USING VIDEO TO IDENTIFY WHAT IS NOT KNOWN IN STUDENTS' MATHEMATICAL THINKING

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Remaining continually curious about students' mathematical thinking is challenging, yet worthwhile, in teaching practice. This paper describes and analyzes two video-based professional learning (PL) activities designed to help teachers go beyond their initial perceptions of what students understand and to identify what else they might learn about students' thinking. The findings suggest the potential of the activities to evoke different types of curiosity about student-thinking and the conditions that may support such questioning.

Keywords: Student-thinking, teacher-curiosity, professional development, early mathematics

Uncovering students' mathematical thinking is accepted as an important part of ambitious mathematics teaching (Lampert et al., 2013). However, the emphasis on immediacy in the classroom often results in teachers lacking the time to be curious about students' thinking and to understand all that is there. Teachers may also not know what else to look for or be curious about when a student seems to understand certain mathematical ideas well. In other words, teachers may not attend to what they themselves do not know about students' thinking. Both these cases may result in teachers paying less attention to student-thinking than needed and responding before having given adequate attention to such thinking. In these challenging situations, it is important for teachers to slow down their own thinking and be curious about the details of students' thinking.

This paper describes two video activities designed to help teachers slow down their own thinking regarding students' mathematical ideas, and support teacher-curiosity about students' mathematical thinking when counting. The first activity helped teachers identify what they could learn about a student's thinking when the student's understanding of early number concepts appeared unclear, and the second helped teachers identify how they might extend what they knew about a student who appeared to have a strong understanding of the concepts involved in the activity. The activities are based on the construct of teacher-curiosity (Anantharajan, 2020) and were part of a professional learning (PL) module designed to support teachers' curiosity about student-thinking.

Literature review and conceptual framework

This study is theoretically based on the approach of Cognitively Guided Instruction (CGI) (Carpenter et al., 1996) and the framework of teacher-noticing of student thinking (Jacobs et al., 2010), which prioritize attending to student-thinking, and using it to determine how teachers can respond to students. The steps involved in these approaches are translated to questions that guide teachers to notice, interpret and respond to student thinking. However, skilled noticing does not automatically help teachers figure out how to respond (van Es & Sherin, 2008). In this context, the framework of teacher-curiosity suggests one way for teachers to respond by introducing specific intermediate steps to help teachers identify what they wish to learn about student thinking, and how they can find out about that aspect of their students' thinking, thus providing a concrete way to move from noticing to responding.

Teacher-curiosity is defined as follows: An instance of teacher-curiosity is one where teachers recognize something as unknown, unfamiliar, puzzling, uncertain, or new in the context of teaching and learning, and feel motivated to initiate inquiry into that instance (Anantharajan, 2020). This conceptualization of teacher-curiosity is based on literature in teacher professional learning, and mathematics education, and understandings of curiosity in philosophy and psychology. Teacher-curiosity is comprised of cognitive, motivational, and active aspects (Audi, 1995, 2017). The cognitive aspect of teacher-curiosity involves an experience of dissonance, not knowing, or confusion and can include surprise, ambiguity, puzzlement, or novelty (Berlyne, 1966; Kashdan, 2004; Lowenstein, 1994) about student-thinking. The motivational aspect addresses the desire to learn more about those aspects of student-thinking that the teacher finds surprising, ambiguous, puzzling, or new, that the teacher wants to learn more about. The active element of teacher-curiosity refers to how teachers go about trying to learn about aspects of student-thinking they are curious about.

Teacher-curiosity can be directed towards many aspects of instruction, including students' thinking. When teachers' curiosity is directed at something they observed or experienced, it may be regarded as 'specific'. For example, if a student counts and says a total number that is one more than the actual total, the teacher may wonder "Did this student count an object twice by mistake or do they not know their number sequence?" While it may relate to understanding a particular instance, specific curiosity may not always be satisfied by information from that instance itself. For instance, in the preceding example, a teacher observing the student in real time would be unable to re-watch the student count. However, they may design other activities or questions to answer the question. When what teachers observe or experience evokes curiosity about something that was not immediately observed or experienced, their curiosity may be regarded as 'diversive.' For example, a teacher may see a student group objects by five and wonder what other numbers the student can group by (Grossnickle, 2016).

The PL at the heart of this study focused on teachers' curiosity about children's mathematical thinking in the domain of counting. Counting involves *counting principles* like one-to-one-correspondence, number-sequence, and cardinality, as well as an understanding of counting *strategies* like grouping (Carpenter et al., 2017).

The current paper focuses on the cognitive aspect of teacher-curiosity. The PL activities were designed to surface the range of potential ideas in student-thinking that teachers could be curious about. Prior work on noticing has focused on identifying what students know and are able to do (Jacobs et al., 2010). The goal of the video activities and corresponding instruments analyzed here was to shift the focus away from determining what students know and can do, to what the teacher-participants *did not* know about what students knew. Approaching children's thinking in this way was intended to provide a starting point to take a stance of curiosity, rather than one of certainty, about student-thinking. The research question addressed here is: How do two video-based PL activities help participants identify what they do not know about students' mathematical thinking?

Methods

The PL focused on Counting Collections. In this activity children count collections of objects. Teachers observe students and ask them to explain their strategies and try to understand students' thinking. The study had six participants. They taught grades TK[1] to 1st grade in three

1 Transitional kindergarten (TK) in the California public school system includes 4-year-olds who turn five after September 2 of the school year, and will enter kindergarten the following year.

schools in California, and four of these participants taught combined K-1 classrooms. They had worked as teachers for between 1 and 20 years, and all of them implemented Counting Collections prior to the PL as well. The PL was informed by research on teacher-learning (Borko et al., 2014; van Es & Sherin, 2008) and supporting curiosity (Kashdan & Fincham, 2004), and was facilitated by the first author. The entire PL comprised of six, weekly after-school sessions of 90-minutes each. Sessions included video-based discussions that were guided by the teacher-curiosity framework. Participants were individually interviewed before and after the PL using a protocol that operationalized the teacher-curiosity framework. The participants were paid a nominal honorarium for their time.

The current paper analyzes participant responses to two video-based activities in the second session of the PL which focused on identifying what is unknown, and therefore possible to be curious about, with respect to student-thinking. Guidelines on planning video-based professional development for teachers (Borko et. al 2014), a framework of criteria for choosing videos portraying students' mathematical thinking (Sherin et al., 2009) and empirical guidelines for interventions to support curiosity (Kashdan & Fincham, 2004) informed the selection of video-clips. Each video contained elements that could potentially push teachers to examine what is and is not known about students' thinking. Each brief video was of a student engaged in a counting activity and who was of a similar age as those the teachers taught. Participants responded individually to instruments developed for these specific video activities. The data analyzed in this paper consist of the responses to these two instruments (Table 1). For this activity the participants were given a research-based list of counting principles and strategies they could refer to as they watched and commented on the student videos.

Video 1: Christian Counts Bears

The purpose of this activity was to draw participants' attention to the unpredictable and the unexpected in students' thinking. The activity was structured in four steps: Provide partial information about student-thinking; invite participants' prediction based on the partial information; provide more information that might be surprising to participants and have participants assess their prediction; and finally, have participants identify what they now do not know about the student's thinking. The conjecture was that participants would want to know what might explain any gaps between their prediction and what the student actually did.

Table 1: Christian Counts Bears Instrument

Questions
[Video paused]
1. Based on what you have seen, what do you expect Christian might do or say next?
2. Why would you expect that?
[Video resumes]
3. Did you see what you expected? [If not, what was different?]
4. Based on this video, what do you not know, or what could you try to find out about Christian's thinking?

In this video, the student, Christian, is seated at a table and a researcher gives him several plastic bears to count. The researcher asks Christian to count all the objects together, count objects by color, and compare the size of different color groups. Christian's counting initially suggests that if objects were to be added to his collection, he would count the entire collection again to find out the total number, rather than remembering how many he previously counted and counting on from there. At one point in the video, the researcher adds more objects to Christian's collection and asks how many he has in total. In the PL, the video was paused just prior to this point and participants were asked to predict in their response instruments (Table 1) what Christian might do next, based on what they had seen so far. The participants then saw the rest of the video, where he seems to count on mentally and says the new total number. Asked how he knew, he explains that he held the previous total in his mind, started to count mentally from the next number and arrived at the new total. This ability was not apparent in the preceding part of the video. Participants were then asked to note whether he did what they had predicted. Finally, participants were asked to reflect and identify what they felt they did not know about Christian's mathematical thinking.

Mohammad Counts Cookies

The purpose of this activity was to explore what else there may be to learn about a student whose understanding is both strong and visible. Unlike the previous activity, here the participants were shown the whole video. They were then invited to propose informed conjectures of what remained unknown but relevant based on the information in the video.

In this video, the student, Mohammad, is seated at a table with a researcher. The researcher asks him to imagine that he has three boxes with five cookies in each. He is then asked how many cookies he has in all. The researcher provides Mohammad a set of blocks which he uses to help him count. He first places three yellow blocks side by side. These may represent the boxes although he does not say so, because he does not count these yellow blocks going forward. On each yellow block he stacks five blocks of other colors. He then counts the first two stacks as 5 and 10. Then he counts on the remaining five by ones. He says that he has fifteen cookies in all. In the video he devises a clear counting strategy and explains his thinking without apparent difficulty. This video gave participants the opportunity to reflect on what they might learn about a student who did not indicate any confusion or struggle. The instrument for this activity had only one question that participants responded to after watching the whole video: Based on what you see in the video, what could you try to find out about Mohammed's thinking?

Coding and Analysis

The data was coded for the type of curiosity that participants expressed, and the mathematical principles and strategies that the participants mentioned in their responses. The responses were also coded for whether participants perceived a strong or partial understanding of the principle or strategy (Table 2). Additionally, for the first video, the analysis also looked at participants' statements about whether or not Christian did what they had predicted.

Inter-rater agreement was calculated with Kappa statistics at the parent and child code levels, and discussion and consensus at the grandchild code level. The Kappa value for the parent and child codes ranged from 0.7-1.0. Agreement values were calculated for the pre- and post-interview transcripts[2]. This instrument and data set operationalized all the elements of the teacher-curiosity framework and was a reliable indicator of the types of responses in more activity-specific instruments like the video response tool that operationalized only the cognitive aspect of teacher-curiosity. After reaching agreement on the interview data the two coders discussed the data from all other instruments, including the instruments analyzed in the current paper to confirm that the

2 In these interviews participants watched a video and responded to questions based on the teacher-curiosity framework, as well as broader questions on the role of curiosity in their own work. The codebook used in the present study corresponds to the questions in the interviews and the elements of the framework.

codebook could be applied to these data as well. The kappa calculations mentioned above were thus for data sources (i.e., interview transcripts) not the focus of this study, but mentioned here because coding those was necessary to developing the codebook used here. Disagreements were discussed to reach consensus.

Table 2: Codes Applied

Type of curiosity	
Specific curiosity	Participant is curious about something that they observe a student do or say.
Diversive curiosity	Participant is curious about something a student might do or say, which they cannot observe at that time.
Mathematical ideas that participants perceive in students' thinking	
a. Counting principles	Principles teachers perceive as constituting child's understanding of counting. (Carpenter et. al. 2017, NRC 2001) Includes: • One-to-one correspondence • Cardinality • Number sequence • Abstraction principle • Order irrelevance • Other principles or concepts (e.g., number conservation, operations, base-ten system, place value).
ai. strong understanding	Teacher implies the child has a comfortable/ fluent/ consistent understanding.
aii. partial understanding	Teacher implies the child is working on understanding, or that the child's understanding is not always consistent.
b. Counting strategies	Strategies children use to count objects. Includes: • Grouping by number • Strategies to keep track • Sorting based on non-numerical qualities • Visual arrangements • Associating number with object • Other strategies
bi. strong understanding	Teacher implies the child has a comfortable/ fluent/ consistent understanding.
bii. partial understanding	Teacher implies the child is working on understanding, or that the child's understanding is not always consistent.

Table 3. Sample of Coded Responses

Question	Bella	Codes	Beth	Codes
2. Why would you expect that?	*He understands that he can touch each one & count it out loud.* *He does not seem to count on.*	ai. One-to-one correspondence; strong understanding bii. Counting on; partial understanding	*He doesn't seem to be keeping track or counting on as he is answering the questions.*	bii. Keeping track and counting on; partial understanding

The responses for each instrument were organized into a single table where each row represented a question in the instrument and each column represented a participant. Thus, all the participants' responses were visible side-by-side. The first author then coded the responses for the mathematical ideas that the participants refer to. Table 3 indicates a sample of two participants' responses to question 2 and the codes applied to Christian Counts Bears.

Results

The coded data indicated multiple patterns. The results for each video are discussed below.

Christian Counts Bears

After watching the first portion of the video, five of the six participants predicted in their written response that Christian would recount the entire collection. This was consistent with what they had observed until that point, where Christian tended to count all the objects each time objects were added. One participant wrote that he would count on from the previous total he counted, if he could remember the total. Four of the six participants also wrote that they expected that Christian would not arrive at the correct total number of objects after counting the final time.

Five participants said he did not do what they expected – in other words, they said their prediction was not correct. One said she had expected that if he counted on he would begin at 14 but he began at 16, and in this sense he did not do what she expected. The aspect of Christian's counting that surprised the participants was his ability in the second portion of the video to count on from a number he held in his mind. This aspect of counting on is what all participants said they could learn more about. For example, Lillian stated in her response, "I would ask him to show me how he counted them all together by adding on" and Hannah stated, "We know he can do the basic idea of counting on. We don't know if he can do it accurately."

Three participants also identified an additional mathematical idea that they wanted to look for and learn about in the student's thinking, namely addition. Bella wondered "Can he add bigger numbers together? What can he add together in his head?"

The participants all expressed two types of curiosity about Christian's thinking. On the one hand, they wished to know more about things that would help explain why he had counted on, and why their own predictions were not accurate. For example, Beth stated, "I would like to find out how high he can count with one-to-one correspondence. (When he counted the 12 bears, he did not have one to one)." The other type of questions were related to what else Christian knew or could do as a kindergartener, or related to the general body of ideas related to number and counting. For example, Stacey wondered, "Would he be able to add together 2 smaller groups of bears?" In summary, the participants used what they had seen upto the point the video was paused to predict Christian's future moves and discovered that their prediction was not accurate. Rather, participants found that Christian's understanding and skills were more complex and advanced than they anticipated. Consequently, participants perceived that there were aspects of his thinking that they did not know about and identified what they could learn about his thinking.

Mohammad Counts Cookies

In their responses after watching the entire video of Mohammad, all the participants went beyond Mohammad's 'correct answer' and identified further aspects of his thinking they could try to learn about. All the participants attended to the details of what they saw Mohammad do and posed questions about what he understood. Lillian wondered if he could subitize the quantity of five blocks. Bella said she would ask him to explain his strategy for "figuring out 15."

The participants all wondered if Mohammed understood how to count by groups of 5 or 10. This ability to skip count or count by grouping can be seen as a precursor to multiplication and the participants may have wanted to determine this before exploring multiplication with him. For example, Hannah wondered, “Can he count by 5s? if there were 25 would he count on starting at 11 still? Or would he make groups of 10? Count by 5s?”

Without a particular element of surprise or dissonance evoked by the video, the types of questions that participants posed were of two types. The first was to clarify aspects of what they saw him do. The questions posed by the participants also make clear that despite completing the task with apparent confidence and giving the right answer, there were aspects of what the student did in the video that were not clear and thus possible to clarify. For example, Beth stated, “He was good at counting on. I would like to know more about how he keeps track.” The other type of questions that participants posed had to do with related mathematical ideas that they could explore with the student, to find out what else he understood and to what extent. For example, Rita wrote, “I could try to find out what he knows about 10s. [Can he] use a 10s frame?” In summary, the participants identified aspects of Mohammad’s thinking that were related to what he was able to do but were not apparent in the video.

Discussion

This paper looks at how two video-based activities helped participants in a PL identify what they could further learn about students’ mathematical thinking. The experience of the participants in the two activities was quite different. For the Christian Counts Bears video, participants were likely curious about what was unknown because of the nature of the video the facilitator chose, the way the viewing of the video was structured (i.e., pausing at an opportune time), and how inviting predictions required participants to process the information they had up to that point. These steps appeared to seed dissonance and give rise to participants’ puzzlement. The dissonance or disequilibrium that resulted from participants predicting what the student could do then finding out that the prediction was not accurate likely evoked participants’ questions about what more they could learn. It was also striking to note that the participants readily stated that the student did not do what they predicted, which indicates a certain humility and openness on the part of the participants to revise their thinking. It also potentially suggests that the structure of the activity may have helped them revise their thinking by focusing on their surprise rather than evaluating whether or not their prediction was correct.

For the Mohammad Counts Cookies video, participants had to draw on their own knowledge and experience to assess the information from the video and imagine what other information they would like to have. The response of the participants affirms Kashdan and Fincham’s (2004) recommendations for interventions to facilitate curiosity, which informed the design of the task. These include developing “tasks that capitalize on novelty, complexity, ambiguity, variety, and surprise,” “enjoyable, group-based activities,” and “tasks that are personally meaningful” (p.490). The video-based PL activities may also provide participants cues for practice, to attend to the unexpected and unknown aspects of their own students’ thinking.

With the Christian Counts Bears video, making an inaccurate prediction and seeing some surprising aspects of the student’s thinking may have provided a starting point to learn more about aspects of the student’s thinking, such as counting on or keeping track of counting, as well as consider other mathematical understandings that may explain what he did, such as addition. The teachers’ questions might have been motivated by a desire to resolve the dissonance they experienced, which can be a strong motivating factor in learning (Chinn & Brewer, 1993).

For the Mohammad Counts Cookies video, participants considered a wide range of ideas they could learn about in Mohammad’s thinking, despite his apparent confidence and correct answer. The structure of the video activity may have helped participants identify what else they could learn about Mohammad's thinking. This includes the sequencing of the video after Christian Counts Bears, which may have primed the participants to reflect on what is known or unknown about the mathematical thinking of the student in a video. This mental priming, as well as the explicit invitation to consider what else the student may know, may have helped participants identify the wide range of mathematical ideas they could explore with this student.

Although participants did not explicitly refer to Mohammad's understanding of multiplication in Mohammad Counts Cookies, they did want to learn about his understanding of grouping and skip counting, which can be a step towards multiplication. Teachers often tend to associate multiplication with older students (Carpenter et al., 2017). However, the participants' responses suggest that activities like these may provide opportunities to slow down, look beyond the correct answer, and uncover the seeds of such "advanced" ideas in younger students as well.

Both types of curiosity that participants displayed towards student-thinking – specific and diversive – implicitly acknowledge that there is more to learn about students' understanding than is initially apparent. The findings however indicate that specific curiosity may be further unpacked into questions that seek two types of answers: information that will help resolve a feeling of dissonance as the result of a surprising occurrence or an incorrect prediction, and information to explain what is observed. The former can help teachers correct misconceptions or assumptions about students, and the latter can help teachers make sense of the knowledge and understanding that a student demonstrates in the moment.

In the context of video-based PL activities, it is possible that some specific questions can be resolved by re-watching the video. For example, Rita asked of Christian's counting, "Where does his 1-to-1 [correspondence] stop?" Watching the video again with this question in mind may help her answer this question. But not all questions can be answered by re-watching the video because the information to answer some of the specific questions that participants asked is not present in the video. They would need to interact with the student and give him more tasks to find out, like asking him to count all bears by ones, or asking him to explain how he keeps track, asking him to count the collection again using the strategy that he just used – all ideas proposed by the participants. Thus, these questions indicate specific curiosity to the extent that what the student does in the video triggers participants' questions though the answers may not be in the video.

On the other hand, diversive questions can help teachers use their observations as a starting point to wonder what else the student may know or understand. In this sense, diversive questions may help teachers plan their response to and next steps even with students who seem to have strong mathematical understandings. In the context of the PL itself, diversive or "what else" questions may be an effective way to close the video activities, so as to invite teachers to continue thinking about what they might learn about a student's thinking, including their own. Developing the capacity to pose these questions can also help teachers approach student-thinking with the attitude that there is always more to learn. PL activities can thus be designed to elicit all these different types of questions about student-thinking.

Limitations and future work

As this is a small dataset, further research is needed to determine to what extent these findings are applicable to other PL contexts. Further, the participants in the study already implemented Counting Collections and were interested in teaching that elicits and responds to student-thinking. It would require further research to understand whether and in what ways teachers with other pedagogical approaches respond to the activities described in this study.

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