SEMANTICALLY LINKED SYNTACTIC LITERACY AFFORDANCES IN SECONDARY MATHEMATICS

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This report details a literacy affordance framework for describing and connecting the ways in which teachers focus their students on the syntactic structures of reading, writing, speaking, and listening in mathematics. This framework is intended to serve as a critical access point for connecting and moving broader research in secondary mathematics teaching towards a sociolinguistic perspective. The framework is applied to a sample of teachers from two U.S. states to indicate ways in which these secondary mathematics teachers currently attend to such literacies in otherwise dialogically orientated lessons. Findings indicate the applicability of the framework as well as the opportunities and shortfalls in how such teachers currently attend to language in secondary mathematics.

Keywords: Classroom Discourse, Communication, Mathematical Representations, Instructional Activities and Practices

It is impossible to disentangle the use of language from the learning of mathematics. Reeves (1990) states, "language is the essential vehicle for transmitting and understanding mathematics in school, for turning experience into thinking and learning" (p. 213). Pimm (1987) goes further, declaring that mathematics *is* a language, and if mathematics is a language then the teaching of mathematics *is* the teaching of language. Research on how reading, writing, speaking, and listening relate to mathematics teaching in sequester is abundant (e.g., Österholm [2006] for reading; Resnick [1982] and Shield and Galbraith [1998] for writing; Chapin and O'Connor [2007] for speaking; Hintz and Tyson [2015] for listening) but, as Gutiérrez and colleagues (2010) explain, studies on how all four such modalities of language *intertwine* to mediate the teaching of mathematics are lacking. Research on multilingual or English learners in mathematics education has striven to promote a multimodal and resource-oriented perspective to the topic of teaching such students (see de Araujo et al., 2018), but broader research in mathematics education is fraught with culturally neutral (at best) or deficit-oriented (at worst) perspectives towards language (Moschkovich, 2010).

My aim with this study is to describe the opportunities which secondary teachers do (or do not) afford students to grapple with the multimodal, multisemiotic language of mathematics. The present study thus describes a literacy affordance framework which recognizes and connects the multimodal dimensions of language in mathematics teaching. Further, the study demonstrates the utility of this framework in the context of twelve secondary mathematics lessons. Specifically, this study seeks to answer the following questions:

1. In what ways do secondary mathematics teachers enact affordances addressing their students' use of syntactic structures to read, write, speak, and listen mathematically?"
2. In what ways do such teachers' instructional affordances semantically link the syntax of these different modes of language?

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Texts, Literacy, and Literacies in Mathematics

One of the challenges in addressing the role of language in mathematics education is the limited definition of literacy within the field. For instance, Draper and Siebert (2010) describe how teachers may not recognize *reading a graph* or *writing an equation* as literacy practices if their conception of literacy is confined to only "fluency in reading and writing [with] print texts" (p. 23). Such restrained conceptualizations of literacy mask opportunities to recognize and study the use of language in mathematics education. This study addresses this concern by integrating consistent and inclusively defined definitions which are meant to better connect ideas of language, literacy, and mathematics.

At the core of this study are the ideas of *texts*, *literacy*, and *literacies*. Although traditional definitions of texts and literacy are limited to a focus on reading and writing printed text (Draper & Siebert, 2010), this work recognizes a more inclusive understanding of such terms. In the present study a *text* is considered any representational object which is intended by its creator to communicate a meaning (Draper & Siebert, 2010; Wells, 1990). Literacy, in turn, can be considered "the ability to negotiate (e.g., read, view, listen, taste, smell, critique) and create (e.g., write, produce, sing, act, speak) texts in discipline-appropriate ways" (Draper & Siebert, 2010, p. 30).

Literacies then are the multiple modes (or "meaning-making systems"; Kress, 2001, p. 11) of texts through which students must navigate during the learning process. These include both primarily receptive (reading and listening) and primarily expressive (speaking and writing) literacies (Aguirre & Bunch, 2012; Bloom, 1974; Draper & Siebert, 2004). Meaning can also be communicated in other ways such as gesture (Arzarello et al., 2009). However, because this study adopts the four primary language demands of reading, writing, speaking, and listening which students face in school mathematics (Aguirre & Bunch, 2012), such modalities fall beyond the focus of this study.

Syntactic Literacy Affordances

Given the current study's definition of texts as representational objects, and literacies as different modes of texts, literacies themselves can be considered representational systems. Goldin (2002) came to a similar conclusion in recognizing the development of representational systems in mathematics as akin to language learning. The current study adopts Goldin's conception of representational systems and flips the focus back to the realm of literacy in mathematics teaching. Of particular relevance is Goldin's (1998, 2002) recognition that representational systems have internal *syntactic* configurations as well as *semantic* relations with other representational systems.

Regarding the syntactic nature of representational systems, Goldin (1998) explains that "To know and be able to construct the configurations formed from characters, and to use the relationships among configurations established by higher-level structures, is one way of giving *meaning* to the characters and configurations in a representational system" (p. 144). Representational systems are not immaculately bestowed with meaning. Rather, understanding and using the syntax of the system fosters that meaning. In the present study this indicates the importance of affording students' opportunities to grasp the syntax of reading, writing, speaking, and listening.

Syntax is traditionally defined as "grammatical relationships among words in a sentence or the structural arrangement among sentences in a passage" (Vacca & Vacca, 2002, p. 381). Given this study's broader definition of text, syntax also refers to valid ways in which symbols or objects that hold mathematical meaning can be procedurally manipulated or configured (Bayaga

& Bossé, 2018; Goldin & Kaput, 1996; Kaput, 1987). A *syntactic literacy affordance* thus occurs when an instructional activity supports students with developing their understanding or use of syntactic structures within the representational systems of reading, writing, speaking, or listening. Specifically, corresponding definitions of such affordances within each relevant literacy are drawn from this overarching definition to form the crux of the literacy affordances framework:

- A **syntactic reading affordance** is when a teacher focuses students on interpreting the syntactic structures of *already-constructed written texts* (representational objects such as written language, graphs, tables, equations, charts, etc.). This instructional move emphasizes how attending to such structures helps to uncover mathematical meaning. Ambiguities of a construct*ed* written text are addressed.
- A **syntactic writing affordance** is when a teacher focuses students on the syntactic structures of *their own written texts*. This instructional move emphasizes how attending to such structures helps to communicate mathematical meaning. Ambiguities while construct*ing* written text are addressed.
- A **syntactic speaking affordance** is when a teacher focuses students on the syntactic structures of *their own spoken texts* (representational language such as explanations, justifications, clarifications, etc.). This instructional move emphasizes how attending to such structures helps to communicate mathematical meaning. Ambiguities while construct*ing* spoken texts are addressed.
- A **syntactic listening affordance** is when a teacher focuses students on interpreting the syntactic structures of *others' spoken texts* (representational language such as explanations, justifications, clarifications, etc.). This instructional move emphasizes how attending to such structures helps to uncover mathematical meaning. Ambiguities of a construct*ed* spoken text are addressed.

Semantically Linked Syntactic Literacy Affordances

Lingering beyond this focus on syntactic literacy affordances are the (previously noted) semantic aspects of representational systems. Indeed, the above definitions of syntactic literacy affordances, with their emphasis on addressing ambiguities within each literacy, are semantic in nature. It would make sense, for instance, for a mathematics teacher to help students *read* the syntax in a graph of a linear function by *speaking* with students about the representation.

This is intentional, as the literacy affordance framework does not describe the modalities used to enact all instruction (which certainly extends beyond the scope of reading, writing, speaking, and listening) but rather which syntax the teacher is *focusing student attention towards* in its relation to mathematical meaning. I fully recognize that teachers and students may be *using* semantic cues as well as syntactic structures within and across literacies throughout their instruction. This emphasis on literacy affordances shifts the conversation away from literacies as they arise with or without the teacher's intention to instead emphasize instructional aspects of literacy that are more directly within the teacher's control.

However, semantic aspects can still be considered through the literacy affordances framework. Kaput (1987) astutely describes the mathematical power in "applying the syntactical properties of a given symbol system's symbol scheme to a new field of reference" (p. 181). In other words, corresponding the syntax of one representational system onto that of a new representational system is "among the key ways that mathematics evolves, both historically and within individuals" (Kaput, 1987, pp. 180-181). This study's framework provides a window into how teachers might promote such correspondences from a literacy standpoint by identifying when teachers address the syntax of *multiple* literacies in relation to a single mathematical text. Such groupings of syntactic literacy affordances are thus considered *semantically linked*.

The Literacy Affordances Framework

When combined, the literacy affordances framework situates syntactic reading, writing, speaking, and listening affordances along similar dimensions as Aguirre and Bunch's (2012) visualization of language demands of reading, writing, speaking, and listening. However, this model stands apart in focusing on the teacher's role in attending to language rather than on the demands of the language itself. Given the importance of semantically linking such affordances, this aspect is centered on the representation of the framework, shown in Figure 1.

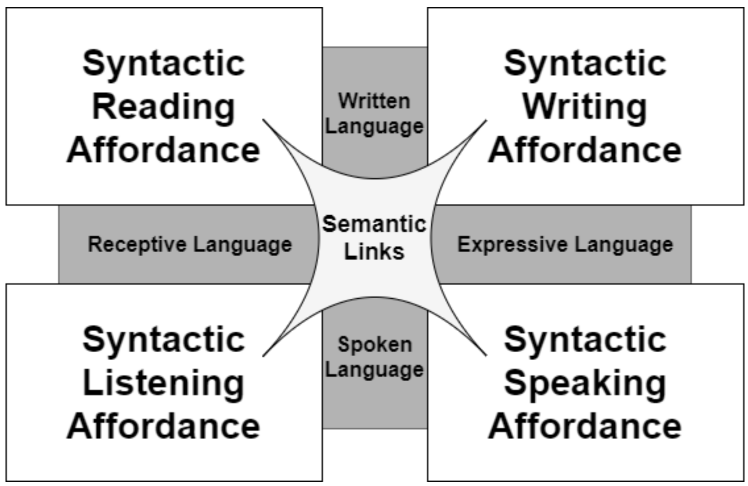


Figure 1: The literacy affordances framework adapted from Aguirre and Bunch (2012)

Methods

Research Setting and Participants

This study is based on recordings collected from 9 secondary mathematics teachers' lessons in two U.S. states. 6 of these teachers taught in a mid-Atlantic state and 3 in a southwestern state. 6 of these teachers identified as white while the remaining 3 teachers identified as Black, Hispanic, and white/Asian, respectively. These recordings were captured as part of the SMiLES project (Secondary Mathematics in-the-moment Longitudinal Engagement Study), which collected student survey data, classroom observations, and teacher and student interviews to understand the role of engagement in secondary mathematics classrooms.

These nine teachers were chosen for the present study because their instruction appeared highly dialogic. Three such qualities of dialogic instruction are the use of high-level tasks (Henningsen & Stein, 1997), opportunities for sharing multiple representations or strategies (e.g., graphs, tables, etc.), and student discourse (Munter et al., 2015). Each of these dialogic qualities had previously been captured in qualitative coding of the classroom videos as part of the SMiLES project (Jansen et al., 2021). High demand tasks were hypothesized to present more opportunities for reading comprehension, while sharing multiple representations and mathematical talk were conjectured to afford more opportunities for writing, speaking and listening. Thus, teachers who enacted dialogically focused instruction were hypothesized to be ideal candidates for this investigation.

The mean of these three qualities was taken for all 156 lessons analyzed for SMiLES and lessons with a mean greater than 2 (out of 3) and with no individual dialogic quality rating lower than 2 were selected for inclusion in this study. Additionally, for teachers who had more than two lessons that met these criteria, only the two lessons with the highest mean were selected so that any one teacher's instruction would not dominate the focus of the results. This left 12 lessons to analyze for the present study.

Classroom Observations

Observations were conducted during the 2018-2019 and 2019-2020 school year. The observed lesson episode for SMiLES was an activity which the teachers believed would be most engaging for students. When teachers would attempt to engage students in learning mathematics, it was hypothesized that they would be likely to also provide a greater number of opportunities for students to engage with or across different literacies. However, the absence of such literacy affordances in these activities would also be valuable, as such results would indicate that these teachers do not necessarily include literacy as part of their conceptions of engaging mathematics instruction. As such, the activities captured in these classroom recordings were well suited for the present study.

Unit of Analysis

Given this study's focus on syntactic literacy affordances – including those which are semantically linked – it was critical to define a unit of analysis that would capture affordances which genuinely correspond with one another and to not confuse these with semantically isolated affordances. For instance, enacting a syntactic reading affordance to support students in interpreting the features of a linear function represented in a graph and then later providing a syntactic writing affordance to support a student in revising a written function equation would not inherently link the two affordances. However, if both the reading and writing affordance attended to how the *slope* of the same function manifests (in the graph and in the equation), then these two affordances would be (from an instructional standpoint) semantically linked.

As such, this study delineated its unit of analysis not only by the overarching mathematical ideas that constitute an instructional task (i.e., Stein & Lane, 1996), but also what Gresalfi et al. (2009) refer to as the task *affordances*, which includes "the ways that mathematical knowledge got constructed – individually, in pairs, with the entire class, and with the teacher" as well as "the ways that the teacher engaged with students around the task as they completed it" (p. 56). The unit of analysis for this study can thus be considered a *textual affordance*, or an instructional moment where one or more texts are being used to communicate meaning about a particular mathematical idea in a particular social context.

Reliability of the Framework

To establish reliability for this study, the author met with two colleagues to test the unit of analysis and the types of syntactic literacy affordances. In the first round of checks, the author identified distinct textual affordances within a sample activity from SMiLES and asked the colleagues to replicate the procedure. Together, these two colleagues correctly identified 100% of the textual affordances that the author had previously identified.

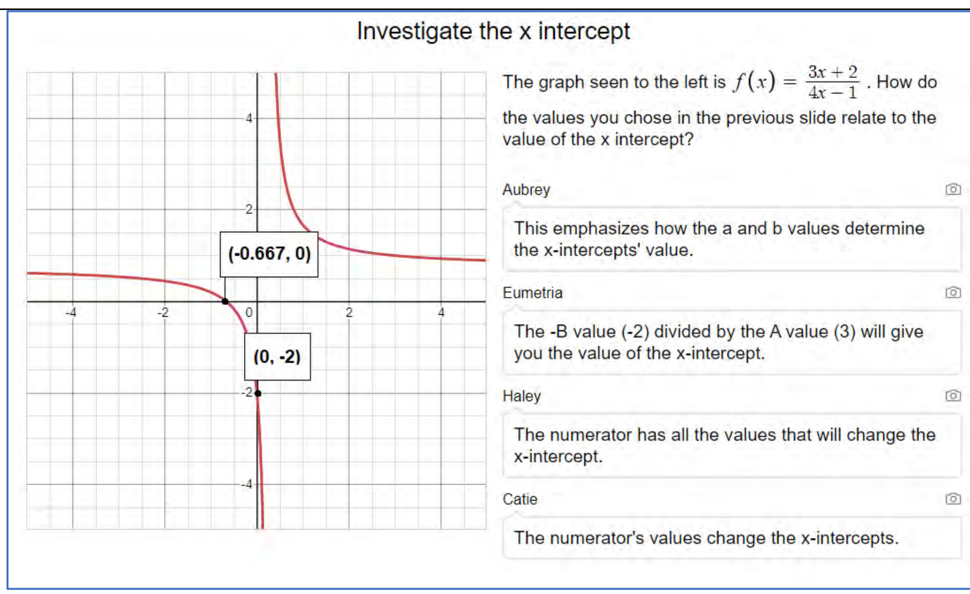
Examples of different syntactic literacy affordances (reading, writing, speaking, and listening) were then identified by the author from a sample observation recording and transcript, including examples where multiple, semantically linked syntactic literacy affordances were present. These same examples were sent to the two colleagues for them to replicate the process of identifying the type of syntactic literacy affordance. Interrater agreement from this process was approximately 85%, indicating a sufficiently reliable framework.

Results

The 12 analyzed observation recordings ranged in length from approximately 15 to 28 minutes. Every activity investigated included examples of syntactic literacy affordances. Fifty-two enactments of syntactic reading affordances and 44 enactments of syntactic writing affordances were found. Eleven enactments of syntactic speaking affordances were present in the analyzed activities as well as one syntactic listening affordance. In addition, 16 examples of semantically linked groups of affordances were also found, although 73 of the 108 syntactic literacy affordances were not linked. Although different amounts of literacy affordances would be expected given that the length of teacher-selected activities varied, these data show the prevalence of such affordances throughout such activities regardless of the length of the activity.

Use of Technology for Whole Group Writing Affordances

Syntactic writing affordances were intentionally defined as focusing students on their own constructed texts. As such, a teacher merely asking a student to describe a mathematical text whole group could not be considered a syntactic writing affordance since the syntax of the written text is not at play but rather the syntax of the student's spoken interpretation of that text. This could have potentially limited whole group syntactic writing affordances to instances where students construct (or reconstruct) their texts in a public space (e.g., a white board) or the teacher has the means to share individual work publicly (e.g., a document camera). For the latter option, the results showed that several teachers used virtual learning platforms (e.g., GeoGebra and Desmos) to enact whole group syntactic writing affordances.



Mrs. Hudson: So a and b were the coefficient of x and the constant term in the numerator. All right. And when you all started to analyze that, there were a couple of you are like -- I'm going to point out -- this one here, where it said, "the -B value, which is (-2), divided by the A value (3) will give you the value of that x-intercept." So, there were a couple of us -- we explained that -- or some of you all even said "if you set the [denominator] equal to 0, and solve for x." I saw some of that. So I'm glad that you all saw the patterns there, but at least you all saw that it is in the numerator and it is the values of the coefficient and the constant.

Figure 2: A whole group syntactic writing affordance

For example, Figure 2 shows how Mrs. Hudson (all names are pseudonyms) provided feedback on students' written explanations of how features of a rational function relate to the x-

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intercept. She contrasted some students who simply referenced "the numerator" against those who described the actual a and b values and the procedure of solving for x. She noted how these students "saw the pattern there, but at least you *all* saw that it [the values used to find the x-intercept] is in the numerator and it is the values of the coefficient and the constant." The ambiguities in some students' displayed writing, such as a response which stated "The numerator's values change the x-intercepts" were drawn out through this syntactic writing affordance.

Semantically Linked Syntactic Literacy Affordances

There were 16 examples of semantically linked literacy affordances in the data. Figure 3 shows an example from Mrs. Barnett's classroom, where she enacted a semantically linked syntactic speaking and listening affordance during a lesson about maximization with systems of inequalities. She posed a question to Jimmy about how it can be known that one combination (or solution) is "the best." She then engages in a revoicing talk move (Chapin & O'Connor, 2007), having other students respond to Jimmy's answer. She focuses students on *how* to effectively listen during this talk move ("the exercise here is, can we restate or rephrase what Jimmy just said? We need to listen carefully.") allowing both Jimmy and his peers to attend to the clarity of his spoken response. Her insistence that Jimmy must "tell me the math that supports that" with his justification further cements the syntactic speaking affordance within this exchange.

Mrs. Barnett:	I've written my question quite large here. You need to convince me that this is the very best combination. We've talked about the fact that, okay, everywhere within this region…is possible, but Jimmy told us that there are better combinations than others. Of course we want to carry the most passengers. But then how do I know?
Jimmy:	If you were to do all the other combinations, you would still find that with that combination of vans, you would not be able to carry as many senior citizens as the (0,5) combination, which would carry 75.
Mrs. Barnett:	Do you guys hear what Jimmy had to say? Can someone restate or rephrase what Jimmy just said? Say it one more time, Jimmy, because now the exercise here is, can we restate or rephrase what Jimmy just said? We need to listen carefully.
Jimmy:	There's no other way to get more than 75 people due to the budget. There are good combinations. (0,5) is the best combination because it gives you the most people that you can carry within the vans.
Mrs. Barnett:	Okay. Can someone restate or rephrase what Jimmy just said?
Michael:	Based on the money you have, you can't get any more 75 senior citizens to be taken in the vans.
Mrs. Barnett:	Jimmy, did Michael adequately rephrase what you said?
Jimmy:	I mean, yeah. It's kind of the main idea of that.
Mrs. Barnett:	There's no larger amount of passengers that can be carried. You're telling me that it's what, 75 people?
Jimmy:	Yes.
Mrs. Barnett:	Can you tell me the math that supports that?
Jimmy:	Five times 15, because each large van carries 15 passengers. If we purchased five large vans, five times 15 is 75.

Figure 3: Semantically Linked Syntactic Literacy Affordances

Nonexamples Syntactic Literacy Affordances

Many instructional moves fell short of focusing students on the syntactic structures of reading, writing, speaking, and listening mathematically. The key definitional piece of the literacy affordances framework that held back additional moves from being classified within it was the need for the move to *emphasize **how** attending to such structures helps to communicate mathematical meaning*. While the teachers in the study often alluded to the fact that different texts do communicate mathematical meaning, these conversations did not always describe how. Indeed, many such interactions between teachers and students could be described as funneling rather than focusing patterns (Wood, 1998). For example, one teacher (Ms. Ellis) asked a student "what direction would this graph open" for a quadratic function written in expanded form ($f[x] = ax^2+bx+c$). The student believed it would open down and when asked "How do you know?" they ambiguously said because it was negative. Ms. Ellis then responded with "What is a negative?" and realized that the student was referring to the c value. At this point Ms. Ellis told him that "c tells you the y-intercept letters. That is correct. What tells you the direction of the graph?" and proceeded to funnel the student towards recognizing that his a value, with the value of 1, would mean that the graph of the function would open upwards.

Although this paper does not stake any claims on the efficacy of this instructional move for supporting the student in matching equations of functions with their graphical representations, the lack of emphasis on the student's mathematical speaking syntax is apparent. Instead of focusing the student on the ambiguity of their one-word responses (e.g., "what do you mean by 'it's negative?' I'm not sure what you are referring to in the equation when you just say 'it.'") the teacher ignored the ambiguity of the spoken syntax (or at most implied its ambiguity by asking "What is a negative") and moved on without addressing how she interpreted it to be so.

Discussion

This study investigated how secondary mathematics teachers support student meaning-making by attending to and linking the syntactic structures involved in reading, writing, speaking, and listening to mathematical ideas. This first required the construction a literacy affordance framework which described – in corresponding terms – how teachers can attend to these four literacies. Such a framework on its own represents a critical step in drawing research on mathematics teaching towards a sociolinguistic perspective (Moschkovich, 2010) by providing a basis upon which to describe teachers' attention to the syntax of language.

This framework was validated and applied across a diverse set of secondary mathematics classrooms, providing an exploratory glimpse into the ways that teachers do (and do not) attend to and connect these literacies. Some findings indicate concerning trends. For instance, the limited findings of spoken language affordances could indicate that, despite the critical role of mathematical discourse and argumentation in mathematics reform movements (see CCSSI, 2010; NCTM, 2014), dialogic instruction (Munter, Stein, & Smith, 2015) is still limited in these classrooms.

However, the results also indicate that these teachers *are* attending to literacy, and that over 30% of the time these affordances semantically link multiple literacies in relation to a particular mathematical idea. The use of technology to attend to student writing in a whole-group setting is also notable. Such whole group opportunities could expose more students' writing to feedback and validation from the teacher and position such students as competent participants in mathematical discourse (Gresalfi et al., 2009).

If, as Pimm (1987) says, the teaching of mathematics is the teaching of language, then the opportunities which mathematics teachers afford for students to engage with literacy warrant the upmost attention. Language, as the core means of our ability to communicate mathematical meaning, dictates not only *what* mathematical meaning is elevated in the classroom but also *who*

Olanoff, D., Johnson, K., & Spitzer, S. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

plays a part in constructing that meaning. Affording students opportunities to develop their language of mathematics is thus a critical piece of affording them the means to mathematical power.

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