A CONCEPTUAL SYNTHESIS ON APPROXIMATIONS OF PRACTICE

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Approximations of practice create valuable opportunities for preservice teachers (PSTs) by engaging them in components of teaching. By reviewing the literature, this study explored PSTs' learning through approximations and the extent approximated practices preserve the complexity – or authenticity – of teaching. A review of 25 empirical studies related to approximations of mathematics teaching indicated that mathematics teacher educators are currently exploring an expansion of opportunities through approximations wherein PSTs could experience a higher degree of authenticity. The existing conceptualization of authenticity emphasizes the complexity of practices but overlooks how approximated practices prepare PSTs for their future teaching. An alternative definition is proposed for the emerging conceptualization of authenticity to highlight how PSTs' learning through approximations prepares them for their future teaching.

Keywords: Preservice teacher education, Instructional activities and practices, Approximations of practice, Authenticity

An effective mathematics teacher education program provides preservice teachers (PSTs) with opportunities to gain practice-based experiences and develop core pedagogical practices for teaching mathematics (Association of Mathematics Teacher Educators [AMTE], 2017). PSTs often gain such practice-based experiences by engaging in *approximations of practice* (referred to as "approximations"), which represent an opportunity for PSTs to engage in aspects of practice with additional support in order to develop their professional teaching skills such as leading classroom discussions, posing purposeful questions, developing lesson plans, etc. (Grossman et al., 2009; Schutz et al., 2018). Research has indicated these approximations are often simplified versions of actual classroom teaching because they: (a) are often enacted in teacher education settings, which cannot fully reflect social and cultural aspects of school contexts: (b) often include only some components of teaching, and (c) are usually enacted with scaffolding (Janssen et al., 2015; Tyminski et al., 2014). Thus, mathematics teacher educators have cautioned that approximations do not preserve the complexity of actual practices and are not necessarily authentic (Grossman et al., 2009). These conceptualizations suggest the following three dimensions in which to measure authenticity: context/setting of practice, nature of practices (e.g., decomposed practices vs. full practices), and scaffolding in practices. These three dimensions are utilized to explore the authenticity of approximations in this paper. While the literature has shown that approximations are valuable for PSTs' learning, the extent to which those approximations prepare PSTs for addressing the improvisational and interactional nature of teaching (Hunter et al., 2015) is still underexplored. Therefore, mathematics teacher education literature was synthesized to explore how approximations of practice create opportunities for developing PSTs' teaching skills and how teacher education programs conceptualize and practice authenticity during approximations. The following research questions guided this study:

1. What teaching practices have been used in approximations-based instructional activities?
2. What influences did approximations of practice have on PSTs' learning to develop their practices?

Olanoff, D., Johnson, K., & Spitzer, S. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

3. To what extent were the approximated practices authentic?

Perspectives

This study is grounded on the practice-based approaches to teacher education and the concept of authenticity in approximations. In recent decades, many mathematics teacher education programs have been designing and implementing instructional activities that provide PSTs with opportunities to engage in aspects of teaching practice (Ball & Cohen, 1999; Zeichner, 2012). Such instructional activities are usually referred to as approximations of practice (Grossman et al., 2009). The actual practices are assembled during approximations either by including only some aspects of practices and/or by providing PSTs with scaffolding; and thus, approximations usually have reduced complexity of teaching (Tyminski et al., 2014). Because of the reduced complexity, educators have stated limitations of the practice-based education and approximations (Zeichner, 2012). As such, simplified practices pose a risk of creating technicians who can only apply sets of routine skills but struggle to adapt them to the school context. To address these limitations, there is an ongoing discussion around how the authenticity in approximations should be conceptualized and practiced. Teacher educators primarily have two perspectives about when and how the complexity of practices should be adjusted during approximations. From one perspective, educators argued that it can be overwhelming for PSTs to begin their teaching with complex practices, necessitating a reduction of the complexity of teaching at the beginning phases of practices (e.g., Bannister et al., 2018; Klein & Taylor, 2017). Klein and Taylor (2017) mentioned that approximations should provide PSTs with opportunities to practice in a context that is different from the natural context as it has reduced complexity. Other educators have argued that simplified practices pose the risk of creating routinely inauthentic practices, and these practices might not be transferable to school contexts (e.g., Campbell & Elliott, 2015).

This discussion about complexity suggests that authenticity is related to the complexity of teaching. Educators have discussed the dimensions of authenticity in several ways. Grossman et al. (2009) proposed the approximations that (a) have less support (scaffolding), (b) are integrated (not decomposed), and (c) have a similar setting to actual classrooms are more authentic than the approximations which involve more support from educators and involve only some components of teaching. Campbell et al. (2020) mentioned that the location of approximations (university classrooms or school classrooms), students who participated in approximations (peers or students), and teacher educator's roles (providing scaffolding or not) would determine the degree of authenticity. Tyminski et al. (2014) and Janssen et al. (2015) also defined authenticity along the continuum of three dimensions: whether or not the practice is decomposed; what the contexts of practice are; and whether or not teachers provided with scaffolding. Based on these definitions, the following dimensions of authenticity are selected: decomposition of practice, setting/context of the practice, and scaffolding.

Decomposition of Practice

Decomposition of practice indicates breaking down the practice into small components to assist PSTs to learn those small portions of practice (Grossman et al., 2009). For example, PSTs may engage in responding to students' thinking, which is only one component of the practice of teacher noticing. In the studies related to approximations, educators often engage PSTs in some components of practices. For instance, Trent (2013) listed some components of teaching that PSTs engaged in: selecting tasks, organizing students for peer or group work, learning to provide feedback to students. One way to decompose the practice is to make small components of

Olanoff, D., Johnson, K., & Spitzer, S. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

practices in a way that PSTs would be able to master practices on those modules (Grossman et al., 2009; Janssen et al., 2015). However, educators (e.g., Campbell & Elliott, 2015; Trent, 2013) have challenged PSTs might not automatically be able to recompose those decomposed practices into complex practices. Since teaching is both improvisational and interactional that is based on students' thinking (Hunter et al., 2015), how those decomposed practices prepare PSTs for teaching is still researchable. Therefore, how the current studies related to approximations are utilizing decomposition in their studies is a focus of this investigation.

Setting or Context of the Practice

The second dimension of approximations—settings— refers to the contexts where the practice is situated. Authenticity is associated not only with the decomposition of practice but also with the setting or context of practice (Campbell et al., 2020). McDonald et al. (2013) defined the school setting where teachers and/or PSTs have opportunities to engage in the full practice as authentic settings and the one where PSTs have restrictions to conduct activities are considered as controlled or designed settings. Since the university-level course setting (i.e., methods course) is different from the actual setting, approximations might lead to the divide between theory and practice (Grossman et al, 2009). Thus, teacher educators have been trying to replicate the social and cultural complexities of actual practices in the university-level setting through approximations (Codreanu et al., 2020). There are some efforts to include responsive teaching in approximations. For example, Campbell et al. (2020) included planted students' errors and asked PSTs to respond to those errors. In this study, how the literature related to approximations have negotiated the difference between university-course settings and actual classroom settings is explored; the factors that the approximation-based literature has highlighted to replicate the social and cultural complexities of the mathematics classrooms are explored.

Scaffolding

Scaffolding during approximations refers to supports that PSTs receive to improve their teaching practices. Since teachers often enact practices independently and do not receive feedback during teaching, the extent to which PSTs receive scaffolding differentiates the approximations from the actual teaching (Grossman et al., 2009). The feedback given to PSTs could impact what they focus on during approximations. Some approximations are more loosely constrained than others. When PSTs have options to choose and approximate the whole practice, it is a loosely constrained approximation (Kavanagh et al., 2020). This study explores several forms of scaffolding and their possible influences in PSTs' learning.

Methods

A conceptual synthesis method was employed to explore the primary concepts and discussions related to approximations (Petticrew & Roberts, 2008). First, inclusion and exclusion criteria were established to identify the literature, which was the data for the study. The literature was then analyzed through top-down and bottom-up interactive modes of analysis (Chi, 1997).

Data Source: Literature Search Procedures

Twenty-five articles were identified as involving approximations of practice with PSTs. The first round of the literature search involved Key Word Processes Search (Depaepe et al., 2013). Two key phrases of, "approximations of practice" and "math," were used to search the literature in Education Resources Information Centre (ERIC) and PsycInfo, which produced a total of 32 results. This number was narrowed down to 13 by eliminating dissertations, non-peer-reviewed journals, conference proceedings, and/or book chapters. Only empirical peer-reviewed journal articles were included in the study because the aim was to understand and analyze (a) how

Olanoff, D., Johnson, K., & Spitzer, S. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

researchers have utilized approximations in teacher education programs, (b) what specific teaching skills PSTs developed by engaging in those approximations, and (c) what challenges are associated with the development of teaching skills. The second-round search involved a Journal Search Method. Articles were selected from *Journal of Mathematics Teacher Education* (JMTE), as it was the sixth most-cited journal in the field of mathematics education in 2017 (Williams & Leatham, 2017) and because it publishes empirical research around teacher education and teacher development. All published issues from the beginning of 2017 to 2020 were accessed to search the relevant articles from the journal. Using the inclusion and exclusion criteria described above, 12 articles were selected. The 25 articles can be found in the reference section of this paper.

Analysis of the Selected Literature

The interactive nature of the top-down and bottom-up approach (Chi, 1997) was utilized to code the articles; by considering one case as each article, teaching practices used in the study, teacher learning from approximations, and dimensions of authenticity were summarized. To answer the first and second research questions related to teaching practices involved in approximations and their influences on PSTs learning, a bottom up approach was utilized; the practices involved in approximations and major findings of each study were summarized. The three already identified dimensions of authenticity (a top-down approach) were used to answer the third research question of exploring authenticity in approximations. Once codes were identified collaboratively, researchers independently coded about 30% of the initially identified articles to calibrate initial coding at the beginning, including the identification of new codes, and any discrepancies between coding (i.e., a bottom-up approach). For example, "analyzing and reflecting on teaching" was a new code identified during independent coding, which was not in the initial sets of codes and was later added to the set of codes.

Findings and Discussions

This section is organized by research questions. Findings for the first, second, and third research questions are answered for the first, second, and third research questions, respectively.

Practices Used in Approximations-Based Instructional Activities

Research indicated the following five primary teaching practices used in the literature related to approximations: curriculum enactment (e.g., Earnest & Amador, 2019; Santagata & Yeh, 2014); eliciting, interpreting, and responding to student thinking (e.g., Bailey & Taylor, 2015; Webel & Conner, 2017); teacher professional noticing (e.g., Amador et al., 2016; McDuffie et al., 2014); organizing classroom discussions (e.g., Ghousseini & Herbst, 2016; Weston et al., 2018); and analyzing and reflecting on teaching (e.g., Auslander et al., 2020; Cooper et al., 2020; Kinser-Traut & Turner, 2018). In almost half of the research reports (13 out of 25 research studies), PSTs were engaged in two or more practices. In particular, the studies that explicitly focused on the teacher noticing and classroom discussion also included the practice of eliciting, interpreting, and responding to student thinking. In the noticing-related literature, researchers broadly assessed which aspects of classroom events (i.e., teacher actions vs. student actions) PSTs attended to and how they responded to those events, including how PSTs attended to and interpreted students' mathematical thinking. For example, Schack et al. (2013) explored the extent to which PSTs noticed students' equitable practices of mathematical learning, which included students' competencies related to mathematical thinking.

The literature indicated that in the approximations related to teacher noticing, PSTs often engaged in the components of noticing, including attending to, interpreting, and responding to

Olanoff, D., Johnson, K., & Spitzer, S. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

student thinking. Some studies that focused primarily on leading classroom discussion involved the practice of eliciting, interpreting, and responding to student thinking. Ghousseini and Herbst (2016) focused on the practice of leading classroom discussions and included interpreting student thinking. Thus, the practice of eliciting, interpreting, and responding to student thinking was the most highlighted practice in the literature related to approximations. Another highlighted practice through approximations was analyzing and reflecting on teaching. PSTs analyzed their own and their peers' teaching to develop reflective practices. Reflective practices include teachers' ability to identify and analyze aspects of classroom actions (e.g., task posing) in which instruction is successful and other areas which they need improvement (Alsawaie & Alghazo, 2010). For example, PSTs reflected on teaching by analyzing classroom practices (Kinser-Traut & Turner, 2018) and by discussing their experiences of sequencing learning activities (Cooper et al., 2020).

Some studies did not explicitly emphasize specific practices; they focused broadly on curriculum enactment without highlighting one or more practices. In these studies, PSTs planned and taught lessons (or parts of lessons) either in a virtual classroom setting (e.g., video simulations; Amador et al., 2016) or in a real classroom setting. For example, in Earnest and Amador (2019), PSTs planned a lesson and enacted the first five minutes of their lessons through the given animation, indicating this approximation did not focus on only one teaching practice.

Types of approximations used to develop practices. Researchers from the identified articles primarily used two mediums of approximations: virtual mediums and role play. Virtual mediums often included enacting lessons using simulation tools (e.g., Earnest & Amador, 2019; Weston et al., 2018) and/or interacting with virtual student characters (Bannister et al., 2018; Webel & Conner, 2017). Role play involved rehearsals of one or more components of teaching, and PSTs rehearsed teaching either with their peers or during field teaching. For instance, in Tyminski et al. (2014), PSTs practiced teacher questioning with their peers in a teacher education classroom setting, while in Santagata and Yeh (2014), PSTs rehearsed teaching in a school setting. PSTs typically used role-playing or simulated classroom scenes to communicate what they noticed or to demonstrate their anticipated student responses with virtual mediums. Estapa et al. (2018) had PSTs use animated software to represent pivotal classroom moments that they noticed from a video lesson. PSTs also engaged in eliciting, interpreting, and responding to student thinking through both role-play and simulations. For instance, in Schack et al. (2013), PSTs interpreted and discussed diagnostic interviews as well as practiced diagnostic interviews (role play) to interpret and respond to student thinking. PSTs often engaged in leading classroom discussion through role-play with their peers. In Ghousseini and Herbst (2016), PSTs chose mathematical tasks that called for reasoning and communication, led classroom discussions in a constructed classroom dialogue, and practiced leading discussions during their field teaching.

PSTs' Learning Through Approximations

The literature revealed that approximations of practice afforded PSTs opportunities for understanding and enacting aspects of practices in a scenario simulating a classroom context. For instance, Campbell and Elliott (2015) mentioned that PSTs identified actual learning goals while role-playing leading classroom discussions. Earnest and Amador (2019) discussed that their approximated practices prepared PSTs for using the curriculum to design instructions; however, those practices could not prepare PSTs for how to select specific materials in their classes. PSTs conceptualized and applied some dimensions of equitable and responsive teaching after they engaged in approximations of responsive and equitable teaching. In responsive teaching, "teachers' instructional decisions about what to pursue and how to pursue are continuously adjusted during instruction in response to children's content-specific thinking" (Jacobs &

Empson, 2016, p. 185). For instance, in Bannister et al. (2018), PSTs began to focus on students' strengths rather than their deficit perspective as they engaged in approximated practices related to deficit thinking. Initially, PSTs highlighted students' mistakes and problems and defined those mistakes as problems, while PSTs began interpreting those mistakes as learning opportunities at the end. PSTs learned to attend to students' thinking and pose tasks to respond to student thinking from approximations (Estapa et al., 2018), suggesting that approximations contributed to cultivating PSTs' ability to develop responsive teaching.

Researchers claimed that approximations related to simulations also prepare PSTs for responsive teaching. PSTs often need to predict both teachers' and students' roles when creating classroom scenes using simulations (Amador, 2017; Schack et al., 2013). For instance, in Earnest and Amador (2019), PSTs created a classroom scene wherein they selected speech bubbles by anticipating students' responses to their questions, requiring them to predict and respond to student thinking. Similarly, in Amador et al. (2016), PSTs anticipated their students' responses and planned for how they would respond to those students. The findings from these research reports indicated that these virtual tools encouraged PSTs to anticipate and analyze students' thinking and specific responses, indicating that the approximations related to virtual simulations also prepared PSTs, to some extent, for responsive teaching (e.g., de Araujo et al., 2015).

These findings indicated that PSTs' learning through approximations was reported in terms of which practices PSTs developed at the end of approximations. Indeed, there was less attention to the extent PSTs are able to transfer the practices learned from approximations to their teaching. For instance, teacher educators have been exploring how PSTs develop noticing skills by comparing what PSTs notice at the beginning and at the end of approximations. Thus, there is a less attention in the literature on how PSTs possibly apply noticing skills in their future teaching. Since approximations have often different settings than the real practices, PSTs' learning at the end of approximations does not necessarily suggest they can improvise the learning in their actual teaching.

Degree of Authenticity in Approximations of Practices

The earlier defined dimensions (decomposition, setting, and scaffolding) were used to explore authenticity in approximations.

Decomposed vs. full practices. A review of the literature suggested the following three types of decompositions in approximations: (a) focused on only some components of a practice throughout the study, (b) began with a component of practice and gradually added more components, and (c) engaged PSTs in full practices without decomposition. The first category of studies focused on only some components of practice throughout practice enactment sessions. For instance, in Bannister et al. (2018), PSTs engaged in learning to notice students' strengths rather than their deficit thinking, which is a component of teacher professional noticing. The second category of studies decomposed a practice into small components by engaging PSTs in one component of practice at a time. For instance, in Estapa et al. (2018), PSTs engaged in a core practice of teacher noticing at the beginning by having PSTs practice only one component of teacher professional noticing, namely attending to classroom events. After a time, PSTs explained what they *attended to* and how they would *respond to* those events. The third category did not decompose practices; they provided PSTs with opportunities to engage in a full practice (e.g., teacher noticing) or in planning and enacting lessons. In Earnest and Amador (2019), PSTs planned and enacted a lesson using simulation technology.

As discussed, some studies provided opportunities for PSTs to engage in recomposed practices at the end of approximations. In the process of recomposition, several components of

Olanoff, D., Johnson, K., & Spitzer, S. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

practices are combined together; thus, recomposed practices are more complex than decomposed practices (Janssen et al., 2015). Even though PSTs engaged in both decomposed and recomposed practices in some studies, they did not have opportunities to learn the ways to recompose practices. Thus, PSTs may not have learned skills of improvising decomposed practices during their teaching. Further, the literature does not suggest which kinds of decomposition is more beneficial in developing PSTs' teaching skills. Consequently, the decomposed practices without concrete ways of recomposition might pose a risk of creating a boundary between teacher education programs and school contexts because PSTs cannot experience the social and cultural complexities of school contexts (Grossman et al., 2009; Campbell & Elliot; 2015).

Less authentic vs. more authentic setting. As defined in the literature, a factor determining the degree of authenticity is the extent to which the context of an approximation is similar to a school classroom setting. The literature suggested that PSTs engaged in the practice of teaching in three different settings: simulated environments (e.g., Amador et al., 2016; Webel & Conner, 2017), teacher education classrooms (e.g., Lampert et al., 2013; McDuffie et al., 2014), and school classrooms (e.g., Santagata & Yeh, 2014; Schack et al., 2013). PSTs often enacted lessons either in a simulated environment or in their classrooms; PSTs considered their peers and/or virtual student characters as their students, suggesting that PSTs could not experience student interactions with the social and cultural complexities of classrooms through approximations.

Researchers (e.g., Janssen et al., 2015) identified simulated environments as less authentic than PSTs' classroom contexts. They claimed that simulated environments do not necessarily preserve the complexity of teaching as PSTs have limited opportunities to develop skills to make moment-to-moment decisions and to respond to their students in real-time while creating animated classroom scenes (e.g., de Araujo et al., 2015). The studies involving role play claimed that they preserved authenticity by engaging PSTs in tasks that were similar to tasks they would do in school settings. For instance, Tyminski et al. (2014) claimed that PSTs were asked to consider and write authentic students' problem-solving strategies. However, in both simulations and in role play, PSTs could not experience ways of understanding and responding to students' cultural and social backgrounds. While the literature has indicated several approximations that provided PSTs with opportunities to practice anticipating and responding to students' thinking, the literature does not suggest how simulated virtual environments and teacher education classrooms prepare PSTs for the social and cultural complexities of school settings.

Scaffolded vs. independent enactment. The literature indicated that PSTs were scaffolded in different ways and in different times (before, after, and/or during lesson enactment). Scaffolding was provided in the form of specific frameworks/protocols, constructed dialogues, instructor-modeled activities, instructor or peer feedback, and lectures and tutorials. For instance, in McDuffie et al. (2014), PSTs were given noticing lenses (i.e., teaching, learning, task, power, and participation lens) to develop their noticing skills. In Ghousseini and Herbst (2016), teacher educators used constructed dialogues and asked PSTs to fill in the portions that were removed from those dialogues. In Bailey and Taylor (2015), teacher educators modeled problem posing in order to enhance PSTs' abilities to elicit and interpret students' thinking. In Leavy and Hourigan, PSTs were given lectures and tutorials on problem-posing skills. Overall, PSTs were scaffolded during approximations, and there were several forms of scaffolding. Since teachers are required to make most instructional decisions individually and independently, scaffolded practices are considered to be less authentic than independently enacted practices (Janssen et al., 2015).

Olanoff, D., Johnson, K., & Spitzer, S. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

Conclusion

As previously mentioned, eliciting, interpreting, and responding to student thinking was the most highlighted practice in approximations, suggesting that teacher educators attempted to prepare PSTs for responsive teaching through approximations. To develop this practice, PSTs were engaged with their peers, students' work samples, and planted students' errors. Review of the literature also suggested that PSTs' learning from approximations were explained in terms of what PSTs learned at the end of approximations. One example of such learning is the gain in PSTs' skills to respond to students' thinking at the end of approximations (Monson et al., 2020). Since teaching is both improvisational and interactional, which is based on students' thinking (Hunter et al., 2015), how these learned skills prepare PSTs for their actual teaching is a critical aspect of PSTs' learning through approximations. Yet, there is less attention on the literature about to what extent PSTs are able to improvise their responsive teaching skills that are learned through approximations to respond to students' thinking in real time during their teaching.

As discussed earlier, the degree of authenticity is a cause for concern in approximations, and researchers have often considered setting, scaffolding, and decomposition as three dimensions determining the degree of authenticity. Some approximations are more authentic than others, depending on what construct is aimed to be developed through approximations. In the literature, approximations involving role playing are characterized as more authentic than approximations involving simulations, which may only be valid for some specific practices. For example, role play would be more authentic than simulation if it aims is to develop PSTs' practice of 'responding to student thinking' because PSTs engage in a setting similar to the actual practice of role playing. However, approximations involving simulations would be more authentic than role playing if the focused practice is "anticipating student thinking" as PSTs anticipate students' responses while creating virtual scenes. Even though approximations are perceived to be less authentic both in terms of the nature of the activities and the setting of the practice, they seem productive for providing PSTs with opportunities to anticipate students' roles in their classes. Researchers have assumed that scaffolding reduces the authenticity of practices because PSTs do not get direct scaffolding during teaching. This study suggests that scaffolding does not always reduce authenticity; when and how PSTs receive scaffolding would determine the degree of authenticity. As such, if PSTs get feedback before or after engaging in a practice, it serves as scaffolding without reducing authenticity. After all, teachers are expected to get feedback and continue to improve their teaching throughout their teaching careers (Conference Board of the Mathematical Sciences, 2012). However, feedback provided in the middle of practice might decrease the authenticity as PSTs' decisions potentially depend on the feedback.

Based on these findings and discussion, teacher educators may not be able to fully determine the authenticity in approximations based on what PSTs learned at the end of approximations because teachers should be "seen as complex, sensible people who have reasons for the many decisions they make" (Leatham, 2006, p. 100). Even though teacher educators might perceive a practice to be transferable to school, novice teachers might not transfer because they struggle to negotiate the power dynamics within schools and teacher education programs (Trent, 2013). In fact, there is a less attention in the literature about the extent to which PSTs transfer the practices learned from approximations in their future teaching, suggesting a need for extending the research to examine the ways in which approximations can be most productive in informing PSTs' future teaching. Thus, the expansion of an alternative definition of authenticity is needed that incorporates the extent to which PSTs transfer the skills learned from approximations in their future teaching. This alternative conception of authenticity proposes that the degree of

authenticity should be based on the extent to which approximations provide a way to recompose small components of a practice in order to improvise practices learned from approximations in school contexts. Collectively, this dimension of authenticity enhances PSTs' ability to enact components of practices in their teaching. With an acknowledgement that this alternative definition does not include all possible dimensions of authenticity, it brings more depth to the current dimensions of authenticity in approximations.

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