AN ANALYSIS OF PATTERNS OF PRODUCTIVE AND POWERFUL DISCOURSE IN MULTILINGUAL SECONDARY MATHEMATICS CLASSROOMS

UN ANÁLISIS DE DISCURSO PRODUCTIVO Y PODEROSO EN AULAS MULTILINGÜES DE MATEMÁTICAS DE SECUNDARIA

William Zahner
San Diego State University
bzahner@sdsu.edu

Lynda Wynn
lwynn@csustan.edu
California State University Stanislaus

Ernesto Daniel Calleros
San Diego State University & University of California San Diego
ernestocalleros@gmail.com

Kevin Pelaez
San Diego State University and University of California San Diego
kpelaez10@gmail.com

We present an analysis of three 9th grade integrated mathematics lessons in which a group of teachers and researchers redesigned a sequence of lessons with the goal of engaging a linguistically diverse group of students in productive and powerful discussions (Herbel-Eisenmann et al. 2013). The three lessons were part of a design experiment. Two lessons were observed during regular school day instruction, and the other lesson was part of an after-school teaching experiment. Drawing on a sociocultural framework and methods of classroom discourse analysis (Cazden, 2001; Pierson, 2008), we analyze how the teachers in the three settings structured whole-class discourse to create opportunities for a multilingual group of students to participate in the discussion and to appropriate mathematical tools for thinking.

Keywords: Equity, Inclusion, and Diversity; Design Experiments; Algebra and Algebraic Thinking; Classroom Discourse

In this paper, we describe results from a design research effort in which a group of teachers and researchers redesigned a sequence of lessons with the goal of engaging ninth graders in academically productive whole-class discussions (Herbel-Eisenmann et al., 2013). The research was situated in a US school where nearly all students were from minoritized communities, most students were multilingual, and about 30% of students were designated as English Learners (ELs). Drawing on the Academic Literacy in Mathematics framework (Moschkovich, 2015), project-specific design principles (Zahner et al., 2021a, 2021b), and research on student learning of linear rates of change (Lobato & Ellis, 2010; Thompson, 1994), we created a sequence of lessons specifically designed to promote student participation in classroom discussions about linear rates of change. In the empirical results below, we show how, in comparison to student engagement before the design intervention, the redesigned lessons led to increased student participation and higher levels of cognitive work in whole class mathematical discussions.

Theoretical Framework & Prior Research

This study is rooted in a sociocultural perspective on learning, where mathematics learning is conceptualized as appropriating problem-solving tools and developing participation in valued mathematical practices, including mathematical discourse practices (Forman, 1996; Moschkovich, 2002). In alignment with this theoretical focus, our analyses focus on classroom discussions and patterns in teacher and student discourse. We also drew upon the Academic

Olanoff, D., Johnson, K., & Spitzer, S. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

Literacy in Mathematics (ALM) framework (Moschkovich, 2015) to create study-specific design principles (Figure 1). In the ALM framework, developing academic literacy in mathematics includes developing forms of mathematical *proficiency*, engaging in mathematical *practices*, and participating in mathematical *discourse*.

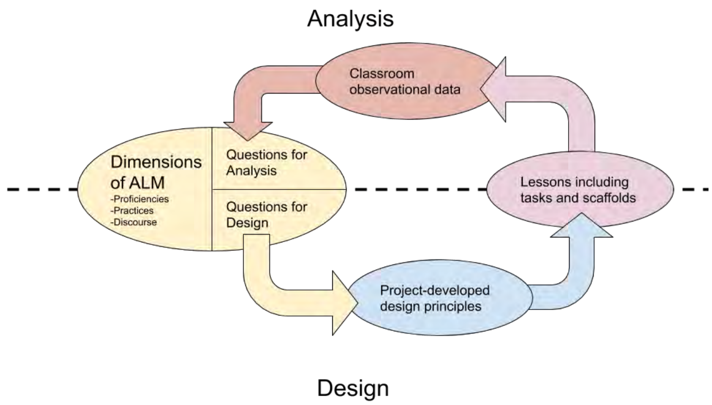

Figure 1: Overview of the Design Cycle. (Image credit Authors year, used with permission)

Multilingual students, including those learning the language of instruction, can learn critical mathematical concepts and reasoning practices through engaging in productive and powerful discussions (Chapin & O'Connor, 2012; Erath et al., 2018; Erath et al., 2021; Gutiérrez, 2002; Khisty & Chval, 2002; Moschkovich, 1999). Following Herbel-Eisenmann et al.'s (2013) definitions, *productive* discussions are those that lead students to appropriate mathematical tools for thinking (i.e., develop mathematical proficiencies and practices in the ALM framework). *Powerful* discussions are those that promote students' participation in and identification with mathematics (i.e., engage in the disciplinary discourse). One way to foster productive and powerful discussions is for students and teachers to engage in authentic dialogue focused on important mathematical concepts (e.g., O'Connor, 2001). Yet, most multilingual students who are classified as English Learners have very limited opportunities to engage in productive and powerful classroom discussions. Instead, these students are often tracked into low-level classes where they have little access to either rich content learning opportunities or the discourse of the discipline (Callahan, 2005; Kanno & Kangas, 2014). Thus, one critical question facing the field is how to transform patterns of classroom discourse in multilingual settings.

The default template for most classroom discussions is the triadic Initiation-Response-Evaluation (IRE) sequence (Cazden, 2001). Initiations are questions or statements. Responses occur after, and in response to, an initiation. Evaluations are moves that offer judgement–either explicit or implicit–on the response and end the IRE sequence (Cazden, 2001; Mehan, 1979). IRE-dominated instruction typically positions the teacher as the authority, thereby limiting students' opportunities to engage in productive and powerful discussions. In this project our goal was to transform patterns of discourse. Yet, we found that, while the ideal transformation of classroom talk may be to create dialogic discussions like the one in O'Connor (2001), achieving such transformation is challenging, possibly due to institutional constraints and the deeply ingrained patterns of discourse in school mathematics (Herbel-Eisenmann et al., 2010).

Short of a wholesale transformation, there are subtle ways to document transformations in classroom talk while still within the IRE framework. For example, it is possible to distinguish between evaluation and follow-up moves in the "third slot" of the IRE sequence (Wells, 1993).

Follow-up moves include asking students to expand on their reasoning, presenting new examples to build on their contributions, and asking for clarification (Pierson, 2008; Wells, 1993). While evaluation moves typically close a sequence, follow-up moves can extend the discussion. When students respond to a teacher's follow-up move, they often provide a justification or explain the reasoning behind their answer – two examples of disciplinary discourse practices (Moschkovich & Zahner, 2018).

Pierson's (2008) analysis of teacher and student talk in 13 seventh-grade mathematics classrooms offers an avenue for unpacking more and less productive uses of talk within the triadic IRE framework. Pierson (2008) developed two coding schemes: (a) one to capture the level of a teacher's *responsiveness*, the extent to which the teacher focused on student thinking, and (b) a second focused on the level of *intellectual work*, the kind of cognitive effort imposed on or requested from students within a teacher's move. Pierson found a positive relationship between more responsive teacher moves and higher levels of intellectual work in teacher talk with growth in students' mathematics achievement as measured by a curriculum-aligned assessment.

The constructs of intellectual work and responsiveness connect to this project's goal of promoting productive and powerful discussions. In a productive discussion, we would expect to see higher levels of intellectual work. In a powerful discussion, we would expect to see higher levels of responsiveness as teachers take up and build upon students' ideas. Thus, we adopted Pierson's (2008) coding schemes to explore whether our design efforts were effective in promoting powerful and productive talk in linguistically diverse classrooms.

In the analysis that follows, we address the following research question: To what extent did each lesson engage multilingual students in productive and powerful discussions? Specifically,

1. What were the levels of intellectual work and responsiveness in teacher moves?
2. What was the distribution and frequency of student participation in whole-class discussion?

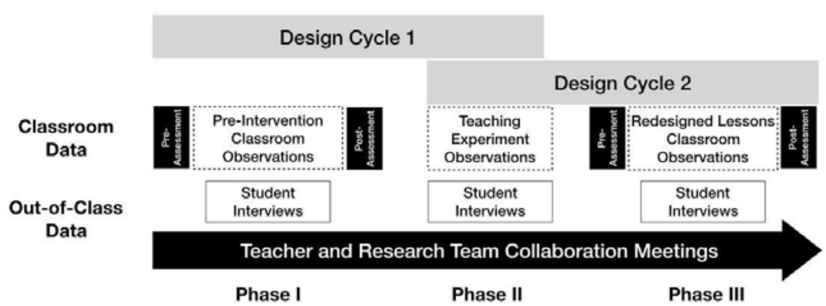

Figure 2. Design Cycles (Image credit Authors year, used with permission)

Methods

The overall framework for this research arose from design research (Cobb et al. 2003). While researchers have identified productive practices in multilingual classrooms (e.g., Chval & Chávez, 2012; Chval et al., 2021), these productive practices appear to be relatively rare in linguistically diverse mathematics classrooms (Callahan, 2005). Therefore, design research was

Olanoff, D., Johnson, K., & Spitzer, S. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

chosen as a method for studying phenomena unlikely to arise without intervention (Cobb et al., 2003). The project included two design cycles spread across three academic years (Figure 2).

Setting

We present data from three lessons recorded in ninth grade Integrated Mathematics 1 (IM1) classes at City High, an urban high school located near the US-Mexico border serving linguistically diverse students. The school was chosen as a research site in order to situate this research in a setting that parallels the inequitable educational experiences of minoritized students in US schools, particularly students who are classified as English Learners (Gándara & Contreras, 2009). At City High, 77% of students were identified as Latinx, 12% Asian, 7% African American, and 4% other. About 89% of students were from low-income families. Thirty percent of all students at City High were classified as ELs. Three City High IM1 teachers joined our research group in redesigning a unit on linear rates of change for Design Cycles 1 and 2.

The Phase I data includes eight class meetings recorded during regular school hours. These lessons were taught by the students' regular teacher, Mr. S, who was certified to teach mathematics and who had taken teacher education coursework related to teaching ELs. Mr. S was bilingual in Spanish and English. He primarily spoke in English during class, and he talked to some students in Spanish during small group work. The Phase II data includes ten lessons taught after school in a Teaching Experiment (TE) setting. The TE lessons were designed by the teachers and researchers. A bilingual researcher with experience facilitating classroom discussions with linguistically diverse students taught the Phase II TE lessons while Mr. S and the other teachers and researchers served as observers. The Phase III data includes ten redesigned lessons taught by Mr. S during regular school hours. In the analysis presented below, we focus on one lesson from each phase, each chosen for analysis because they feature a pivotal concept in the design experiment unit—introducing average rate of change. All whole-class discussions and talk among one small group of students were transcribed. Further, students were invited to participate in the language of their choice across all three phases. This invitation was made explicit in Phase II. In Phases I and III, students could use the language of their choice, but this option was not emphasized. In this analysis, we narrow our focus to whole-class discussions.

Redesign

The design principles and illustrations of the redesigned lessons are presented in Authors (Zahner et al. 2021a, 2021b). In brief, the main foci of the redesign effort were developing student participation in productive and powerful discussions through (a) adopting a coherent mathematical focus across the unit and strategically using problem contexts, (b) designing a unit of lessons with intentionally integrated mathematical and language development goals, and (c) integrating language and discourse supports including technology and mathematical language routines (Zwiers, 2017) throughout the unit.

Analysis

To start our analysis, we coded the transcripts as whole-class and small group interactions, noting the time spent in each participation structure. We noted that in the Phase I lesson (Pre-intervention), 41 of 48 minutes (~85%) were whole-class interactions, 36 of 63 minutes (~57%) in Phase II (TE), and 46 of 77 minutes (~60%) in Phase III (Redesigned lesson). We then examined discourse patterns during these whole-class interactions, coding each teacher- and student-turn of talk as I, R, E, or F, allowing up to two codes per turn of talk since teachers often offer an evaluation and then initiate a new question in one turn.

Next, we used Pierson's (2008) responsiveness and intellectual work coding schemes for analyzing each teacher's talk during whole-class discussions. Pierson's (2008) responsiveness coding scheme categorized each teacher follow-up move (F) into one of four levels of responsiveness: Low, Medium, High I, or High II. The level of each follow-up was determined based on whether the move addressed a student's comment, whose idea (teacher's versus student's) was the focus, and whose reasoning (teacher's versus student's) was displayed. Pierson's (2008) coding scheme for intellectual work had four categories: Low Give, High Give, Low Demand, and High Demand. The two Give codes were for teacher moves in the third slot (E/F) of an IRE/F sequence of talk that provided information, whereas the two Demand codes were for teacher moves that requested information from the students. Teacher moves that both supplied and requested information were double-coded with a Give and a Demand category. The designations Low and High for Give and Demand codes depended on the type of information being supplied or requested in a teacher's move. Low was for basic information, whereas High was for more elaborate information intended to extend mathematical reasoning.

Consistent with Pierson's coding mechanism, we only coded talk with a math focus (e.g., we did not code segments of classroom management). We coded every instance of talk in the third slots (E/F) of the IRE/F sequences as either Low, Medium, High I or High II responsiveness. We expanded Pierson's intellectual work coding scheme to include both the first or third slots (I or E/F) of the IRE/F sequences as either Low Give, High Give, Low Demand, or High Demand, allowing for double-coding of single turns with both Demand and Give codes when applicable. We chose to include the first slot (I) of the sequence because we were not working with predetermined questions as the teachers were in Pierson's study. At each stage, coding was done by one researcher and then the research team met and reviewed the coding to discuss each code, consider questions, and reach consensus.

Results

Evaluation and Follow up Moves

During the Phase I lesson, Mr. S's most common move during the third slot of the IRE sequence was evaluation, occurring in 94% of coded moves. After noticing this trend, we made transforming the pattern of IRE discourse a target of our design efforts. Our aim was to encourage teachers of the Phase III lessons to use more follow-up moves, such as pressing for reasoning or asking students to elaborate on an idea (Chapin et al., 2009). This pattern of discourse was modeled during the Phase II TE lessons, during which 53% of the researcher's E/F moves were follow-ups. As indicated in Figure 3, this form of discourse appeared to be taken up by Mr. S in the Phase III lesson, where 36% of Mr. S's turns in the third slot of triadic IRE discourse were follow-up moves rather than evaluations.

The level of intellectual work coded in the teachers' talk was relatively consistent across Phase I, Phase II, and Phase III. In general, the majority of teacher moves (both Gives and Demands) were coded as Low. Many of the Low Demand turns were questions that had a known answer and that could be answered without offering an explanation or justification. The Low Give moves included providing information without explanation or justification. Figure 4 shows a summary of Intellectual Work across the three classrooms.

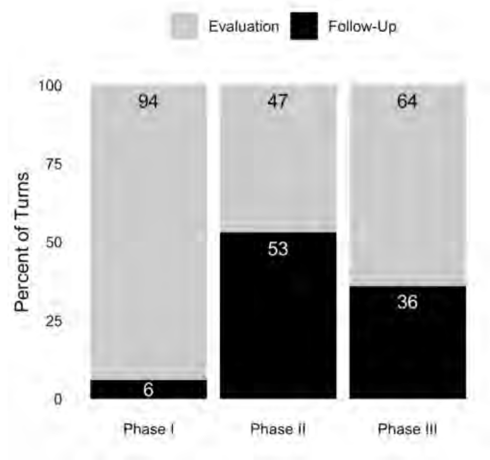

Figure 3. Percent of "third slot" turns that were evaluation or follow-up

Intellectual Work

Despite this consistency, there was one notable shift in the level of intellectual work in the Demand category. During the Phase I lesson, 14% of the teacher's questions were coded as High Demand. This increased to 29% in the Phase III lesson. Thus, while the overall proportion of High Demand turns in each lesson remained under 30%, the proportion of High Demand moves doubled from the Phase I to the Phase III lesson.

There was a complementary shift in the Give category. The proportion of High Give moves decreased from 13% in the Phase I lesson to only 4% in the Phase III lesson. One possible interpretation of this unexpected decrease can be attributed to the teacher using talk moves (Chapin et al., 2009) such as rebroadcasting student input rather than providing high-level explanations. Following the coding scheme, rebroadcasting moves were coded as Low Give. In a sense, Mr. S may have been trying to shift the authority to the students by not engaging in lecture, which decreased the level of Give moves.

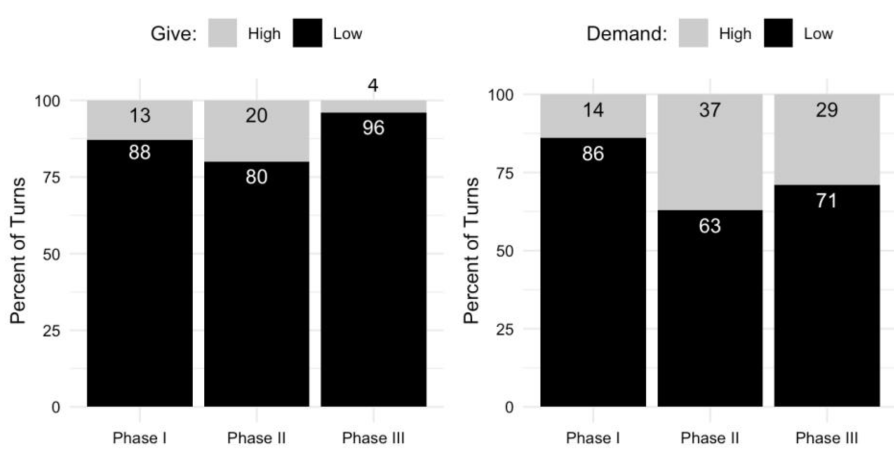

Figure 4. Intellectual Work (Give and Demand) across the three learning environments

Olanoff, D., Johnson, K., & Spitzer, S. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

Responsiveness

In parallel with the results for intellectual work, the coding for the level of responsiveness of the teachers' discourse showed two trends. First, across all three lessons, the majority of the teachers' talk was coded at a low level of responsiveness. Second, despite this trend, there was also a notable increase in the proportion of turns coded at the high level of responsiveness. Over half of Mr. S's talk in the Phase I and Phase III lessons were coded Low in responsiveness (see Figure 5). However, the proportion of teacher talk that was coded as high in responsiveness (combining the categories High I and High II) increased from 14% to 25%. With this increase, the pattern of responsiveness in the teacher's talk in the Phase III lesson was relatively similar to the pattern of discourse in the TE lesson.

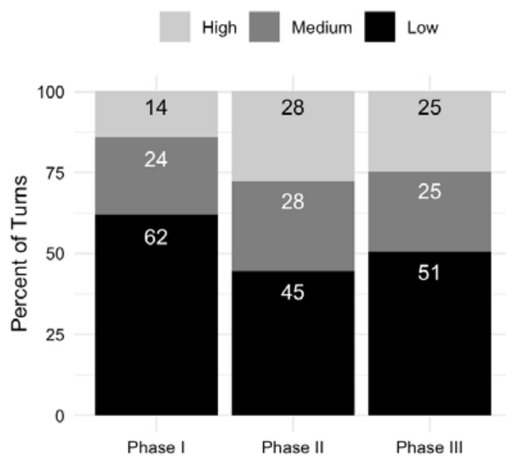

Figure 5. Coding Results for Responsiveness

Distribution and Frequency of Student Participation

Recall that powerful discussions are those which build students' identification with mathematics and broaden student participation (Herbel-Eisenmann et al., 2013). Thus, we were curious about who was contributing and how often in the whole-class discussions. Looking at the patterns of who talked, we were able to characterize the proportion of the talk by the teacher versus students in the whole-class setting. We were also able to identify how many unique students made a contribution to the whole-class discussion. Table 1 shows a total count of the number of coded teacher and student turns, the unique number of students who were called upon by name to contribute to the whole-class discourse, and the number of times each of the called upon students contributed to the whole-class talk.

Table 1. Patterns in Classroom Discussion Participation

	Phase I	Phase II	Phase III
Coded Turns	345	497	447
Students present	28	12	33
Number of unique student speakers called by name	7	12	10
Average turns per identified student	2.5	24	7.3
Number of choral responses	55	7	40
Unidentified student turns	109	0	89
Teacher turns	163	202	230

Olanoff, D., Johnson, K., & Spitzer, S. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

Looking across the two lessons recorded during regular school hours (Phase I and Phase III), we saw approximately the same proportion of students who were called upon to participate: seven out of 28 students (25%) in the Phase I lesson and 10 out of 33 (~30%) in Phase III. Yet, comparing these lessons, one striking trend was that the students who contributed spoke an average of 7.3 times in the Phase III lesson and only an average of 2.5 times in the Phase I lesson. Thus, for the students who were called on to participate, the average number of turns per student was higher in the Phase III lesson than in the Phase I lesson. As might be expected for the much smaller class in the Phase II TE, all 12 of the students who were present were called upon to contribute to the whole-class discourse, and students tended to have more frequent contributions.

In addition to the identified student turns in whole-class talk during the Phase III lesson, students also had multiple opportunities to talk during small group discussions: a substantially higher proportion (38%) of class time was devoted to group work (compared to only 15% in the Phase I lesson). Therefore, the counts of student turns presented in Table 1 underreport the amount of student talk in the Phase III lesson. Our analysis also revealed that during the designated small group times in the Phase I lesson, very little time was dedicated to group discussion and was actually used for individual seat work. Therefore the counts in Table 1 are more likely reflective of the total number of student turns in the Phase I lesson.

Discussion

The trends we identified in the levels of intellectual work in these three lessons lead to questions about what may explain the relatively frequent incidence of low give and low demand during Phases II and III. One possible explanation is that repeating, one of the talk moves from Chapin et al., (2009), was coded as a low level of intellectual work. Yet, we wondered if this coding accurately captured the effects of these moves. Alternatively, the teachers of these lessons used fewer High Give responses to provide space for the students to supply explanations and needed information to each other. In this respect, we consider this trend from decreasing High Give evaluations or follow-ups to increasing the number of High Demand evaluations or follow-ups as a signal of providing students with more mathematical authority. Finally, an additional alternative to consider is that, in our redesigned lessons, some evaluation and feedback was built into the Desmos activities we created for these lessons, which may have reduced the need for the teacher to voice these moves and be picked up in our analysis.

Considering teacher responsiveness to student thinking, we found that our Phase II and Phase III lesson designs helped the teachers increase the proportion of medium and high levels of responsiveness to student thinking. While we would like to have seen this improve even more, we found that change to classroom discourse in this setting has been gradual. Recall that each phase occurred in a different school year. The ability to document a change in teaching practice even after several months had passed since the TE intervention is noteworthy, and speaks to the potential of our redesigned lessons to support teachers in engaging multilingual students in mathematical discourse.

In our next analyses, we plan to look more closely at the small group interactions that take place both during designated group work segments and those side conversations that take place in small groups during the whole-class discussion. Our preliminary analyses indicate that much more mathematical discussion is happening student-to-student than one finds when focused on the whole-class discussions.

Acknowledgments

This material is based upon work supported by the National Science Foundation under Grant No. (1553708). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

References

Callahan, R. M. (2005). Tracking and high school English learners: Limiting opportunity to learn. *American Educational Research Journal, 42*(2), 305–328.

Cazden, C. B. (2001). *Classroom discourse: The language of teaching and learning* (Second Edition). Portsmouth, NH: Heinemann.

Chapin, S., & O'Connor, M. (2012). Project Challenge: Using challenging curriculum and mathematical discourse to help all students learn. In C. Dudley-Marling & S. Michaels (Eds.), *High-expectation curricula: Helping all students succeed with powerful learning* (pp. 113–127). Teachers College Press.

Chapin, S., O'Connor, M., & Anderson, N. (2009). *Classroom discussions: Using math talk to help students learn* (2nd ed.). Math Solutions.

Chval, K. B., & Chávez, Ó. (2012). Designing math lessons for English language learners. *Mathematics Teaching in the Middle School, 17*(5), 261–265. https://doi.org/10.5951/mathteacmiddscho.17.5.0261

Chval, K. B., Smith, E., Trigos-Carrillo, L., & Pinnow, R. J. (2021). *Teaching math to multilingual students, grades K-8: Positioning English learners for success*. Corwin.

Cobb, P., Confery, J., DiSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*, 9–13.

Erath, K., Ingram, J., Moschkovich, J. N., & Prediger, S. (2021). Designing and enacting instruction that enhances language for mathematics learning – A review of the state of development and research. *ZDM Mathematics Education, 53*(2) DOI .

Erath, K., Prediger, S., Quasthoff, U., & Heller, V. (2018). Discourse competence as important part of academic language proficiency in mathematics classrooms: The case of explaining to learn and learning to explain. *Educational Studies in Mathematics*, 99(2), 161–179.

Forman, E. (1996). Learning mathematics as participation in classroom practice: Implications of sociocultural theory. In L. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), *Theories of mathematics learning* (pp. 115–130). Mahwah, NJ: Lawrence Erlbaum.

Gándara, P. C., & Contreras, F. (2009). *The Latino education crisis: The consequences of failed social policies*. Harvard University Press.

Gutiérrez, R. (2002). *Beyond essentialism: The complexity of language in teaching mathematics to Latina/o students.* American Educational Research Journal, 39, 1047–1088.

Herbel-Eisenmann, B. A., Steele, M. D., & Cirillo, M. (2013). (Developing) teacher discourse moves: A framework for professional development. *Mathematics Teacher Educator, 1*(2), 181–196.

Herbel-Eisenmann, B. A., Wagner, D., & Cortes, V. (2010). Lexical bundle analysis in mathematics classroom discourse: The significance of stance. *Educational Studies in Mathematics*, *75*, 23–42.

Kanno, Y., & Kangas, S. E. N. (2014). "I'm not going to be, like, for the AP": English language learners' limited access to advanced college-preparatory courses in high school. *American Educational Research Journal, 51*(5), 848–878.

Khisty, L. L., & Chval, K. B. (2002). Pedagogic discourse and equity in mathematics: When teachers' talk matters. *Mathematics Education Research Journal*, *14*, 4–18.

Lobato, J., & Ellis, A. B. (2010). *Developing essential understanding of ratios, proportions, and proportional reasoning for teaching mathematics: Grades 6-8.* National Council of Teachers of Mathematics.

Mehan, H. (1979). *Learning lessons: Social organization in the classroom*. Cambridge, MA: Harvard University Press.

Moschkovich, J. N. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics, 19*, 11–19.

Moschkovich, J. N. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning, 4*, 189–212.

Moschkovich, J. N. (2015). Academic literacy in mathematics for English learners. *The Journal of Mathematical Behavior*, *40*, 43-62.

Moschkovich, J., & Zahner, W. (2018). Using the academic literacy in mathematics framework to uncover multiple aspects of activity during peer mathematical discussions. *ZDM Mathematics Education*, *50*(6), 999–1011. https://doi.org/10.1007/s11858-018-0982-9

O'Connor, M. C. (2001). "Can any fraction be turned into a decimal?": A case study of a mathematical group discussion. *Educational Studies in Mathematics, 46*, 143–185.

Pierson, J. (2008). *The relationship between teacher follow-up moves and mathematics learning.* [Doctoral Dissertation]. University of Texas.

Thompson, P. W. (1994). The development of the concept of speed and its relationship to concepts of rate. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 179–234). SUNY Press.

Wells, G. (1993). Reevaluating the IRF sequence: A proposal for the articulation of theories of activity and discourse for the analysis of teaching and learning in the classroom. *Linguistics and Education, 5*(1), 1–37. https://doi.org/10.1016/S0898-5898(05)80001-4

Zahner, W., Calleros, E. D., & Pelaez, K. (2021a). Designing learning environments to promote academic literacy in mathematics in multilingual secondary mathematics classrooms. *ZDM – Mathematics Education*, *53*(2), 359–373. https://doi.org/10.1007/s11858-021-01239-0

Zahner, W., Pelaez, K., & Calleros, E. D. (2021b). Principles for curriculum design and pedagogy in multilingual secondary mathematics classrooms. In A. A. Essien & A. Msimanga (Eds.), *Multilingual education yearbook 2021* (pp. 235–255). Springer International Publishing. https://doi.org/10.1007/978-3-030-72009-4_13

Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). *Principles for the design of mathematics curricula: Promoting language and content development.* Stanford University UL/Scale. http://ell.stanford.edu/content/mathematics-resources-additional-resources