HOW PROPORTIONAL REASONING IS PRESENTED IN U.S. AND KOREAN MATHEMATICS TEXTBOOKS

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The purpose of this study is to examine how proportional reasoning is introduced and developed in two widely used U.S. and Korean mathematics textbooks for grades 6-7. Seven research-based frameworks that identify student learning opportunities for understanding of proportional reasoning were used to analyze the textbooks. The results showed that American textbooks include more problems that require explanations and make use of more effective contextual and number structure of problems than Korean textbooks. In contrast, Korean textbooks make a shift from providing highly contextualized problems to presenting abstract and purely computational problems, which aligns with the process of concreteness fading. In addition, Korean textbooks contain more unique types of topics and representations.

Keywords: Proportional reasoning, textbook analysis, cognitive demands of math tasks

Proportional reasoning has been recognized as a key concept in mathematics for middle grades students to develop (Common Core State Standards Initiative, 2011). However, research indicates that students have considerable difficulties understanding proportional reasoning because they tend to apply additive or subtractive thinking processes rather than multiplicative processes (Karplus, Pulos, & Stage, 1983). Students' difficulties with proportional reasoning may be largely attributed to the quality of their learning environments, such as textbooks that influence what is to be taught and what students learn (Alajmi, 2009; Stigler, Fuson, Ham, & Kim, 1986; Weiss, Pasley, Smith, Sanilower, & Heck, 2003). Given the important role of textbook in mathematics teaching and learning, this study, focusing on the case of proportional reasoning, examines learning opportunities presented in representative American and Korean textbooks. The study aims to compare various aspects of the structure and sequence of the lessons on proportional reasoning, and the characteristics of the problems presented in the lessons in American and Korean textbooks. Specifically, the study addressed the following questions: (1) When and how are ratios, rates, and proportional reasoning introduced and developed in American and Korean textbooks?; (2) What similarities and differences are observed in the content of ratios, rates, and proportional reasoning in American and Korean textbooks?

Theoretical perspectives

Student difficulties and recommended strategies for proportional reasoning

Proportional reasoning has been seen as a cornerstone of secondary mathematics curricula because it is important for understanding of percentages, gradient, trigonometry, and algebra

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(NCTM, 2006). Accordingly, students' concepts of proportion have long been a focus of mathematics education research and have been explored about students' errors and difficulties in relation to proportional reasoning tasks (Lo & Watanabe, 1997). One of the roots of difficulties with proportional reasoning is that students have a weak understanding of the part/whole relationships described in fraction notation. Students often struggle with solving some proportional reasoning problems that involve the use of fraction notations (Norton, 2006).

Research has consistently emphasized students' difficulties with proportion and proportion-related tasks and applications and explored pedagogical ways to improve students' development of proportional reasoning (Behr, Harel, Post & Lesh, 1992; Lo & Watanabe, 1997). First, research has recommended to contextualize problems in real-life situations, as it can activate students' familiar experiences and informal knowledge for sense-making (Resnick & Omanson, 1987). Representing proportion concepts by using various models rather than numbers and symbols may increase students' conceptual understanding. Thus, concreteness fading, which refers to the process of beginning with concrete representations and then fading into more abstract ones, is found to be effective in developing students' conceptual understanding (Goldstone & Son, 2005). In addition, providing problems with high levels of cognitive demand gives students more opportunities to think and reason in given mathematical tasks. Research indicates that using high-level and cognitively complex tasks is important to develop the capacity to think, reason, and solve problems (Stein & Lane, 1996). Furthermore, it is also recommended to provide proportion tasks in a wide range of contextual (e.g., part-part-whole, scaling, well-chunked) and number (e.g., integer or non-integer answers) structures so that students can apply multiplicative thinking into various types of situations (Lamon, 1999). We use these research-based instructional strategies to identify student learning opportunities for understanding of proportional reasoning.

Textbook comparison

Analyses of mathematics textbooks have examined textbooks across countries and have brought many alternatives and insights to the field for improving instruction on challenging mathematical ideas. Prior international comparative studies have shown that curricula in Asian countries contain more tasks that are framed in concrete and real-life situations and provide more cognitively difficult problems, compared to the U.S. For example, Murata (2008) examined the presentation of addition and subtraction in the U.S. and Japanese textbooks and found that Japanese textbooks included more contextualized problems than the US textbooks, which mainly utilized computation problems. Similarly, Ding and Li (2010) compared Chinese textbook series with the two U.S. series on the topic of distributive property and found that the main problem context was computation problems in the U.S. textbooks, whereas it was word problems in the Chinese textbooks. Son and Senk (2010) also compared Korean and American textbooks with standards-based and traditional American textbooks and found that Korean textbooks include more problems that required students to explain than American textbooks. However, some studies have shown inconsistent results [see Fan and Zhu (2007), Li (2000), Hong and Choi (2014), Son and Senk (2010)]. While some studies revealed that American textbooks contained more problems with higher level cognitive demand, problems that required students to provide explanation, and multiple representation than either Chinese or Korean textbooks, other studies reported different findings. Examining whether these findings are consistent with the results of the previous international comparative textbook studies in the present study will enhance the current understanding of what students learn in the U.S. and Korea.

Lischka, A. E., Dyer, E. B., Jones, R. S., Lovett, J. N., Strayer, J., & Drown, S. (2022). Proceedings of the forty-fourth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Middle Tennessee State University.

Methods

Representative and widely used American and Korean textbooks were chosen for this international comparative analysis. For American textbooks, *Eureka Math* or Engage NY modules (EM) (www.engageny.org) were chosen for its popularity (Opfer, Kaufman, & Thompson, 2016). There is only one set of textbook series developed on the national Korean curriculum standards by the Ministry of Education (KM).

The textbook analysis in this study focused on two aspects of textbooks: (1) the structure of the lessons and topics, and (2) the nature of the problems. For the analysis of the textbooks' structure of the lessons and topics, the introduction and development of the concepts of ratio, rate, and proportional reasoning as well as topics arrangement were examined. The analytical framework shown in **Table 1** was utilized to analyze the nature of the problems in depth. The analytical framework consists of the following seven categories: concrete fading (Ding & Li, 2010), cognitive demand (Stein, Smith, Henningsen & Silver, 2000), perspectives (Beckmann & Izsak, 2015; Shield & Dole, 2012; Thompson, 1994), task types (Cramer, Post, & Currier, 1993), contextual and number structure (Lamon, 1993), response types (Charalambous et al., 2010; Mayer et al., 1995), and problem solving difficulty (Hsu & Silver, 2014).

Table 1. Categories and subcategories of analytical framework

Analytical framework	Subcategories
1. Concreteness Fading	Word Problem Visual Representation Word Problem with Visual Representation Abstract
2. Cognitive Complexity	Memorization Procedures without Connections Procedures with Connections Doing Mathematics
3. Perspectives	Multiple Batches Variable Parts
4-a. Contextual Structure	Well-chunked Part-part-whole Associated Sets Stretcher/Shrinker Symbolic
4-b. Number Structure	I-I-I I-W-I I-B-I I-B-N N-B-N N-N-I N-N-N

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4-c. Response Type	Numerical Answers Only Algebraic Expressions Required Explanation Required
5. Problem-solving Difficulty	Easy Medium Hard

Problems are coded as instances, according to the definitions of each analytical framework. After coding, we counted the frequency of all the problems in each subcategory of the analytical frameworks. A Microsoft Excel spreadsheet was used to record the frequency and percentage. The number and percentage of problems that were demonstrated in each subcategory were recorded in the spreadsheet. The counts and percentages of problems were summed, and are reported in the Findings section.

Summary of findings

The nature of problems with ratio, rate, and proportional reasoning

Table 2 presents the total number of problems in both textbooks counted. In total, there are 679 problems and 236 problems in EM and KM, respectively. Further, when the frequency and percentage distribution of total problems were categorized per concept, EM present percent problems most frequently and ratio problems least frequently. In contrast, KM include proportional reasoning problems most frequently and percent problems least frequently.

Table 2. Total frequency of problems in Korean and EM

EM (n=679)						KM (n=236)					
Grade (Module)	Ratio (%)	Rate (%)	Percent (%)	Proportional reasoning (%)	Total	Grade (Vol.)	Ratio (%)	Rate (%)	Percent (%)	Proportional reasoning (%)	Total
6 (1)	80 (34)	109 (47)	45 (19)	0 (0)	234 (100%)	6 (1)	48 (53)	25 (28)	17 (19)	0 (0)	90 (100%)
7 (1)	15 (9)	28 (16)	0 (0)	133 (75)	176 (100%)	6 (2)	5 (6)	0 (0)	0 (0)	80 (94)	85 (100%)
7 (4)	0 (0)	0 (0)	269 (100)	0 (0)	269 (100%)	6 (2)	0 (0)	0 (0)	0 (0)	61 (100)	61 (100%)
	95 (14)	137 (20)	314 (46)	133 (20)	679 (100%)		53 (22)	25 (11)	17 (7)	141 (60)	236 (100%)

Concreteness fading and visual representation types

Table 3 shows the frequency and percentage of concrete and abstract problems in EM and KM. The results showed that both textbooks provide most problems in concrete contexts (e.g.,

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EM: 93% and KM: 81%). For abstract problems, only 7 % of the total problems in EM were situated in purely mathematical contexts, while 19 % in KM were framed in abstract contexts. It may seem that EM used more concrete problems than KM. However, it should be noted that the majority of problems in KM was word problems with visual representation, whereas the greater part of problems in EM was word problems. In KM, there were 45 % word problems with visual representation, 33% word problems, and 3% visual representation problems. In EM, 50 % word problems, 28% word problems with visual representation, and 15 % visual representation problems. This may indicate that KM situate the majority of their problems in concrete situations by using visual representation, while EM contextualize their problems through word problems.

In addition, the process of concreteness fading is not obvious in EM. Although concrete representations outnumbered abstract ones in EM, word problems were used most frequently (50%) followed by word problems with visual representation (28%) and visual representation problems (15%). This trend indicates that the frequency of concrete representation types used may not necessarily indicate the transfer from concreteness to abstractness. In contrast, the frequency of concrete representation types used in KM decreases in the following order: word problem with visual representation (45%) - word problem (33%) - visual representation problem (3%). This may show that there is a concreteness fading process within the concrete problems in KM. Moreover, given that KM contained 45% word problems with visual representation, 33 % word problems, and 19% abstract problems, there was a gradual fading process from concrete to abstract representations across problem types. Research shows that although providing learning opportunities in more concrete representations may activate students' familiar experiences for sense-making (Resnick & Omanson, 1987), making connections between concrete and abstract representations than just using concrete representations is found to be more effective in developing students' conceptual understanding (Goldstone & Son, 2005). This may imply that KM may be more advantageous in facilitating students' conceptual development on such abstract concept as proportional reasoning.

Table 3. The frequency and percentage of concrete and abstract problems

	Concrete				
	Word problem (%)	Visual representation (%)	Word problem with visual representation (%)	Abstract (%)	Total (%)
			EM		
Grade 6 M1	98 (42%)	37 (16%)	70 (30%)	29 (12%)	234 (100%)
Grade 7 M1	65 (37%)	28 (21%)	72 (41%)	2 (1%)	176 (100%)
Grade 7 M4	177 (65%)	28 (10%)	51 (19%)	16 (6%)	269 (100%)
Total	340 (50%)	103 (15%)	193 (28%)	47 (7%)	679 (100%)
			KM		
Grade 6 Vol.1	38 (42%)	5 (6%)	37 (41%)	10 (11%)	90 (100%)
Grade 6 Vol. 2	25 (29%)	1 (1%)	21 (25%)	38 (45%)	85 (100%)
Grade 6 Vol. 2	15 (25%)	1 (1%)	45 (74%)	0 (0%)	61 (100%)
Total	73 (33%)	7 (3%)	98 (45%)	47 (19%)	236 (100%)

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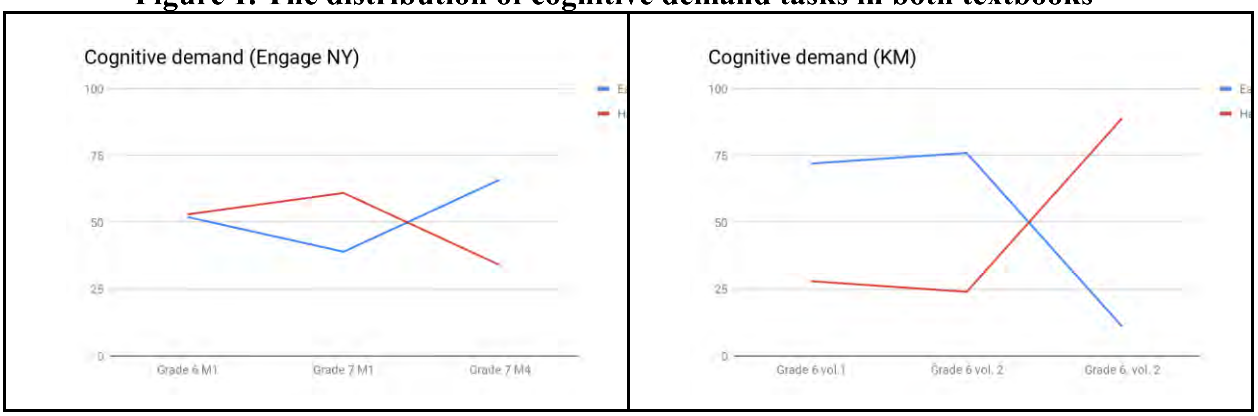
Cognitive demand and problem-solving difficulty

The majority of the tasks is procedures without connections and procedures with connections in both textbooks. Problems with the highest level cognitive demand, doing mathematics, are little represented in both textbooks. EM have a higher percentage of low cognitive demand tasks (54%) than that in KM (42%). This result does not align with the finding from Hong and Choi (2013) reporting that the majority of problems in American and KM require lower level cognitive demand and more than 80% of problems only require simple algorithms or formulas.

Note that EM have more “procedures without connections” (43%) than “procedures with connections” (37%), while KM have more “procedures with connections” (39%) than “procedures without connections” (36%). This may show that KM contain more cognitively challenging problems that require students to use their understanding of concepts and underlying principles and procedures. However, it also should be noted that the problems in KM are generally either a problem with a series of easy subproblems. For example, the subproblems require students to follow at least four steps to complete the problem in EM, while the subproblems in KM ask for one step to complete the problem. Also, based on the analysis of problems in terms of problem solving difficulty, the majority of problems in KM (88%) are easy-level, which requires only one step to complete the problem, whereas EM contain four times more problems that are at least medium-level (49%), which consisted of two or four steps, than KM (12%). This indicates that EM are expected to complete tasks with more steps than Korean students.

Figure 1. The distribution of cognitive demand tasks in both textbooks



Perspectives: Multiple Batches vs Variable Parts

Table 4 illustrates the percentage distribution of problems based on the perspective of multiple batches and variable parts drawn from Beckman and Izsak (2015). A higher percentage of problems that utilized the multiple batches perspective than the variable parts perspective in both textbooks. EM included a higher percentage of problems with the multiple batches perspective (61%) than KM (24%). By contrast, KM included a higher percentage of problems that utilized both the multiple batches and variable parts perspective (36%) than EM (5%). This may show that EM focus on the development of the multiple batches perspective, while KM intend to develop both perspectives. Different from Beckman, the results of our study show that KM, as the curriculum of one of the mathematically high-performing countries, not only utilized the multiple batch perspective, but also the variable parts perspective in developing students' proportional reasoning ability through providing the problems that utilized both perspectives.

Lischka, A. E., Dyer, E. B., Jones, R. S., Lovett, J. N., Strayer, J., & Drown, S. (2022). Proceedings of the forty-fourth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Middle Tennessee State University.

Table 4. Frequency and percentage of multiple batches and variable parts perspectives

Grade	Multiple Batches	Variable Parts	Both	Neither	Total
			EM		
6 (1)	95 (41%)	13 (6%)	29 (12%)	97 (41%)	234 (100%)
7 (1)	93 (53%)	0 (0%)	0 (0%)	83 (47%)	176 (100%)
7 (4)	229 (85%)	7 (3%)	3 (1%)	30 (11%)	269 (100%)
Total	417 (61%)	20 (3%)	32 (5%)	210 (31%)	679 (100%)
			KM		
6 (1)	25 (28%)	0 (0%)	35 (39%)	30 (33%)	90 (100%)
6 (2)	3(4%)	6 (7%)	49 (57%)	27 (32%)	85 (100%)
6 (2)	52 (85%)	0 (0%)	2 (3%)	7 (12%)	61(100%)
Total	80 (24%)	6 (3%)	86 (36%)	64 (27%)	236 (100%)

Contextual and number structure

We further explored contextual and number structure in missing value problems to explore the learning opportunities for proportional reasoning concepts. We found that the main contextual structure of missing value problems in EM is the stretcher/shrinker problems (47%), whereas the symbolic problems (1%) were minimally represented. In KM, the majority of problems are well-chucked (32%) and part-part-whole (30%) problems, while the least frequently represented problems were associated sets (9%). In EM, all four semantic types were evenly utilized in grade 6. The well-chunked and stretcher/shrinker type were most frequently utilized in grade 7. This may suggest that EM began with a balance of all four semantic types and then moved to the stretcher/shrinker problem type in missing value problems. In contrast, KM initially used the stretcher/shrinker problem type, and then heavily relied on using the part-part-whole and the well-chunked problem type. This finding may show that that EM utilized more appropriate contextual structures of their missing value problems than KM, as their students develop conceptual understanding of proportional reasoning, based on the level of difficulty.

Table 5. Percentage of contextual structure of missing value problems in both textbooks

Grade (Module)	Well Chunked (%)	Part-Part-Whole (%)	Associated Sets (%)	Stretcher /Shrinker (%)	Symbolic (%)	Total (%)
			EM (n=192)			
6 (1)	28 (32)	18 (21)	18 (21)	20 (23)	3 (3)	87 (100)

Lischka, A. E., Dyer, E. B., Jones, R. S., Lovett, J. N., Strayer, J., & Drown, S. (2022). Proceedings of the forty-fourth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Middle Tennessee State University.

7 (1)	29 (60)	5 (11)	1 (2)	13 (27)	0 (0)	48 (100)
7 (4)	0 (0)	0 (0)	0 (0)	57 (100)	0 (0)	57 (100)
Total	57 (30)	23 (12)	19 (10)	90 (47)	3 (1)	192 (100)
KM (n=90)						
Grade (Vol)	Well Chunked (%)	Part-Part-Whole (%)	Associated Sets (%)	Stretcher /Shrinker (%)	Symbolic (%)	Total (%)
6 (1)	1 (5)	7 (31)	1 (5)	13 (59)	0 (0)	22 (100)
6 (2)	6 (14)	20 (45)	6 (14)	3 (7)	9 (20)	44 (100)
6 (2)	22 (92)	0 (0)	1 (4)	1 (4)	0 (0)	24 (100)
Total	29 (32)	27 (30)	8 (9)	17 (19)	9 (10)	90 (100)

Discussion and Implications

This study showed notable similarities but also striking differences between EM and KM. The goal of cross-cultural comparison is to know in what measure the learning opportunities provided by textbooks get enacted in classroom practice and student learning. Our findings indicated that how this comparative study of textbooks may contribute insights to improve the learning environments of proportional reasoning. Korean textbooks' emphasis on the process of concreteness fading and skillful use of high level cognitive demanding problems are consistent with prior findings that Asian students are involved in more meaningful and desirable material to learn mathematics. Korean approaches in developing students' explicit understanding, such as the unique construction of lessons, may be helpful for textbook designers in America and other countries. Our study also showed that EM provide more opportunities for students to solve mathematics problems with complex number structures, but also to explain and reason about the problems than KM. EM encourage their students to be independent in solving mathematics problems by asking them to create visual representations to justify their reasoning. These findings seem to conflict with the findings that Asian textbooks present more problems requiring explanation and problems with multiple visual representations than American textbooks. Developers of KM can benefit from EM' approaches in using various strategies, such as a wider range of number structures and visual representations, and stressing more critical thinking.

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