

DOING MATH IN THE DIGITAL AGE: AN ANALYSIS OF ONLINE MATHEMATICS PLATFORMS

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We present a typology for characterizing online student-facing mathematics platforms that examines how they position students as learners, exemplified by 9 commonly used platforms. We identify three types of student learning experiences: instruction and practice, practice and support, and conceptual games and activities, and describe each one in terms of the relationships among instructional guidance, student agency, and the mathematical rigor of tasks. We find that within and across categories, there is substantial variation in cognitive demand and student agency, offering implications for further research, school decision makers and platform designers.

Keywords: elementary school education, instructional activities and practices, technology, curriculum.

Introduction

Increasingly, teachers and school systems in the U.S. are using a range of digital resources and tools to supplement regular mathematics instruction (Choppin, et al., 2014; Kauffman, et al., 2020). In this paper, we look closely at one type of resource that we call *online student-facing platforms* (OSFPs). These platforms represent an array of software programs, such as Dreambox, IXL, or Zearn, that have what Choppin et al. (2014) refer to as “individual learning designs.” We present an analysis of 9 commonly used mathematics OSFPs and propose a typology for characterizing the nature of the learning experiences they offer elementary students.

Our interest in OSFPs has arisen out of evidence that students in the U.S. are using them in increasing numbers and as a substantial portion of their weekly mathematics learning. In our own interview study of elementary teachers’ use of curriculum resources (Remillard et al., under review), 9 out of the 10 teachers in the U.S. reported assigning OSFPs at least once/week. In 3 cases, students used them 3 to 5 hours a week during dedicated periods and 5 teachers reported that OSFP use was dictated by school policy. These platforms can be understood as an often overlooked, but significant, component of mathematics learning. From this perspective, our analysis aims to consider the types of mathematics learning experiences available to students in OSFPs in U.S. elementary schools.

In order to achieve this goal, we draw on frameworks typically used to examine classroom instruction and features of digital resources. While research on digital tools often separates out components, we consider how components work together to frame comprehensive student learning experiences. Building on analyses of similar platforms in Choppin et al. (2014), Kay

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and Kwak (2018), and Cayton-Hodges et al. (2015), we consider how these platforms frame and structure teaching and learning interactions.

Our study contributes to what is known about OSFPs in a number of ways. First, by analyzing them using a teaching and learning framework, we suggest that students' experiences with supplemental tools matter for their mathematics learning. We do not assume that OSFPs should function in the same way or accomplish the same goals as classroom instruction, and we understand them to be supplemental in nature and useful for particular instructional goals. Nevertheless, given the increasingly extensive use of these platforms, understanding their affordances and constraints is important for teachers and researchers when considering their impact on students. Second, OSFPs are often thought of as similar in design and approach. By making visible the mathematical and pedagogical positions of some of the most commonly used OSFPs, our analysis demonstrates the diversity of these resources.

Background and Theoretical Framing

Our theoretical approach integrates frameworks that inform research and practice in mathematics teaching and learning with research on design components of digital learning tools that impact students' mathematical learning opportunities.

Summaries of research on mathematics teaching and learning speak to four critical domains of importance: a) mathematics content, b) types of learning experiences, c) learners' dispositions, and d) the role of teacher. There is general agreement that ideal *mathematics content* should be rigorous, meaning it should integrate procedural knowledge with conceptual understanding and opportunities for application of knowledge to familiar and novel contexts (NRC, 2001; Stein, et al., 1996). Recall of facts and fluency with procedures are both necessary but are only useful when connected with meaningful applications. We also know that *student learning* is increased when students are actively involved in tasks that require them to think through problems, make decisions on how to solve them, monitor their progress, and struggle with mathematical tasks (Hiebert and Grouws, 2007). These types of problem-solving experiences support students in developing *productive dispositions*, which include how students see themselves as mathematics learners in terms of identity, mindset, and agency (Boaler, 2016; Jackson, 2009; NRC, 2001). Research on the *role of the teacher* in supporting students' learning of rigorous mathematics through active engagement and fostering the development of productive dispositions is extensive. Several important themes stand out: Teachers play a critical role in scaffolding student learning, not by reducing task rigor, but by ensuring that all students can access the task (Jackson et al., 2013) and responding to students' needs during productive struggle. Some distinguish between "just-in-case" support, which provides all students with guidance prior to students engaging with problems, and "just-in-time" support which provides targeted guidance to particular students when they need it, allowing more opportunity for productive struggle (Dixon, 2020). This type of effective scaffolding is informed by knowledge of student understanding produced by ongoing, short-cycle formative assessment (Black & Wiliam, 1998; Copur-Gencturk & Rodrigues, 2020). By continuously monitoring students' understanding of a problem, teachers have the opportunity to provide scaffolds to students as necessary.

Research on the affordances of digital tools is still under development, and few studies have examined technological tools with an eye toward how they might contribute to mathematics education (Kay & Kwak, 2018). No studies focus exclusively on what we are calling OSFPs, instead classifying them as one type of broader collection of digital apps or tools. We build our

analysis, in part, on a small set of studies that attend to the learning experience supported by these platforms or other digital mathematics apps, but do so in substantially different ways.

Clayton-Hodges, Feng, and Pan (2015) developed a framework for analyzing and assessing mathematics apps, which proposed four dimensions: a) quality of mathematical content (mathematical accuracy and richness), b) feedback and scaffolding, c) richness of interactions (modes of interaction and item types), and d) scoring and adaptability. To a large extent, Choppin and colleagues' (2014) analysis of digital curriculum programs hones in on the "richness of interactions" category. They focus on how learners interact with the platforms and found that they generally engage learners in one or more of three distinct types of activities: a) view video presentations, b) practice procedures that have been demonstrated, and c) manipulate representations to solve problems. Kay and Kwak (2018) offer the dimension of purpose as an additional category. Based on a review of research on mathematics apps, they found five different purposes: instructive, practice, constructive, productive, and game-based. They also offer a list of eight characteristics around which the apps they examined tended to vary, many of which overlap with or add detail to Clayton-Hodges et al.'s (2015) dimensions: types of learning valued, quality of the content addressed, clarity of learning goals, usability, engagement, adaptability to differing levels, mode of feedback, and opportunities for collaboration. Each of these classification systems contributed to the development of our analytical framework. Because we were interested in the learning experience offered by the OSFPs, we selected dimensions from these frameworks that most aligned with research on the nature of mathematics teaching and learning. These are described in the following section.

Design and Methods

The data for this study come from an analysis of student-facing mathematics platforms that are frequently used in the United States. We constrained our selection to platforms that students use by logging on and working individually as a supplement to primary mathematics instruction. We began with the OSFPs identified by teachers in a related study on teachers' use of digital resources (Remillard et al., under review). We then added platforms based on reports of OSFPs most commonly used by teachers in the United States (Kauffman, et al., 2020) and our own awareness of available platforms with unique features. After completing the first phase of analysis, we searched for additional OSFPs to test the viability of our emergent typology. In this paper, we report on a reduced subset of 9 platforms that exemplify the range and variation of each category.

To begin our analysis of each OSFP, we immersed ourselves in the student experience. Through completing multiple tasks and exploring the learning pathways of the platforms, we became familiar with the program organization, types of tasks, instructional supports, and responses to correct and incorrect student entries. We also read and watched instructional and promotional materials for teachers and reviewed teacher resources to understand the intended purposes of the platform features. Although the platforms included a number of teacher-facing features, we restricted our analysis to student-facing components, given our aim of understanding the student learning experience. From this phase of analysis, we wrote detailed memos that summarized the characteristics and features of each OSFP and outlined the overall nature of mathematics teaching and learning available in each platform.

Based on our initial exploration of the platforms, we developed a set of categories that roughly aligned platform features with constructs from the literature on mathematics teaching and learning and incorporated key dimensions included in analyses on platforms and apps. These

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are summarized in Table 1. The majority of categories were emergent, developed through successive rounds of description, comparison, and refinement. The levels of cognitive demand, introduced by Stein et al. (1996) and widely used in subsequent studies, were the only *a priori* categories applied.

Table 1: Categories and descriptions for OSFP analysis

Constructs	Platform Features	Categories
Mathematical Rigor	Cognitive demand (Stein et al., 1996)	Memorization (Mem); procedures without connections (PWOC); procedures with connections (PWC); doing mathematics (DM)
	Types of tasks	Examples: Replicating procedures, problem solving, creating models, test-like questions, conceptual activities, mathematical games
Instructional Guidance features	Instructional supports	How direct instruction is provided through videos or interactive lessons
	Responsive supports	How just-in-time supports are provided through feedback, hints, corrections, and optional instruction
	Learning pathway	How student’s pathway through tasks is determined (type of adaptive tools)
Student agency/dispositions	Student decision making	How students monitor/guide their own work; availability of student choice

Many platforms we analyzed included a range of features, multiple task types, and several modes of interaction, making categorization challenging. For the purpose of this study, we focused on the most prominent features and approaches related to critical domains of teaching and learning mathematics, discussed earlier, in each OSFP. Many platforms included some timed practice of computation facts, which varied little across platforms and are not considered here. In addition, platforms used a range of gamification or engagement strategies (e.g., points, badges, characters, music, avatars, non-mathematical games), which we did not include in our analysis.

By analyzing the platforms using emergent categories and short descriptions, we were able to explore patterns in the combinations of features across different platforms. When grouped according to these patterns, we observed the potential for different learning experiences for students. We identified three primary types of *student learning experiences* offered by the platforms: Instruction and Practice, Practice and Support, and Conceptual Games and Activities. These categories overlap with and add depth to analytical frameworks from Choppin et al. (2014), Clayton-Hodges et al. (2015), and Kay and Kwak (2018).

Findings

We organize our findings around the three types of student learning experiences that emerged from our analysis. As illustrated by Table 2, the instructional guidance, student agency, and mathematical rigor of platforms intertwine to shape a distinct student experience. Within each type, we found variation across the subcomponents, which we illustrate through several exemplar platforms.

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Table 2. Description of three OSFP types with exemplar platforms.

Student Learning Experience Categories	Nature of Interaction	Student Agency	Task Types with Levels of Cognitive Demand (Stein et al. 1996)
<i>Instruction and Practice</i> (Dreambox, iReady, Zearn)	Videos, sometimes with interactive tasks, followed by practice with identical tasks with hints or explanations	Minimal	Replicating procedures to create visual models and/or solve test-like word problems Mem, PWC
<i>Practice and Support</i> (IXL, Khan Academy, Prodigy, Study Island)	Practice tasks with hints or explanations; support may be corrective or conceptual	Choose type and amount of support	Test-like questions. Mem, PWOC, PWC
<i>Conceptual Games and Activities</i> (Beast Academy, Math Playground)	Concept-building mathematical games and activities; guided lessons or self-discovery of increasingly complex concepts.	Choose problem solving approach	Students solve game-like or progressively complex challenges Mem, PWC, DM

Instruction and Practice

Our first OSFP type, *instruction and practice*, demonstrates concepts or skills through direct or guided instruction and then has students replicate them through highly similar procedural tasks with little student agency. Within this type, we found several variations. Dreambox and iReady follow a gradual release approach to instructional guidance (I Do-We Do-You Do), where students first watch a demonstration, then participate in filling out incremental, predetermined steps, and then practice the same steps on their own. Zearn’s approach is more guided, using interactive videos, during which the student is frequently asked to answer questions, interpret models, or demonstrate understanding along a conceptual pathway before they practice independently.

In terms of mathematical rigor, all 3 platforms employ visual models to connect procedures to underlying concepts (PWC tasks). iReady also includes mathematical games that allow students to apply their learning in a game context, for example by adding weights (negative) or balloons (positive) to a submarine to move it along a vertical number line and pass through a goal. Dreambox, however, uses a didactic approach of demonstrating each activity twice with a focus on completing steps. which reduces the rigor level, despite the use of interactive visual models.

We found student agency in these platforms to be minimal or superficial, as they require students to move through all lesson components in order. For example, in Zearn, video lessons start playing automatically, but students can skip through explanations of the steps, and in Dreambox students can select the topic of the next task while still proceeding down the same lesson path. Several of these platforms offer individualized learning paths based on initial diagnostics, though we found that in Dreambox students are often forced to practice many tasks at the same level despite having repeated correct answers. Zearn has no adaptivity; all students complete the same tasks in the same order.

Overall, the approach of *instruction and practice* OSFPs reflects a perspective that learning involves replicating procedures correctly in repeated practice, with iReady and Zearn at the more

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conceptual end of the spectrum, as students are introduced to conceptual models, and Dreambox at the more procedural end, with students using visual models in a rote manner. These OSFPs use a just-in-case model and appear to position students as passive learners, who may view mathematics as a set of steps to memorize.

Practice and Support

OSFPs that we describe as *practice and support* have students begin by directly practicing tasks and receiving instructional support only when they choose it or demonstrate need through incorrect responses. Students may access written/visual or video instruction at any time, encouraging them to select the level of support that they need. There is still substantial variation among platforms in this type, shaped by how the mathematical rigor of tasks and related instructional guidance work in tandem to modulate the level of student agency.

In *practice and support* platforms, tasks are formatted like test questions, including multiple choice, multi-select, fill-in-the-blank, drop-down, and interpreting or creating visual models. This range of task type, which includes word problems and visual models, reflects those commonly used in online assessments and textbooks in the U.S. We found that *practice and support* platforms vary in both the cognitive demand of tasks (procedural or conceptual) and the nature of instructional guidance (corrective or conceptual), often in tandem. At one end of the spectrum, Study Island provides tasks, videos, and explanations that focus on completing rote procedures without conceptual understanding (PWOC). The accompanying student feedback is corrective, providing a generic description of the solution strategy and then the correct answer. At the other end, IXL and Khan Academy use more complex tasks, based on visual models and alternative algorithms to develop conceptual understanding (PWC). Students are supported by conceptual feedback in the form of optional, stepped-out hints, explanations, and conceptually-focused video lessons. Khan Academy gives students a menu of support options and also uses a point system to encourage them to try solving first on their own. Prodigy lies in the middle, with more rigorous test-like questions, but the hints provide a small amount of supporting information or indicate the early steps of a solving strategy, and it only provides corrective feedback.

Practice and support platforms offer a higher level of student agency than *instruction and practice* platforms because they allow students to determine when and how to seek instructional guidance. Yet there is still a substantial range in student agency among *practice and support* platforms, depending on both the task complexity, the kinds of support offered, and students' level of choice in accessing the supports.

Overall, the components of *practice and support* OSFPs work together to suggest a model of learning in which students are encouraged to try solving problems independently and seek or be given help if they need it. This design positions students as active learners when practicing or figuring out a range of problem types and, in some platforms, gives them the opportunity to self-monitor their understanding. While IXL and Khan Academy offer somewhat more cognitively complex tasks and supports than Prodigy, all three use an underlying just-in-time model that begins with student practice.

Conceptual Games and Activities

OSFPs that we categorize as *conceptual games and activities* invite students to make sense of tasks and experiment with problem solving strategies. Many of these tasks offer minimal or no language introducing the problem or its objectives, inviting students to explore.

These platforms offer two types of tasks: problem progressions and logic games. Problem progressions start with a simple problem that requires only basic computation or memorization to solve and then provide increasingly more complex problems that organically lead to multi-step

or algebraic thinking. For example, one Math Playground progression begins by showing two identical candies with a total cost of 6¢, which students drag to the 3¢ jar. The task quickly progresses to involve multi-step algebraic thinking to find the cost of an unknown candy when a 24¢ candy and two identical unknowns together cost 52¢, all without any instructions or demonstrations (doing mathematics; DM).

Math logic puzzles and games rely on an understanding of procedures but encourage conceptual thinking (PWC). Beast Academy and Math Playground both contain logic games where a limited set of numbers must be arranged to produce correct sums/products along horizontal, vertical, or diagonal lines, supporting flexible number sense. Additionally, some of the operational automaticity games in Math Playground support flexible number sense, for example by having students select different pairs of bubbles that have a sum of 8. (These flexible automaticity games are also available in iReady).

The instructional guidance features differ in our two exemplars. In Math Playground, feedback is minimal and corrective (e.g., a beep indicating a wrong answer), with no hints, explanations, or instruction. However, the responsive support is also unlimited; students can make endless attempts using different strategies until they find the correct solution. Beast Academy, meanwhile, provides instructional support that is more similar to a *practice and support* platform, where students may seek help at any time by clicking on a video or an illustrated lesson, and students are shown correct answers with brief explanations after two attempts.

Overall, the *conceptual games and activities* OSFPs suggest a view that students learn through solving increasingly complex tasks, using their own strategies, with minimal or no instruction. While Beast Academy provides a higher level of instructional guidance than Math Playground, the creative and rigorous tasks in both platforms position students to have agency in determining their own solution paths.

Discussion and Significance

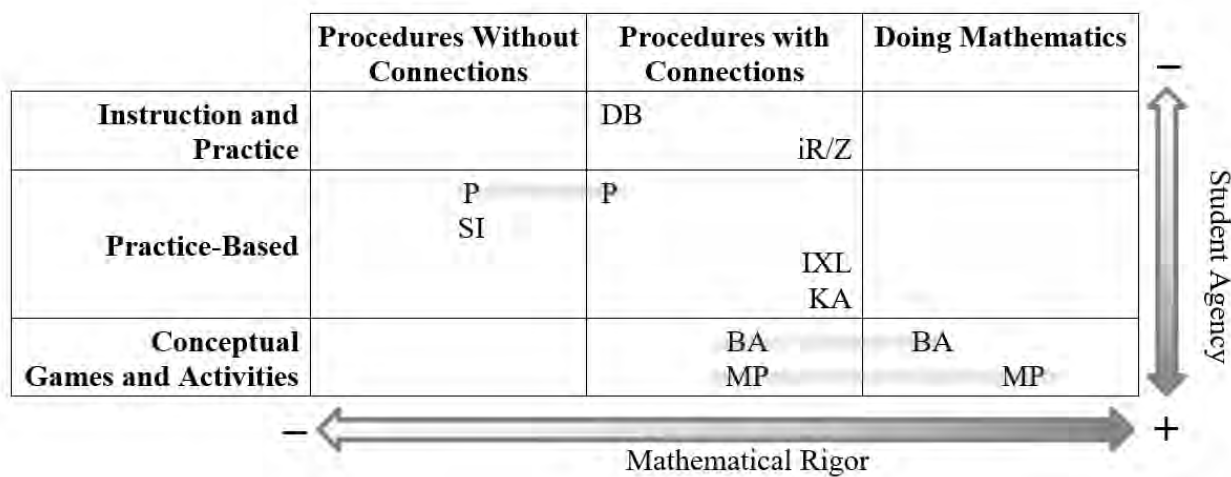
Given the prominent use of OSFPs in U.S. classrooms, one aim of our study was to understand and make visible the types of learning experiences available to students when using these platforms. Though OSFPs are often overlooked as supplements to a core curriculum, the hours students spend with them has the potential to substantially shape their ideas about the nature of mathematics and themselves—whether they see themselves as active or passive learners, and whether mathematics involves replicating procedures, understanding them, or creatively solving problems (Boaler, 2015; Skemp, 1978).

Our typology offers a framework for analyzing OSFPs beyond the 9 that we showcase here, supporting researchers and school decision makers in understanding the affordances of these resources. While building on prior work that compared platform components and features (Cayton-Hodges, et al., 2015; Choppin, et al., 2014; Kay & Kwak, 2018), our analysis takes a more integrated approach. By putting components and analytical categories for conceptualizing mathematics teaching and learning in relation to one another, our framework offers a fuller picture of how platforms create mathematics learning experiences.

In particular, our approach illustrates the utility of our three analytical categories for examining OSFPs, instructional guidance, mathematical rigor, and student agency, while also highlighting how they work in relation to one another to shape a particular learning experience. There are substantial differences, for example, in the overall design of the three types we identified in how they position the nature of mathematics and students' roles. Moreover, we

found that differences in how these components are combined leads to further variation, with the potential for substantially different students’ experiences.

We have attempted to capture the way variation in mathematical rigor and student agency interact in our nine exemplar platforms in Figure 1. Moving horizontally, the platforms increase in mathematical rigor from left to right. Gray bars indicate platforms with tasks at two levels of cognitive demand. For example, Prodigy (P) contains tasks at both the PWOC and PWC levels. Student agency is placed along the vertical axis, increasing from top to bottom. Each platform is placed in relation to both continua and shown within our analytical types. In addition to showing how the platforms we analyzed differed, the figure demonstrates how the different categories influence the overall learning experience.



Key: DB = Dreambox, iR = iReady, Z = Zearn, P = Prodigy, SI = Study Island, KA = Khan Academy, BA = Beast Academy, MP = Math Playground. Gray bars indicate platforms with tasks at two levels of cognitive demand.

Figure 1: Relationship between student agency and mathematical rigor across and within each OSFP category.

Finally, we argue that the use of student agency as an analytical lens when examining digital platforms draws attention to important and unexamined aspects of OSFPs. Student agency differs qualitatively across the types of platforms and is supported by different features. In *practice and support* platforms, students are encouraged to monitor and manage their work on tasks, getting support only when they need it. In *conceptual games and activities* platforms, students can develop agency through sense making and directing their own approach to problem solving. Both types of agency can support students to develop productive dispositions and mindsets related to mathematics learning (Boaler, 2016). *Instruction and practice* platforms, on the other hand, provide limited attention to this consequential aspect of students’ mathematical identity development, though Zearn suggests that this can be increased through interactive lessons where students complete some steps before they are modeled. We recommend that increasing opportunities for the development of student agency would be a fruitful path for designers of OSFPs to pursue.

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