# DEVELOPMENT OF A MATHEMATICS DISCIPLINE-SPECIFIC LANGUAGE SCALE

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*Existing studies have defined and assessed disciplinary literacy, mathematical literacy, and general academic language. However, there is a need to define and assess mathematics discipline-specific language (MDL), particularly for elementary school teachers. Therefore, the purpose of this study was to develop a research instrument to assess the MDL of elementary school teachers. The final instrument developed through iterative analysis included 20 items on a 4-point Likert-like scale distributed between three distinct MDL categories: technical, symbolic, and visual. Instrument validity was confirmed using Confirmatory Factor Analysis with the set of 211 video recordings and corresponding lesson plans of mathematics lessons taught by pre-service elementary school teachers enrolled in a graduate special education program.*

Keywords: Mathematical Knowledge for Teaching, Preservice Teacher Education, Teacher Knowledge

## Purpose of the Study

Mathematical literacy is essential for solving problems encountered in today's rapidly evolving world. It is defined as "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned, and reflective citizen" (OECD, 2009, p. 14). Mathematically literate students should know how to do mathematics and how to speak the language of mathematics. This is not learned by simply memorizing definitions; but rather, by using the language of mathematics in their learning experiences (Hill, et al, 2008). The Common Core State Standards for Mathematical Practice emphasize the importance of developing mathematical literacy and discipline-specific mathematics language (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

Discipline-specific language is defined as "the decontextualized oral and written language used within a specific discipline or profession with specialized vocabulary, syntax, and discourse patterns" (Zhang, 2014, p. 40). Mathematics discipline-specific language (MDL) is different from everyday language (Machaba, 2017). MDL is used to encode information and it allows members of the discipline to communicate in discipline-specific ways (Bernstein, 1996). Examples of MDL include vocabularies, symbols, and notations. A lack of appropriate MDL can hinder student learning and become a source of misconceptions carried over to later mathematics education (Di Domenico, 2014; Köse, 2008). It is vital that attention be given to MDL beginning at the elementary level (Siffrinn & Lew, 2018) since high-quality early mathematics instruction serves as a sound foundation for later learning in mathematics (Darling-Hammond & Bransford, 2005). In order to develop students' MDL, elementary school teachers must be proficient in their own MDL.

Studies that assess teachers' knowledge of MDL have been mostly conducted at the secondary level (Colwell & Gregory, 2016; Di Domenico, 2014; Spires et al., 2018) and in other content areas (Cisco, 2016; Feez & Quinn, 2017; Ruzycki, 2015), but it remains unclear what MDL

looks like in the lower grades. Therefore, the purpose of this study was to develop and validate an instrument, titled the Mathematics Discipline-specific Language Scale (MDLS), for assessing the MDL of elementary school teachers. This study was guided by the following research question: What are constructs of MDL for elementary school teachers?
This study addresses the conference themes by resolving the dissonance between what is expected of elementary school teachers' MDL and the lack of the assessment instrument through the process of developing and refining the MDLS.

## Literature Review and Theoretical Frameworks

MDL is a major component of disciplinary literacy (McConachie & Petrosky, 2009; Fang, 2012). Analysis of literature on disciplinary literacy theory provided background information on what is known about MDL of elementary school teachers. Disciplinary literacy is defined as "the use of reading, reasoning, investigating, speaking, and writing required to learn and form complex content knowledge appropriate to a particular discipline" (McConachie & Petrosky, 2009, p. 70). Shanahan and Shanahan (2014) proposed that disciplinary literacy should be introduced as early as elementary school. Therefore, elementary school teachers must develop MDL to make their instruction more accessible (Siffrinn & Lew, 2018).
McConachie and Petrosky's (2009) disciplinary literacy framework defined criteria necessary for students to develop discipline-specific literacy in the core subject areas. This framework suggests the following components of literacy: reading, reasoning, investigating, speaking, and writing. Gee (2012) suggested that language incorporates behaving, interacting, thinking, reading, speaking, and writing. Therefore, McConachie and Petrosky's (2009) speaking and writing components of disciplinary literacy can be categorized as language. In this study, McConachie and Petrosky's (2009) framework was adapted to focus solely on the language component of disciplinary literacy for the discipline of mathematics. The discipline of mathematics has a language of its own that is functional for constructing knowledge and reasoning in the subject. Further, Fang (2012) suggested that different language patterns of mathematics can be categorized as technical, symbolic, and visual. Fang (2012) defines technical language as discipline-specific grammatical features, structure, and vocabulary. Symbolic language is represented by mathematical symbols that are used to describe relationships between mathematical objects. Based on topics covered in elementary school mathematics curriculum, the following symbols are learned in grades K-5: basic operations signs, the plus sign ($+$), the minus sign ($-$), the multiplication signs ($\times$, $\cdot$, or $*$), the division signs ($\div$ or $/$), the relation signs ($=$, $>$, $<$, $\geq$, and $\leq$), the fraction notation ($\frac{\square}{\square}$ or $\square/\square$), the place value signs (decimal point . and coma , ), units of measurement signs (feet ′ and inches ″), grouping symbols (parentheses, brackets, braces), and money signs ($ and ¢). Visual language at this level is represented by number lines, number paths, array and area models, strip diagrams, schematic diagrams, drawings, tables, and graphs. These three categories of MDL were used to classify the patterns of MDL in this study.
Theoretical frameworks need to be tested in practice, and in research that is accomplished through development of research instruments that are consistent with the theory. There are various research instruments that assess MDL of teachers (Stanford: Center for Assessment, Learning, & Equity, 2016; Hill, 2010). However, most of these instruments focus on a single aspect of the language. For example, the Teacher Performance Assessment (edTPA) evaluates pre-service teachers' precision in language, where MDL precision is defined as being accurate

with definitions and symbols in labeling, measurement, and numerical answers (Stanford: Center for Assessment, Learning, & Equity, 2016). The Mathematical Quality of Instruction (MQI) Coding Tool assesses pre- and in-service teachers' explicitness of mathematical terminology and technical language fluency as part of overall instruction quality rather than focusing specifically on quality of MDL (Hill, 2010). In the MQI, fluency is defined as the density of MDL during periods of teacher talk. The MQI also defines explicitness as teachers' accurate use of technical terms. Further, most research instruments focus on the teacher's ability to support language development of students, rather than assessing the teacher's own MDL. In order to assess the quality of teachers' MDL, this study adapted assessment criteria from edTPA and MQI, e.g., precision, fluency, and explicitness for each language category specifically for the MDL of elementary school teachers.

This study's theoretical framework for the development of the MDLS aligned the three assessment criteria of MDL quality for each of the three language categories (Figure 1).

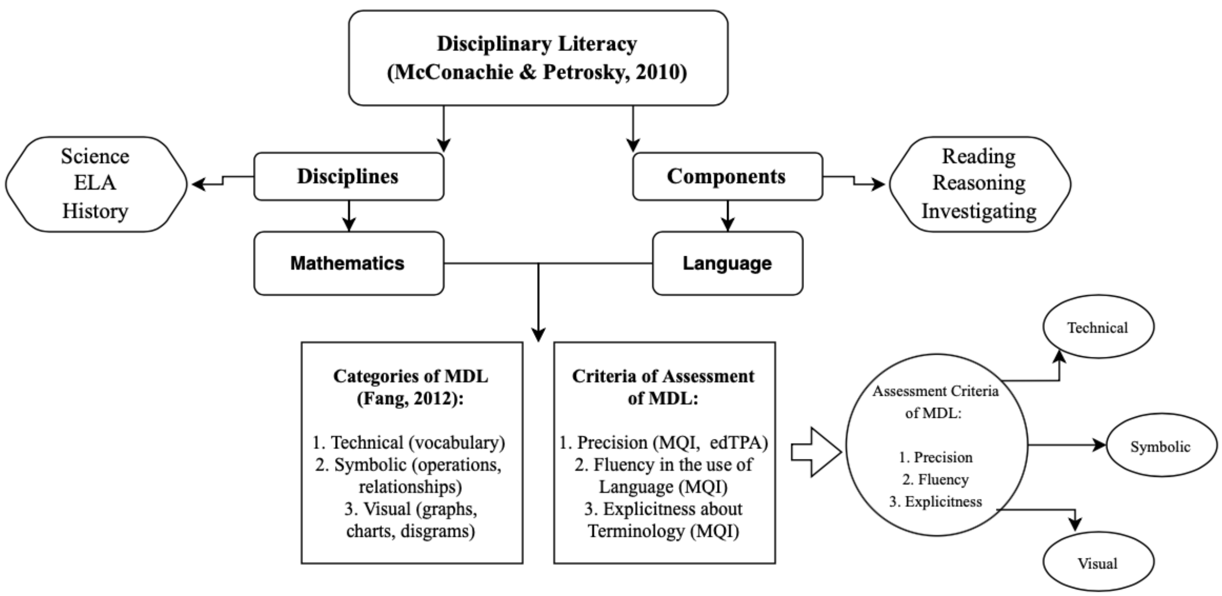


**Figure 1: Framework for Development of the MDLS**

## Methods

Development of MDLS items was guided by the criteria defined by Hathcoat, Sanders, and Gregg (2016). First, category-specific items had to describe directly observed characteristics of MDL in video recordings and corresponding lesson plans. Second, the following practices were used: 1) generating twice as many items as needed, 2) making items simple and specific, and 3) ensuring that items are unidimensional and easy to read.

Explicit phrases that were found in the literature in relation to MDL characteristics were collected and analyzed to generate the preliminary statements for the MDLS items. Further, these statements were revised into performance-based statements. Then, each item was classified into one of the three categories of MDL. A 4-point Likert-like scale (never, rarely, often, always) was used in the MDLS to measure frequency of occurrence for each MDLS item. Iterative process of item revisions at this stage aimed to ensure that each item was observable and independent of others.

The initial version of the MDLS was sent to two mathematics literacy experts to evaluate the content validity. Based on their feedback, wording of several items was revised for clarity. The reliability and construct validity of the MDLS were tested on video recordings and corresponding lesson plans developed by pre-service elementary teachers enrolled in a special education graduate program and collected over the course of six years in grades K-5 mathematics classrooms in urban public schools in Northeast USA.
In order to test inter-rater reliability, two raters were trained to use the MDLS. An initial round of scoring included independent assessment of a set of thirty video recordings and corresponding lesson plans. Pearson correlation analysis was used on this set of scores. Debriefing with the raters was conducted and based on debriefing, additional revisions to wording of items and the structure of Likert-like scale were made. This version of the MDLS was then independently scored by the same two raters on a new set of thirty video recordings and corresponding lesson plans. Inter-rater reliability was tested again using Pearson correlations on this set of scores. Construct validity was tested using a Confirmatory Factor Analysis (CFA). The CFA was conducted in SPSS 28 using varimax rotation with three factors according to the number of categories of MDL defined by the theoretical framework. The goodness-of-fit of this model was tested using SPSS AMOS 28. This procedure led to removal of several items. Internal consistency of the final MDLS was tested using the split-half reliability method in SPSS 28.

## Results

Initial MDLS consisted of 27 items, with nine items in each category (Table 1). Based on the feedback from the experts, the wording of three items in technical language (TL4, TL6, and TL8), two items in symbolic language (SL14 and SL17), and all items in visual language was revised for clarity (see Revision 1 in Table 1).

**Table 1: Iterative Analysis of MDLS**

| Label | Initial Version | Revision 1 | Revision 2 | Final Version |
|---|---|---|---|---|
| TL1 | The teacher states/writes numerical answers that are relevant to the problem's context. | The teacher states/writes numerical answers that are relevant to the problem's context. | The teacher states/writes numerical answers that are relevant to the problem's context. | |
| TL2 | The teacher uses discipline-specific words to provide clear definitions. | The teacher uses discipline-specific words to provide clear definitions. | The teacher uses discipline-specific words to provide clear definitions. | |
| TL3 | The teacher correctly states/specifies units of measurements (when applicable). | The teacher correctly states/specifies units of measurements (when applicable). | The teacher states/specifies units of measurements, or puts meaning to the numerical value. | The teacher states/specifies units of measurements, or puts meaning to the numerical value. |
| TL4 | The teacher correctly uses at least two different terms to describe the same mathematical idea (when applicable). | The teacher correctly uses at least two different terms to describe the same mathematical concept (when applicable). | The teacher makes connections between mathematical concepts. | The teacher makes connections between mathematical concepts. |

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| TL5 | The teacher discusses the meaning of used discipline-specific words. | The teacher discusses the meaning of used discipline-specific words. | The teacher discusses the meaning of used discipline-specific words. | The teacher discusses the meaning of used discipline-specific words. |
| TL6 | The teacher compares and contrasts everyday language with technical language. | The teacher compares and contrasts everyday language with mathematical concepts. | The teacher compares and contrasts everyday language with at least two mathematical concepts. | The teacher compares and contrasts everyday language with at least two mathematical concepts. |
| TL7 | The teacher provides correct explanations. | The teacher provides correct explanations. | The teacher provides correct explanations (when applicable). | The teacher provides correct explanations (when applicable). |
| TL8 | The teacher correctly uses terms to describe mathematical ideas. | The teacher correctly uses mathematical terms to describe concepts. | The teacher uses mathematical terms to describe concepts. | The teacher uses mathematical terms to describe concepts. |
| TL9 | The teacher uses mathematical language as a vehicle for conveying content. | The teacher uses mathematical language as a vehicle for conveying content. | The teacher uses mathematical language as a vehicle for conveying content. | The teacher uses mathematical language as a vehicle for conveying content. |
| SL10 | The teacher uses symbols correctly. | The teacher uses symbols correctly. | The teacher uses symbols correctly (when applicable). | The teacher uses symbols correctly (when applicable). |
| SL11 | The teacher correctly uses the equals sign. | The teacher correctly uses the equals sign. | The teacher correctly uses the equals sign. | |
| SL12 | The teacher uses mathematical notation to provide clear definitions. | The teacher uses mathematical notation to provide clear definitions. | The teacher uses mathematical notation to provide clear definitions. | The teacher uses mathematical notation to provide clear definitions. |
| SL13 | The teacher discusses the meaning of the symbols used. | The teacher discusses the meaning of the symbols used. | The teacher discusses the meaning of mathematical symbols. | The teacher discusses the meaning of mathematical symbols. |
| SL14 | The teacher makes connections between symbols and mathematical ideas. | The teacher makes connections between symbols and mathematical concepts. | The teacher makes connections between symbols and mathematical concepts. | The teacher makes connections between symbols and mathematical concepts. |
| SL15 | The teacher supports the meaning of the equal sign as relational rather than operational. | The teacher supports the meaning of the equal sign as relational rather than operational. | The teacher supports the meaning of the equal sign as relational rather than operational (when applicable). | The teacher supports the meaning of the equal sign as relational rather than operational (when applicable). |

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| SL16 | The teacher uses mathematical symbols when providing explanations. | The teacher uses mathematical symbols when providing explanations. | The teacher uses mathematical symbols when providing explanations. | The teacher correctly uses mathematical symbols when providing explanations. |
| SL17 | The teacher correctly uses symbols to describe mathematical ideas. | The teacher correctly uses symbols to describe mathematical concepts. | The teacher correctly uses symbols to describe mathematical concepts (when applicable). | |
| SL18 | The teacher uses simple, concise language to connect symbols to their meaning. | The teacher uses simple, concise language to connect symbols to their meaning. | The teacher uses simple, concise language to connect symbols to their meaning. | The teacher uses simple, concise language to connect symbols to their meaning. |
| VL19 | The teacher selects visual representations that are appropriate for the structure of the problem. | The teacher selects mathematical models that are appropriate for the structure of the problem. | The teacher selects mathematical models that are appropriate for the structure of the problem. | |
| VL20 | The teacher correctly labels elements of a visual representation. | The teacher correctly labels elements of a mathematical model. | The teacher labels elements of a mathematical model. | |
| VL21 | The teacher correctly converts visually represented information into mathematical notation. | The teacher correctly converts mathematical models, or visually represented information, into mathematical notation. | The teacher converts mathematical models, or visually represented information, into mathematical notation. | The teacher converts mathematical models, or visually represented information, into mathematical notation. |
| VL22 | The teacher states what the visuals represent. | The teacher states what the mathematical models represent. | The teacher states what the mathematical models represent. | The teacher states what the mathematical models represent. |
| VL23 | The teacher links visual representations with quantities in the problem. | The teacher links mathematical models with quantities in the problem. | The teacher links mathematical models with quantities in the problem. | The teacher links mathematical models with quantities in the problem. |
| VL24 | The teacher includes only necessary details in visual representations. | The teacher includes only necessary details in mathematical models. | The teacher includes only necessary information in mathematical models. | |
| VL25 | The teacher correctly uses visual representations to | The teacher correctly uses mathematical models to describe concepts. | The teacher uses mathematical models to describe concepts. | The teacher uses mathematical models to describe concepts. |

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| | describe mathematical ideas. | | | |
| VL26 | The teacher uses discipline-specific words when describing visual representations. | The teacher uses discipline-specific words when describing mathematical models. | The teacher uses discipline-specific words when describing mathematical models. | The teacher uses discipline-specific words when describing mathematical models. |
| VL27 | The teacher uses different visual representations for different types of problems. | The teacher uses different mathematical models for different types of problems. | The teacher uses different mathematical models for the same problem. | The teacher uses different mathematical models for the same problem. |

The Pearson *r*-values for Revision 1 of MDLS ranged from .63 to .86. All items with *r*-values below .7 were discussed with the raters during debriefing and further revisions were made (see Revision 2 in Table 2). The raters also suggested changing the original 4-point Likert-like scale that was based on frequency of occurrence to a 4-point scale based on quality of MDL (not evident, incorrect, somewhat correct, correct). For the scoring with Revision 2 of MDLS, the Pearson *r*-values ranged from .71 to .89 confirming high inter-rater reliability (Asuero, Sayago, & Gonzalez, 2006). The remaining video recordings and corresponding lesson plans were divided between the raters to complete the scoring using this version of MDLS.

In order to confirm the construct validity of MDLS, CFA on the set of 211 video recordings and corresponding lesson plans was completed in SPSS 28 using principal component analysis extraction method with varimax rotation. Loadings less than .42 were suppressed which is consistent with assumption about significant loadings for this sample size (Guadagnoli & Velicer, 1988). Five items (TL1, TL2, VL19, VL20, and VL24) that did not load to their specific categories were removed.

The goodness-of-fit of this three-factor model was examined using maximum likelihood estimation performed in SPSS AMOS 28. The analysis resulted in a significant chi-square, $\chi^2(206) = 658.0$, $p < .001$. The comparative fit index (CFI = .834) and the Tucker-Lewis index (TLI = .813) were both below the accepted values. In addition, the standardized root mean square residual (SRMR = .082) and the root mean square error for approximation (RMSEA = .102) were higher than acceptable. In order to improve the fit of the model, two items (SL11 and SL17) that had very high modification indices were removed. The final model (Figure 2) resulted in a significant chi-square, 2(167) =368.7, $p < .001$, although this can be sensitive to the sample size.

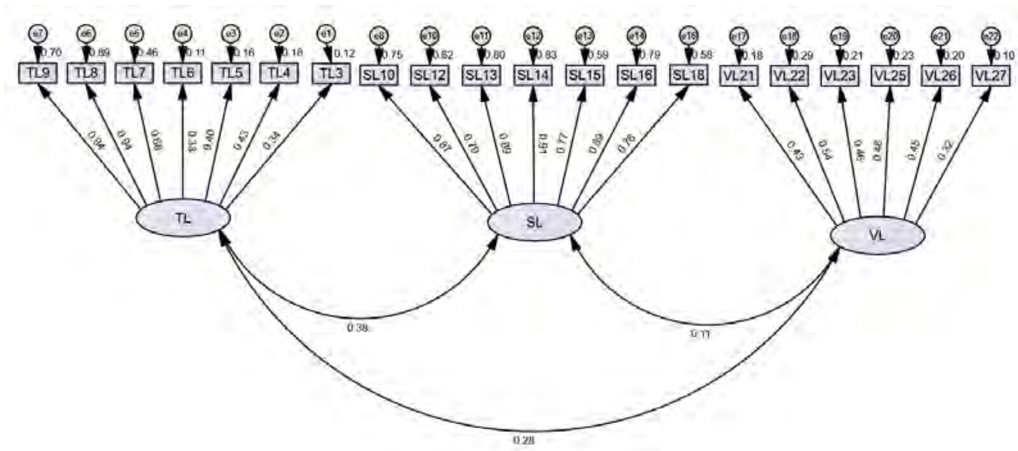


**Figure 2: AMOS Path Diagram for a Final 3-factor Model of MDLS**

The ratio $\chi^2 / df = 2.208 < 3.0$ indicates a good fit (Kline, 2005). The comparative fit index (CFI = .90) shows relatively good fit and the Tucker-Lewis Index (TLI = .886) is just a little below the acceptable value of .9, which suggests a reasonable fit (Bentler, 1990). Moreover, both SRMR = .047 < .08, and RMSEA = .076 < .08 are acceptable. Based on these indices, the three-factor model has a reasonable fit and CFA procedure confirmed the three factors matching the three theoretical categories of MDL.

The final version of the MDLS consists of 20 items with seven items in technical language and symbolic language categories each, and six items in visual language category (Table 1). Cronbach's alpha was used to test internal consistency of the final scale. The value of alpha for symbolic language subscale ($\alpha$ = .954) is excellent, for the technical language subscale ($\alpha$ = .795) is good, and just a little below acceptable for visual language subscale ($\alpha$ = .594) with very good overall value of 0.897 indicating acceptable internal consistency of the MDLS.

## Conclusion

This study developed and validated a quantitative instrument for external assessment of the MDL of elementary school teachers. To the best of our knowledge, this is the first instrument that defines constructs of MDL for elementary school teachers. Another significance of this study is that through validation of MDLS it confirmed three categories of MDL suggested in the theoretical framework by Fang (2012). Thus, this study contributes to the field by bridging the gap between the theoretical definitions of MDL and the practical measurements of MDL in the field. The process of iterative development and analysis of the MDLS items led to the higher clarity of MDL categories and clear distinction between them.

The results of this study have practical implications for teacher education and professional development programs. The MDLS could be used to assess gaps and deficiencies in the MDL of preservice and in-service elementary school teachers and therefore to support their MDL development through focused programs. Further studies are needed to develop better understanding of the visual language category and to analyze how the teachers' MDL influences student learning of mathematics. Although this study made progress in operationalizing different categories of MDL, more work is also needed to operationalize qualities and correctness of MDL. Therefore, future studies will 1) identity levels of the MDLS using the criteria of precision, fluency, and explicitness, and 2) connect these levels to the three categories of MDL.

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