and the need for specific research that examined the complexity of culture, language, and socioeconomic status (NRC, 2007). This report summarized the cognitive aspects of LPs in science and pointedly expressed the absence of context, culture, and other sociopolitical constructs.

In a 2011 paper, an expansion of her 2010 plenary response, Empson questioned what LTs afforded, foregrounded, and obscured, which parallels the current conference theme of dissonance and harmony. I appreciated that this more detailed analysis considered both promises and pitfalls of LTs by making the case that learning is as much contextual and social as it is cognitive. Empson went on and acknowledged that teaching was a relational act that "depends fundamentally on interpersonal relationships of trust and respect." (Empson, 2011, p. 587). What was underdeveloped in this paper was the explicit unpacking of the word *contextual* and the historical and political nature of those contexts. Additionally, when children's differences were alluded to in the text, the words race, gender, culture, language, sexual orientation, or ability were not explicitly mentioned. When we do not intentionally name the different elements of students' intersectional identities we can inadvertently reify some scholars' beliefs that LTs only need to focus on "cognitive differences." I argue that while this critique pushed for the inclusion of context (and culture to some degree) in LT research, a sociopolitical could have strengthened this critique. Just as Empson argued that learning cannot be separated from teaching, I argue that teaching cannot be separated from the teacher. Since teachers hold a range of beliefs and biases

about who can do mathematics and who should be afforded opportunities for "rigorous" mathematics, neglecting to use a sociopolitical lens secures an oppressive, anti-black, anti-immigrant, anti-poor, anti-LGBTQ, ableist system. We must explicitly engage teachers' beliefs as we center context and culture in conversations about LTs.

In their examination of equity in LT/LP research, Delgado and Morton (2012) analyzed existing research using a cognitive constructivist framework. They noted that this framework was aligned with their definition of equity. These authors pushed against "equity for all" (or dominant framing of equity) and embraced a postmodern definition of equity that "acknowledges existing inequities in society [and] proposes responsive, individualized attention to students in order to compensate for past lack of opportunities and to promote social justice" (Delgado & Morton, 2012, p. 205). As they examined LT/LP research that explicitly considered "issues of equity," the authors found that student populations lacked diversity (or didn't report any demographic information), only focused on common themes amongst students' mathematical ideas instead of capturing all students' ideas, and failed to consider how students' family and community knowledge shaped their engagement in tasks and resulting movement through a trajectory. The authors concluded their paper and stated,

> Research groups developing LPs and LTs should ideally include advocates for certain groups of students, for example, an expert on special education and team members that are deeply knowledgeable about the culture of minority students. Developing learning progressions and learning trajectories that do not address inequity in educational opportunities in math and science for students will only exacerbate the current problem. As the learning sciences, science education, and mathematics education fields continue to negotiate and define the nature of LPs and LTs, an expansion to include equity concerns at the forefront can greatly benefit groups that have been traditionally underserved. (Delgado & Morton, 2012, p. 209)

LTs and Equity

In this section, I present three ways "equity" has been addressed in LT-based research. First, I highlight Sarama & Clements' body of work as an example of a dominant framing of equity as it primarily focuses on access and achievement (Gutiérrez, 2007). I also discuss how their attention to equity has shifted over time to include some critical framings of equity. Next, I discuss equitable uses of LTs by highlighting two cases: LTBI and Suh et al. (2022). After showing how Clements and Sarama's work influenced my dissertation study and led to the development of the E-LTBI framework, I transition to the work of Suh and colleagues who built from our LTBI findings and intentionally embedded equity in their LT-based PD model. I conclude by highlighting the work of Zahner and Wynn (2021), who centered equity in an attempt to address gaps in representation in LT development.

Dominant Framing. One area of LT & equity research focuses on how LTs can offer access and support achievement for minoritized students. The body of work of Clements and Sarama represents decades of research and tens of millions of dollars of grant funding from large-scale funders (e.g., The National Science Foundation and Institution of Education Sciences), which led the development of curriculum (Clements & Sarama, 1998; Sarama & Clements, 2019), conferences, and the creation of research centers. Because these scholars' definition of LTs and the resulting body of work has been so influential in LT-based research, I draw on it as one example of a dominant framing of equity. Consider the large-scale randomized trial that was conducted and published in several venues (Clements et al., 2013; Sarama et al., 2012). The authors noted that they chose their research site because "children from low-resource communities and who are members of ethnic and linguistic minority groups demonstrate

significantly lower levels of mathematics achievement than children from higher-resource, nonminority communities" (Clements et al., 2012, p. 2). The authors went on to note that their LT-based PD model, "include[d] guidelines for promoting equity through the use of curriculum and instructional strategies that have demonstrated success with underrepresented populations" (Clements et al, 2012, p. 4). Findings from this and similar studies indicated that African-American students in their experimental groups scored significantly higher than their counterparts in control groups. One conclusion of this study was that "centering instruction around LTs may focus teachers' attention on students' thinking and learning in mathematics rather than their memberships in ethnic groups and thus avoids perceptions that negatively affect teaching and learning" (Clements, Sarama, Wolfe & Spitler, 2012, p. 26). In their 2014 book chapter, Clements and Sarama stated, "several "gold standard" randomized control trial studies have shown that curricula and professional development based on learning trajectories increase children's achievement more than those that do not." (p. 7). They went on to say, "learning math at an early age is critically important for young children, especially those from disadvantaged communities" (Clements & Sarama, 2014, p. 8).

Clements and Sarama's attention to and expression of equity continued to grow and expand throughout the years. They sharpened their perspective by explicitly addressing six myths about LTs, three of which are germane to this analysis (Clements & Sarama, 2017/2019). First, they argued that LTs are asset-based because they help teachers recognize and build upon students' thinking. Next, they defended critics' notions of LTs being narrowly focused by noting that LTs are "deeply constructivist" and address broad ranges of ideas. Finally, they stated, "Learning trajectories are expressly built to be adaptable to different cultures, groups, and individuals. One important adaptation is for different cultures. Learning trajectories take funds of knowledge from all communities seriously and encourage using such funds" (Clements & Sarama, 2017/2019, p. 2).

More recently, Clements et al. (2020) suggested that teachers who know how to use the three components of an LT are better suited to understand the complexity of early mathematics content and offer instruction that is more closely aligned with students' current conceptions, thus providing more robust mathematics experiences for all children. They stated that such environments are necessary for "vulnerable children who live in poverty, are members of linguistic and ethnic minority groups, or…children with disabilities" (Clements et al., 2020, p. 1). They suggested that early-childhood teachers could benefit from sustained PD focused on learning trajectories that also included direct support for engaging children with learning disabilities. The authors ended this paper by announcing their STEM Innovation for Inclusion in Early Education Center, which they noted is a critical step in ensuring equity and excellence in early STEM experiences. Ongoing work from this team continues to suggest that LT-based PD positively impacts students from "low-resource communities" (Sarama, Clements & Guss, 2021).

As I followed this and other bodies of LT-based research over the years, I observed how the discussions of context and culture have both evolved by expanding the attention given to equity and, in some cases, remained stagnant by only considering equity in relation to student achievement. I have paid particular attention to how students were described and positioned in these and other studies. I would encourage Clements and Sarama to consider how the language they used to dispel myths in their 2017/2019 resource document may be at odds with how often students and their communities are referred to as low resource, vulnerable, and minority. Although those phrases were often used as demographic descriptions, doing so without a

sociopolitical lens may serve to reify the deficit orientations Clements & Sarama seek to disrupt. It is also important to note how context was included (e.g., as a description or as a mediating variable in a statistical model, etc.) and whether or not the context of the study was situated in the historical context of schools and schooling in the United States (e.g., critical explanations of sociopolitical factors that intentionally created disadvantaged or low-resource communities). Finally, I invite the reader to carefully consider how this dominant view of equity and its evolution was inextricably tied to cognition, which overwhelmingly excluded critical discussions of context and culture.

Equitable Usage of LTs. A second area of equity and LT research focuses on the ways LTs are used in instruction. The Learning Trajectories Based Instruction (LTBI) research project (for which I was a graduate research assistant) is one such example (Sztajn et al., 2012). This study used Clements and Sarama's early number, counting, addition, and subtraction LTs in a multi-year research project with K-2 teachers. In our 2015 paper *From implicit to explicit: Articulating equitable learning trajectories based instruction*, my colleagues and I argued that although we initially considered our LT-based research with teachers to attend to issues of equity, what we learned in our work caused us to reconsider some of those assumptions (Myers et al., 2015). Similar to other cognition-focused teacher learning models, our project centered students' thinking, disrupted notions about the "traditional sequencing of mathematics," and created space for students' individual thinking to emerge and be positioned as valuable along various trajectories. Teachers in our study deepened their knowledge of K-2 mathematics content, appreciated the language the trajectory afforded them, and began to recognize and value a range of students' mathematical contributions (Edgington, 2012; Myers, 2014; Wilson et al., 2015). In this sense, one assumption of our work built on Clements et al. (2012) suggestion that by using LTBI, teachers would see that all children were capable of mathematics, potentially reducing their focus on other demographic factors.

Unfortunately, the suggestion that focusing on cognition could reduce attention to other factors did not hold true in our work, highlighting how good intentions and race-evasive approaches are not enough to effect radical change across diverse groups (Rodriguez, 2003). As a result, we saw LT language taken up and used oppressively such as when teachers replaced the language of "low students" and "high students" with LT-based vocabulary (e.g., the low students being renamed the direct counters). Results from my dissertation highlighted that most LT-based work aligned with a dominant approach to equity (Myers, 2014) and that the elements that connected to Gutiérrez's (2007) critical axis were shallow or maintained dominant framing. Moreover, in my paper titled, *The unintended consequences of a learning trajectories approach*, I reported on a teacher, Elizabeth, who possessed several deficit orientations about her students. This teacher also shared that she wasn't confident in her own mathematics knowledge and therefore taught science instead of mathematics to her kindergarten students. Although this teacher made "gains" in her content knowledge and began offering LT-based instruction in her classroom, several issues emerged. A primary concern was that this teacher's deficit orientations about students overshadowed what she learned about LT-based instruction. This teacher ultimately used knowledge gained in the PD to justify retaining kindergarten students from minoritized groups, and the LT-based language she acquired in our sessions provided "credibility" to her decisions as a teacher (Myers, 2015).

It is important to note that my dissertation study sought to examine what equity may look like as a *by-product* of participating in LT-based PD, as equity was not explicitly centered in the PD design. Building upon findings from LTBI-based research, Suh and colleagues considered that

LT-based PD was not enough to disrupt teachers' beliefs about minoritized students. Therefore, she and her team intentionally embedded equity in their LT-based teacher learning study design, simultaneously centering cognition, context, and culture. They engaged teachers in professional development focused on LTs, asset-based instruction, and cognitive demand (Suh et al., 2022). They hypothesized that pairing LTs with equity-oriented and anti-deficit frameworks for noticing (Louie et al., 2021; van Es et al., 2022) would help teachers recognize students' multiple knowledge bases. They suggested that this framework would support teachers to assign mathematical competence to their students. The authors noted that teachers in their study moved beyond discussing "gaps" in students' understanding to using what they referred to as "strength-based language" (Suh et al., 2022). It is unclear from these findings if the shift from "gap-based language" to "strength-based language" reflected a change in teachers' beliefs or if teachers were merely using the "new language" that had become normalized in the professional learning space, potentially as a proxy for previously held viewpoints (Myers et al., 2013; Wilson et al., 2017). The field can benefit from continued research that examines how similar findings (e.g., strength-based language) relate to broader conceptions of socially just and anti-oppressive learning environments.

Representation in LT Development. A final area of equity and LT research focuses on who is used in constructing LTs. In noting the absence of diversity in the student population of much LT-based work, Zahner & Wynn (2021) conducted clinical interviews with 23 multilingual students using LT-based tasks focused on proportional reasoning and linear functions. This study of twenty-five ninth-grade students (primarily Latinx, Asian, and African American), ten of which were multilingual, provided valuable insights into how the linguistic complexity in mathematics tasks impacted how students approached tasks and explained their reasoning, thereby influencing their potential "ranking" or placement on a content-focused LT. Their findings call into question the role of language in previous large-scale work conducted to create, norm, and validate LTs. Given that many initial LT-based studies did not consider language or the linguistic complexity of tasks in the study design, we are left to wonder how students interpreted tasks. Even when scholars acknowledged demographics (e.g., language status) in large-scale studies they typically neglected to present analysis around how language mediated performance on standardized tests. Zahner and Wynn's research is an example that culture and context can be centered while simultaneously investigating cognition.

Summary

For organizational purposes, I presented these two sections (critiques of LTs and equity in LT-based research) separately. Readers should note that some of this work occurred concurrently and that some of the shifts in how equity was presented in LT-based research were in direct response to the ongoing critiques of equity in LT research (e.g., Empson, 2010, NRC, 2007, Sztajn and Wilson, 2019), LT-based conferences and working groups, and other conversations that have continued in the field. Despite a small shift in how scholars have attended to equity in their LT work, cognition is still the focus, and I argue that we have not yet met the call of centering equity in LT research as both a political act and a collective responsibility. I also note that in order for an LT-based re-set to happen in ways that honor Aguirre et al.'s (2017) call, we must pause to unpack a) how discussions of "equity" have (or have not) evolved in LT-based work, b) whether we have been intentional about acquiring the knowledge necessary to make the shifts in genuine ways, c) if our work focuses on the full humanity of students' experiences and moves beyond acknowledging their test scores, and d) how we have chosen to foster and cultivate critical collaborations that value the expertise of a range of scholars, colleagues, and

families. In what follows, I describe PCTM and suggest that this framework can help us ask the complex, sociopolitical questions needed (Gutiérrez, 2013) for us to examine LT-based as we consider how cognition, context, and culture are necessarily entangled.

Using PCTM to Critically Consider Cognition, Culture, and Context

In honoring the conference theme and Aguirre et al.'s (2017) call to be intentional about discussing interlocking systems of oppression with colleagues, I suggest that Political Conocimiento in Teaching Mathematics (PCTM) can provide the field with a lens to examine LT-based research and ask complex questions. Because PCTM explicitly engages content, pedagogy, social context, and politics, it is suitable for examining LT-based research across its three primary domains (e.g., development and validation, informing instructional tools, and professional learning with teachers). For too long, large-scale research projects (e.g., LTs and Cognitively Guided Instruction) have been conducted in mathematics education and taken up by the mainstream without engaging a sociopolitical lens during conception. Equity-oriented scholars have then dedicated their careers to proving how the initial work was not grounded in equity while also considering how to "re-mix" the research and curricula to meet their justice-oriented agendas (Maldonado et al., 2022). My goal in offering this framework is to call for an end to this two-phased approach that centers cognition first and then leaves equity-oriented scholars to consider culture and context. After describing PCTM, I unpack questions posed in the conference theme to illuminate the power of this theoretical framework.

Political Conocimiento in Teaching Mathematics

PCTM is a theoretical framework that highlights the unique ways that mathematical knowledge for teaching (MKT), pedagogical content knowledge (PCK), knowledge of students, and political knowledge are entangled (see Figure 1) to produce a unique way of *knowing* that teachers (and researchers) need to consider as they teach and conduct research, especially with historically marginalized students. Here, I list two elements of this framework that make it useful to critically analyze LT-based teaching and learning. First, PCTM explicitly links content, pedagogy, knowledge with students and communities, and political knowledge (e.g., power dynamics) and situates the entanglement of each of these in a social and historical context (Gutiérrez, 2012). This linkage does two things. One, it dispels the "false dichotomy" between mathematics and equity that some scholars assert (Aguirre et al., 2017), making it inconceivable to conduct research on cognition and teaching without attending to context and culture. Two, this linkage asserts that political knowledge is not merely added to the other dimensions. On the contrary, engaging a political lens causes us to re-examine the other components (Gutiérrez, 2012). As such, one would not consider developments related to content and pedagogy (e.g., develop a LT-based curriculum) and then question how to use it with diverse learners. Instead, this framework embraces the tensions that exist when these individual pieces are entangled, thereby allowing us to ask richer questions and reflect on decisions we make to foreground or background different dimensions (Myers, Gutiérrez & Kokka, in press). Using such a framework at the onset of LT-based research would have eliminated many of the critiques that came later since culture, race, language, communication patterns, task design, teacher and researcher identity, etc., would have all been considered in the development and validation phases. Embracing this tension allows us to see that dissonance and harmony can coexist while we also question who experiences dissonance and who experiences harmony (Gutiérrez, 2009a).

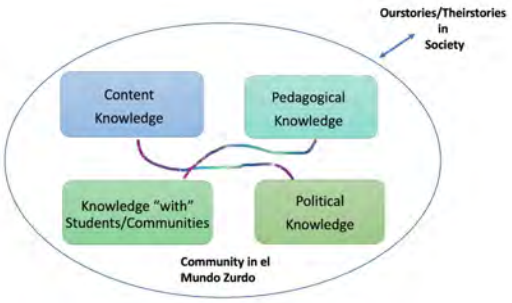

Figure 1. Political Conocimiento in Teaching Mathematics (Gutiérrez, 2012)

Second, the word conocimiento is drawn from the Spanish verb conocer. The Spanish word *saber* represents a "fixed" kind of knowing (e.g., to know facts, how to follow steps, to know a piece of information). The verb *conocer,* however, represents a fluid type of knowing that is contextual (e.g., to know a person, to know of a place). This distinction is critical as *conocer* asserts that context and conditions are essential to framing *how* we know (Gutiérrez, 2017). This kind of knowledge prevents us from objectively knowing "low-income students," "minority students," or "English language learners" in a fixed or homogenous way and, as such, applying a "best practice" to that population with the hopes of minimizing an achievement gap. A teacher or researcher using PCTM as a lens would first question how a "best practice" was developed. That person would ask, "best for whom? Best under which conditions? Best to what end? Best for what outcome?" Using this type of framework also problematizes the treatment of groups of students as static objects of our research and suggests that knowledge must be co-constructed with learners, families, and community partners. Reframing knowing as relational also reminds us that how we *know* is always changing because our environments and sociopolitical contexts are ever-changing. Moreover, as scholars, our *knowing* about LTs is also fluid as we continue to re-negotiate our work in light of new understandings about our intersectional identities, new scholarship, etc. PCTM is undoubtedly a powerful theoretical framework that the field can use to consider complex questions similar to those posed in this year's conference theme. In the next few paragraphs, I consider two of the thematic questions through the lens of PCTM.

How does LT-based teaching challenge a settled mathematics learning status quo?

Using PCTM allows us to answer this question in two ways. First, it is important to acknowledge the historical and political contexts that afforded the "mathematics learning status quo" to be created and maintained for so many years, leading to dissonance. Our field can benefit from engaging in discussions around how mathematics has been socially constructed in a way to maintain systems of oppression (Gutiérrez, Myers & Kokka, 2022, in review). Using PCTM forces us to unpack the political nature of the status quo, understand who is negatively affected by it, and develop a comprehensive approach to address the problem instead of accepting it as the norm or rushing to a "quick fix."

Second, PCTM allows us to consider how questioning mathematical content (thereby questioning mathematical content knowledge) can lead us to work toward harmony. Given that the four elements of the PCTM framework are entangled and that the resulting knowledge is relational, pulling one thread in an attempt to disrupt a status quo necessarily challenges our ways of knowing, allowing us to consider the other dimensions and reimagine mathematics learning more broadly. For example, several studies demonstrated that LT-based professional development supported teachers in understanding the complexity of mathematics content, how underlying ideas were connected, that mastery was not a prerequisite for more sophisticated

ideas, and that informal understandings were valuable prerequisites for building more complex ideas. This work is important. What was missing from those conversations is specific research on how changes in teachers' MCK interacts with the other elements of PCTM, to potentially disrupt other status quos in mathematics teaching and learning. For example, does the "asset-based" lens teachers acquired during LT-based PD support them in reframing their understanding of the mathematical practices (e.g., centering a range of communication and argumentation styles, debunking traditional notions of precision) through a sociopolitical lens? Moreover, even when teachers develop an "asset-based" lens as a result of LTs and constructed counternarratives about students' thinking, what did this mean for how teachers recognized and valued students' humanity? Did "disrupting inequity" in mathematics thinking lead teachers to advocate for greater change? And did changes in beliefs, if any, persist over time? If LT-based teaching continues to treat mathematics as disconnected from students, families, communities, and contexts, how can we expect to truly eradicate inequity in mathematics teaching and learning?
How does LT-based teaching have an impact on society more broadly, beyond individual mathematics classrooms and school districts?

This question naturally engages each element of the PCTM framework as we consider how LT-based teaching and learning (content and pedagogical knowledge) *might* impact society (students, communities, politics) more broadly (our/theirstories). I argue that, to date, much of what we have seen in LT-based teaching supports long-standing notions that mathematics is a neutral and culture-free domain. As we examine mainstream curricula and approaches to teaching mathematics, we see that there is still a focus on drill and memorization, even though worksheets have been traded for digital tools. PCTM helps us see that school-based mathematics is still privileged at the expense of home, community, and place-based mathematical knowledge. We have not considered how to use our collective power to disrupt standardized testing and its oppressive effects. Instead, much LT-based work has been advertised in support of helping our students perform better on tests. PCTM can support scholars in thinking about using LT-based research to "play the game" and "change the game" (Gutiérrez, 2009b). This question reminds us that because teaching happens in classrooms, which are housed in schools, which are located in communities, which are a part of society, any classroom level teaching and research ultimately has an impact on these other spaces. Whether that impact upholds the status quo or redistributes power is a question scholars delve into while remembering our moral obligations (Stephan et al., 2015) and embracing a "productively self-critical" disposition (Kilpatrick, 2013, p. 73 as cited in Larnell et al., 2016).

Conclusion

Before I close, I return to the case of Elizabeth, who used what she learned in LT-based PD to justify retaining historically marginalized students in kindergarten. As I mentioned, this teacher showed growth in her content knowledge. She also used what she learned to attend to students' thinking and plan next steps aligned with the trajectory. But when her "content-focused reform efforts" didn't produce the results she expected to see on students' quarterly benchmark tests, deficit narratives entered the conversation (e.g., if they weren't eating free breakfast in the morning we could do extra practice, their parents don't spend enough time with them at home). What was missing? How did Elizabeth need to be supported to question the usefulness of an LT-based approach across all students? What tools did Elizabeth need to support her so that she could ask *questions about the nature of standardized testing* instead of asking *questions about her students and their families*? Was Elizabeth ever providing "equitable and high-quality instruction" if these comments were indicative of her beliefs about students? And what does it

mean that Elizabeth chose to retain students instead of advocating for them? Frameworks like PCTM help us ask these questions about Elizabeth's case. Part of our moral obligation as scholars is to ensure that we consider how cognition, context, and culture are entangled and the type of professional learning experiences teachers need to understand the political nature of their work, not just the cognitive aspects. LT-based PD alone is insufficient for ensuring equity and justice.

Mathematics teaching, learning, and research are political acts (Aguirre et al., 2017; Larnell et al., 2016). And as such, we must use care in conducting our work and consider what's at stake when we don't approach our research critically. In this paper, I built the case for using PCTM as a theoretical framework to examine LT-based research to support the hard re-set needed if LTs are ever going to be relevant in creating a more humane and just mathematics experience for historically marginalized students. I also submit that it is necessary to pause, reflect, and engage in the self-work and education needed to prepare for this re-set. This paper contributes to that pause by adding to critical discussions about LT-based research and suggesting a theoretical framework that can support our collective efforts. Despite increasing explicit attention to equity in LT research, a sociopolitical lens is still needed as we grapple with considering LTs at the intersection of cognition, context, and culture. We need to continue to unpack the various definitions of equity that guide our work and engage in conversations across research paradigms to build critical LT-based research models. And while we cannot change the LT research that has come before us, we can strengthen our commitment to equity and justice by asking more complex questions that critically hold cognition, context, and culture together.

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