SUPPORTING DISCUSSION PRACTICE IN MATHEMATICS METHODS: APPLICATIONS OF WHOLE-CLASS SCAFFOLDING

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Discussion is a practice used in mathematics methods courses to support prospective teachers' pedagogical ideas. Underexamined but central to the development of instructional activities, including discussions of teaching, are mathematics teacher educators' (MTEs') tacit and explicit theories of learning and teaching. We report findings from a self-study of three MTEs' discussion practice in methods courses. Data sources include transcripts of MTEs' dialogic analysis of their discussion practice and evidentiary maps based on instructional artifacts. We argue that whole-class scaffolding serves as a tacit theory informing MTE discussion practice. We support this argument using evidence that our discussion practice was driven by prospective teachers' move toward independence and by layering instructional activities.

Keywords: Discussion, Preservice Teacher Education, Teacher Educators

Learning theories (Casey et al., 2018) and professional experiences (Leikin, 2020) have been shown to inform mathematics teacher educator (MTE) practices. Furthermore, professional practices shape the "learning potential" (Grossman et al., 2009, p. 2090) of instructional activities. These findings link theories and experiences of MTEs to learning opportunities provided to prospective mathematics teachers (PTs). Calls for the study of MTE work (Grossman et al., 2009; Lee & Mewborn, 2009) have resulted in descriptions of MTE practices (e.g., van Es et al., 2014), instructional activities (e.g., Tyminski et al., 2021), and MTE knowledge (e.g., Beswick & Goos, 2018) with a few studies exploring MTEs' professional growth (Krainer et al., 2021). In this paper we address MTEs' professional growth of discussion practice, providing an example of how the learning potential of discussions of teaching are developed. Beyond providing descriptions of discussion practice as useful models for MTEs, unpacking theories that inform such practice links MTEs' learning to teach about teaching to PTs' opportunities to learn mathematics teaching.

In this paper we use self-study methodology to identify tacit theory that informs the discussion practice of three white MTEs' teaching mathematics methods at different institutions to elementary and secondary PTs. The three MTEs' are relational (Kitchen, 2005) constructive teachers (Steffe & D'Ambrosio, 1995) of mathematics teaching who utilize relationships with PTs and evidence of their pedagogical concepts to inform instructional decisions. Relational teacher educators view relationships as central to creating opportunities to learn about teaching. Constructivist teachers view learning as an intersubjective activity that includes learner discussions in a collaborative community (Kastberg, 2014; Steffe & D'Ambrosio, 1995). Taken together relational constructivist MTEs view whole class discussions of teaching as opportunities to theorize about teaching and problems from experience.

As relational constructivist MTEs, we use whole class discussions to support PTs' teaching practices. Self-study of our discussion practice resulted in new approaches to posing discussion questions (Kastberg et al., 2019) and creating phenomenological conditions (Lischka et al., 2020) supportive of discussions of teaching. Yet we continued to struggle to facilitate discussions with

PTs, instead often creating recitations during which PTs' shared ideas without addressing the ideas of others. To expand the domain of potential action (Brown & Coles, 2020) available in planning and facilitating discussions of teaching in mathematics methods courses we were guided by the question: *What tacit theories inform mathematics teacher educators' discussion practice in mathematics methods courses?*

Background and Literature

Loughran (2014) defines pedagogy as "two complementary aspects of knowledge and practice: teaching about teaching and learning about teaching" (p. 275) kept in relation. Loughran's definition situates pedagogy as a theory that maintains a relationship between teaching about teaching and learning about teaching. Such theories in mathematics teacher education have been used to design and implement instructional activities for PTs (Kastberg et al., 2018). Making theories that inform MTEs' practice explicit (Mewborn & Stanulis, 2000) is essential to expanding possibilities for instructional choices and engaging in discussions of those choices beyond simply modeling practices (Teuscher, et al., 2016). In this paper we focus on tacit theories that inform MTEs' whole class discussions of teaching.

Discussion is defined as human interaction to address "a question of common concern" (Dillon, 1994, p. 8) through an exchange of ideas (Alexander, 2019) and an examination of differing viewpoints (Kim & Wilkinson, 2019). Discussion differs from recitation in that recitation involves sharing ideas without engaging others in those ideas while discussion involves a sharing of ideas in which others take up those ideas and add to or counter them (Dillon, 1994). We define discussion as a talk strategy MTEs use to support development of PTs' pedagogical concepts (Simon, 2008). MTEs' use of discussion practice is informed by their "knowledge, theories, and understandings" (Pinnegar & Hamilton, 2009, p. 16) to develop "knowledge from practice" (Pinnegar & Hamilton, 2009, p. 17) as a way of "knowing to" (p. 18) engage PTs in a given practice.

Existing reports of MTEs' facilitation of discussions have identified practices associated online discussions (McDuffy & Slavit, 2002), ambitious teaching (Kazemi et al., 2016) and teaching videos (van Es et al, 2014). MTEs' facilitation of pedagogical discussions (Lischka et al., 2020; Kastberg et al., 2021) is also informed by phenomenological factors (Dillon, 1994) such as a sense of community and relevant common experiences. MTEs' discussion practices, including posing discussion questions, and supporting interpretation of the question, impacts the form (i.e., IRE, recitation, discussion) and content of the PTs' talk (Kastberg et al., 2021). Evidence teachers use to support claims made during discussions of teaching informs MTEs' discussion practice, yet findings are mixed. Steele (2005) identified teachers' use of experience to support claims in discussions of mathematics teaching while Dick et al. (2018) suggested teachers did not provide evidentiary support for claims made during discussions of their mathematics teaching. Such research illustrates that teachers' address of discussion prompts focused on teaching are conditional and respectful of the authority of experience (Munby & Russell, 1994), thus raising questions about how MTEs' can initiate and sustain discussions of teaching that move beyond sharing ideas to taking up and countering the ideas of others.

One theory that may support MTE's discussion practice draws from linking whole-class scaffolding and dialogic teaching (Bakker et al., 2015) including discussion. Smit et al. (2013) asserts that whole-class scaffolding involves interpreting and responding to learners' understandings and needs while fostering independence (Visnovska & Cobb, 2015). Whole-class scaffolding is "layered, distributed and cumulative" (Smit et al., 2013, p. 829) and takes place before, during, and after whole-class interactions. Bakker et al. (2015) illustrates ways whole-

class scaffolding can inform dialogic teaching including discussions. Keys to whole-class scaffolding include understanding that instructional activities produce layers of opportunities for developing concepts while supporting movement toward engaging in activities independently. These two key components of whole-class scaffolding align with the existing reports of MTE work that illustrate the importance of PTs' movement toward independent teaching (Grossman et al., 2009) and the need to use pedagogies of practice in concert (Ghousseini & Herbst, 2016).

Methodology and Methods

Self-study methodology is a form of empirical practitioner research focused on improving practice in context. As practitioner research (Borko et al., 2007), self-study methodology supports inquiry into pedagogy of practice. Self-study is self-initiated, improvement-aimed, interactive, uses qualitative methods, and defines validity as based in trustworthiness (LaBoskey, 2007). Our study of tacit theories that inform discussion practices was initiated to inquire into how we planned for and enacted discussions of teaching in mathematics methods with a focus on improving our discussion practices. Beginning in 2015, our collaboration has included weekly meetings to discuss the development of instructional activities and practices. As a collaborative self-study group of MTEs from three different US institutions, we are critical friends (Schuck & Russell, 2005) who undertake scholarly inquiry (Lee & Mewborn, 2009) of our practices.

Our institutional missions range from teaching-focused to research-intensive and our program foci span elementary to secondary teacher certification. Signe's discussion study was guided by the question: How do children learn mathematics? Alyson's discussion study was guided by the question: What is the role of mathematics teachers related to social justice and equity? Susan's discussion study was guided by the question: How does cognitive demand of tasks and knowledge of children's mathematical thinking inform planning instruction? Studying our practice, we identify "living contradictions" (Whitehead, 1989, 41) between our intended and working models of practice.

Critical friendships serve as dialogic communities where dialogue serves as a process of coming to know. Pinnegar and Hamilton (2009) align the scientific method, action research cycle and dialogue as processes of coming to know in different methodologies. In self-study methodology dialogue involves expressing ideas in conversations to be "accepted and elaborated or rejected, rephrased, questioned, or ignored" (p. 87). Participants in the dialogue "may provide evidence, examples, representations, metaphors, or analogies in support of or opposition to the idea or as a way to synthesize and integrate the idea with others" (p. 87). Such dialogues in mathematics teacher education produce "theorizing" about MTE practice that "extends the range of possible behaviors, by dwelling in the details of the experience of teaching and considering the details on a more general level" (Brown & Coles, 2020 p. 99). Assessing the trustworthiness or quality of qualitative research (Grant & Lincoln, 2021) involves planning for and committing to making transparent authenticities that emerge in the study. In the case of self-study, ontological authenticity is central as researchers gather evidence of knowledge of self that illustrates what is learned through dialogic analysis and other analytical methods.

Within the methodology of self-study we used three qualitative analytic methods: analytical dialogues (Guilfoyle et al., 2007), evidentiary maps, and descriptive coding (Saldana, 2016). (1) We engaged in 8 analytic dialogues of our discussion practice in fall 2020 (Covid-19 hyflex teaching). Conversations focused on "coming to know" how we used discussion in our teaching that served as the basis for "action" (Guilfoyle et al., 2007, p. 1111) in our ongoing discussion practice. At the conclusion of this period we had formed a collection of categories that informed our discussion practices. (2) In spring 2021 we used course artifacts (recordings of whole class

discussions, class summaries, assignments, and PT work samples) to create evidentiary maps (see Table 1 for an excerpt) of the "structure of events" (Jordan & Henderson, 1995, p. 57) in our fall 2020 discussion practice. Each map was analyzed using categories from the dialogic analysis. Confirming or contradicting evidence from the dialogic analysis was identified. (3) Descriptive coding (Saldana, 2016) of transcripts from analytical dialogues was used to triangulate results from analytic dialogues and evidentiary maps by linking the findings from the analytic dialogues to evidence in the transcripts of our dialogues. Analysis revealed evidence of commonalities across three MTEs' discussion practices. Movement in these analytical methods proceeded from dialogue to course artifacts and back to dialogue. These three analytic methods created an evidentiary basis for findings common across three MTEs' discussion practice and related contexts.

Findings

This section describes two components of whole-class scaffolding (Smit et al., 2013) that influenced our discussion practice: (1) the move toward independence in mathematics methods courses, and (2) layering learning activities driven by the move toward independence. Data from the three authors' practices were used to derive the findings. This paper uses examples from Susan's pedagogy of discussion practice drawn from transcripts of dialogic conversations and evidentiary maps to illustrate our findings for the research question: What tacit theories inform MTEs' discussion practice in mathematics methods courses? The findings are structured to first present an integrated view of Susan's practice in the form of a vignette. Based on the vignette and with example from our critical friend conversations we highlight each component of scaffolding, beginning with layering of instructional activities and turning to the move toward independence.

Susan's Vignette

One learning goal in Susan's elementary mathematics methods course was to understand cognitive demand of tasks to support PTs' lesson planning. She reasoned that distinguishing between demands of tasks, such as those focused on producing answers and those focused on sense-making, would support PTs' design of problem-based lessons. Susan planned for a discussion on cognitive demand of mathematics tasks for the third class.

The day before Susan's first class, Alyson identified the importance of anticipating in planning for discussions of teaching, just as we would for teaching mathematics.

> Alyson: So you are seeing these different perspectives that might come out from the PTs. Would it help if you thought through those different perspectives, and have some ideas of what you might want to draw out or probe a little more deeply? . . . It's like anticipating different directions that the discussion might go and being ok with different outcomes based on what they're bringing to the table. (Conversation 08-31-2020)

As a result, Susan considered how her first day activities, including eliciting PTs' memories of mathematics tasks, might inform the planned whole class discussion of cognitive demand (Table 1, Class 3). How might PTs associate tasks with children's mathematical thinking? Susan anticipated experiences PTs might share, and purposely planned instructional activities for the first two classes (Table 1) and the autobiography assignment to elicit PTs' ideas about tasks and mathematical thinking. As planned, PTs' shared experiences and perspectives on types of thinking involved in learning mathematics. Some were expected, such as timed facts tests associated with memorization. Others were unexpected necessitating modifications to support connections between PTs' experiences with mathematics tasks and mathematics learning.

Table 1: Excerpts from Evidentiary Map

Date	Event	Description
Sep 1	Class 1 Online	Think-Pair-Share: What are your memories of learning mathematics? First assignment: mathematics autobiography due two days later.
Sep 3	Class 2 In-Person	Small group activity: Examine K-5 mathematics curriculum standards. List verbs; classify levels and types of thinking.
Sep 15	Class 3 Online	From memories: identify thinking involved in learning mathematics. Introduce the Cognitive Demand framework (Smith et al., 1998). Small groups: examine tasks using the framework; whole class: share a claim about a task and *discuss level of cognitive demand*.
Sep 17	Class 4 In-Person	Reviewed verbs from Class 2 and mathematics tasks from Class 3. Small groups: What helps determine if a task involves lower- or higher- cognitive demand? Individual reflection prompt.
Sep 22	Class 5 Online	Revisit tasks from Class 3: Describe cognitive demand. Follow-up: choose a task and make a video explaining the cognitive demand.
Sep 23 – Nov 24	Individual Conferences	Consultations with PTs on lesson plans with attention to cognitive demand of tasks.

During Class 3 Susan initiated a discussion on cognitive demand of tasks by asking groups of PTs to make claims about a small collection of tasks. She knew this was risky given the limited time for the development of phenomenological characteristics (Dillon, 1994) needed for productive discussions. PTs shared claims about the cognitive demand of different tasks, but little evidence was offered. No PT challenged another PT's claim, a move that might have sparked a discussion. Instead, the PTs took turns sharing. Susan described the class activity as a recitation and turned to redesigning instructional activities for Class 4.

Our conversations focused on benefits and limitations of introducing PTs to professional language such as "cognitive demand." Alyson described the importance of introducing professional vocabulary, like cognitive demand, to support PTs' descriptions of teaching.

> Alyson: I still think that we share some of the best practices in different ways. As part of building that common experience and building opportunities for [PTs] to have language like the cognitive demand idea puts words to descriptions that they can then think about . . . I guess I feel like we are spending a lot of time developing that language.
>
> Susan: This whole thing about language and cognitive demand, . . . I don't think they [PTs] have thought about it [cognitive demand] even though I know they are introduced to levels of thinking [in previous classes] but when they come to math methods it's [cognitive demand of tasks] not connected to that [prior experience with levels of thinking]. I'm trying to connect to language other classes have been using. (Conversation, 09-21-20)

Susan describes her effort to connect language from other courses to cognitive demand.

Susan re-designed her instructional activity for Class 4 in response to PTs' association of mathematical tasks and cognitive demand. Susan had hoped to create a discussion of cognitive demand during Class 3 that would connect cognitive demand of tasks and student thinking. Instead PTs' shared tasks with rationales that exhibited limited understanding of the elements of

cognitive demand. Susan adapted her next instructional activity to provide PTs with opportunities to explicitly describe task characteristics in relation to specific elements of cognitive demand.

Susan's initial lessons were designed to gather evidence of PTs' experiences and views of teaching and learning mathematics. Subsequent instructional activities were designed to support PTs to connect their experiences, cognitive demand of tasks, and types of thinking the tasks might elicit from children. Although Susan planned a discussion of cognitive demand of mathematics tasks for Class 3, the PTs had not yet developed connections between their experiences, demand of tasks, and opportunities for children's mathematical thinking. The PTs' responses allowed Susan to make sense of their ideas about cognitive demand. Susan used the PTs' ideas to re-design subsequent instructional activities to focus on connecting cognitive demand and task characteristics (Table 1). Susan did not intend the Class 3 discussion of cognitive demand to result in associating a level of cognitive demand for each task, but to relate levels of cognitive demand to task features and opportunities for children's mathematical thinking. Susan viewed these connections as essential in PTs' lesson planning later in the semester.

Moving Toward Independence

Our conversations about how to provide learning opportunities to support discussion focused on the importance of modeling PTs' ideas and designing instructional activities that would prepare PTs for planned discussions. We focused on developing meanings for key terms in the discussion questions and the significance of such questions in learning to teach. We reasoned from our experiences teaching mathematics, that we could use models of PTs' ideas about teaching (cognitive demand, learning, and social justice) to design instructional activities. Further, using PTs' ideas, we could facilitate the development of connections among ideas about teaching mathematics. This idea is illustrated as Signe describes her movement to using PTs' vocabulary, "way of talking," in instructional activities and discussions.

> Signe: When I finally did use the PTs' way of talking, they had something to say, and they knew they were going to be attended to when they were talking about it, because it was a significant idea. And, it matters when you're trying to do your planning for discussion. (Conversation 11-10-2020)

As we supported PTs to link their "way of talking" to key terms in mathematics teaching, such as cognitive demand in Susan's case, we noticed our assumptions about discussions and PTs' move toward independence. Signe described the need to move on to teaching other concepts, but our awareness that PTs would need to apply learned concepts in their teaching remained.

> Signe: For a while, it [conceptual understanding] was kind of a mystery, like they kind of used some of the words that I used ... but then the rubber meets the road and you get to the final discussion. . . . only that's not the final word, because I have their lesson plans, I have their concept summary . . . So there's not an end, because they're going to do more stuff with that idea, but I'm not going to focus on it anymore. I felt like I got to where I needed to get to, to be able to help them with facilitation of their lessons. (Conversation 11-10-2020)

Our conversations continued to focus on how layers of instructional activity related to planned discussion questions and supporting PTs' planning and teaching of mathematics lessons.

Layering to Support Discussions

Central in our conversations was how we could provide multiple opportunities for PTs to connect their knowledge and experience to key ideas during discussions of teaching. Susan's constructivist pedagogy informed her design and re-design of layers of instructional activity to support PTs' ideas about cognitive demand. Susan provided opportunities for PTs to create ideas about cognitive demand of mathematics tasks and opportunities for learners of mathematics from PTs' experiences. As she created models of PTs' ideas about cognitive demand, Susan designed layers of activity for PTs' to use their ideas in examining mathematics tasks.

Susan, Signe, and Alyson began the semester with dates for discussions on the syllabus. As these dates came, all three MTEs identified the need to build understanding of the concepts so that PTs could meaningfully engage in discussions of the concepts. Our conversations revealed how initial target dates for discussions were shifted back repeatedly to accommodate layering. Signe described trying to find the parameters for the discussion and understandings needed.

> Signe: The first stage of a discussion is to understand everyone's experience and futz around with the parameters of the discussion. Maybe I can't have 'where does knowledge for mathematics teaching come from' until they go into the field. But I can 'have how do students learn math' before that because they've been working on it.
>
> Alyson: moving that to the end, you'll have more shared experience to have that conversation, by the end of the semester. (Conversation 09-07-2020)

PTs needed time to build their ideas about key concepts through experiences with layers of instructional activities provided. We realized that often we planned whole class discussions too early before the PTs had developed ideas about teaching. Our premature rush to discuss as in Susan's Class 3 effort to connect experiences, task characteristics, cognitive demand, and children's mathematical thinking, positioned PTs as experts with knowledge of pedagogical concepts and ways to describe that knowledge. As Susan's evidentiary map illustrates, layers of instructional activities are created as MTEs' gather evidence of PTs sense making and re-design subsequent activities. These layers provide opportunities for PTs, but also suggest that MTEs' discussion practice is informed by the interpretation of PTs' pedagogical concepts.

Discussion

Findings confirm that components of whole-class scaffolding (Bakker et al., 2015; Smit et al., 2013), including movement toward independence and layering of instructional activities, informed our discussion practice. The significance of this finding lies in the potential of MTE theories (Casey et al., 2018), professional experiences (Leiken, 2020), and practices (Grossman et al., 2009) to shape the "learning potential" (p. 2090) of instructional activities. Explicit knowing of theories that inform discussion practice expands the possibility space (Brown & Coles, 2020) for MTE's decisions in planning and facilitating discussions of teaching. Findings from self-study of teaching provide evidence beyond descriptions of practice that serve as models of scholarly practice (Lee & Mewborn, 2009) to identify and describe factors that influence MTE's professional growth (Krainer et al., 2021).

Existing research exploring teacher discussions of practice provides evidence of the challenges MTEs' face in facilitating discussions. Of significance is whether (Dick et al, 2018) and how (Steele, 2005) teachers support claims about teaching during whole class discussions. This illustrates a central challenge in facilitating discussions where talk moves beyond sharing ideas to the consideration of those ideas by others who add to or counter them. Susan's effort to initiate a discussion of cognitive demand illustrates how two interrelated components of whole-

class scaffolding informed her discussion practices. First, efforts to engage the class in discussions of cognitive demand were informed by the need for PTs to plan and teach lessons at the end of the term. In the practical work of methods teaching, dates for field experience and practicum are fixed in the schedule. PTs must be able to function independently when those dates arrive. This practical consideration in methods teaching informs decisions about when PTs must use ideas about teaching relevant to planning mathematics lessons. Susan knew that she needed to prepare the PTs to plan lessons (Table 1, Individual Conferences). Second and related to this practical press toward independence, is the layering of instructional activities as informed by evidence of PTs' pedagogical concepts. Susan knew almost immediately during the discussion of tasks and cognitive demand in Class 3 that a discussion would not be possible. PTs did as she asked and shared tasks and levels of cognitive demand, but there was no adding on or countering of PTs' claims. The evidence of PTs' sense making informed a redesign of instructional activities for Class 4 (Table 1) to provide additional opportunities for the development of connections between tasks, cognitive demand, and opportunities for children's mathematics thinking. Susan's discussion practice illustrates the interconnected nature of components of MTE discussion practice informed by whole-class scaffolding. Movement toward independence drives the development of instructional activities, yet evidence of PTs' thinking encourages MTEs to redesign instructional activities creating layers that provide opportunities for PTs to develop new versions of pedagogical concepts like cognitive demand of mathematics tasks.

Findings from our self-study support the claim that characteristics of whole-class scaffolding, including move toward independence and layering of instructional activities, were embedded in our planning for and facilitation of discussions. We do not claim that we use these components in particular ways or at particular times in our discussion practice. In addition, we do not make claims about coordination of whole-class scaffolding and other theories (e.g., Kitchen, 2005; Steffe & D'Ambrosio, 1995) used explicitly in planning for and implementing discussions of teaching. Instead, we claim that in the development of discussion practice, we attended to the practical work of teaching about teaching by providing opportunities for PTs' to learn about teaching. Additional work is needed to address how MTEs' use theories in concert in planning for and facilitating whole class discussions. For example, how do Susan's explicit theories of relational teacher education (Kitchen, 2005) and constructivist teaching (Kastberg, 2014; Steffe & D'Ambrosio, 1995) support and inform her development of instructional activities that would support PTs' concept of cognitive demand? Questions like this assume that theories are used in concert, but perhaps some theories or components of theories are used for planning, while others are used to facilitate PTs' discussions of teaching.

The two components of whole-class scaffolding, move toward independence and layering instructional activities, illustrate one tacit theory which informed our discussion practice. We claim these components contribute to the integrative concept of whole-class scaffolding (Bakker et al., 2015) that helps us maintain the complementarity between teaching and learning (Loughran, 2014) while informing our discussion practice. Structured this way, MTEs' discussion practice is informed by components of whole-class scaffolding (Bakker et al., 2015). Although we do not claim that all MTEs build discussions on a theory of whole-class scaffolding, we do claim that unpacking explicit and tacit theories that inform MTEs' discussion practice will contribute new knowledge of MTE growth of professional practice called for by Krainer et al. (2021). Findings from such studies have the potential to support MTEs beyond providing models of instructional activities described in studies of professional practice (Tyminski et al., 2021) to understanding the diversity of knowledge and experience that drives

the decisions involved in professional practices. Such findings address in part how MTEs' tacit theories play a key role in the "learning potential" (Grossman et al., 2009, p. 2089) of instructional activities used in professional practice, while providing evidence of the "integration between knowledge and practice" (Krainer et al., 2021, p. S11).

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