ESTABLISHING STUDENT MATHEMATICAL THINKING AS AN OBJECT OF CLASS DISCUSSION

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Productive use of student mathematical thinking is a critical yet incompletely understood dimension of effective teaching practice. We have previously conceptualized the teaching practice of building on student mathematical thinking and the four elements that comprise it. In this paper we begin to unpack this complex practice by looking closely at its first element, establish. Based on an analysis of secondary mathematics teachers' enactments of building, we describe two critical aspects of establish—establish precision and establish an object—and the actions teachers take in association with these aspects.

Keywords: Classroom Discourse, Communication, Instructional Activities and Practices

The Association of Mathematics Teacher Educators [AMTE] Standards (2017) argued that an important component of whole-class instruction is the "intentional discussion of selected and sequenced student approaches... to move students through a trajectory of sophistication toward the intended mathematics learning goal of the lesson" (p. 16). This argument is supported by other related recommendations (National Council of Teachers of Mathematics [NCTM], 2014) that have highlighted the importance of productively using students' mathematical thinking as part of whole-class instruction. There are many different ways that teachers can productively use students' mathematical thinking, however, and these ways are determined, at least in part, by the nature of the thinking itself (Stockero et al., 2020). It has been posited that some instances of student thinking are of particular importance and that using them productively can be especially advantageous (Leatham et al., 2015).

That said, taking advantage of such instances requires coordinating a complex collection of teaching practices, and there is evidence that certain aspects of these practices do not occur naturally in whole-class instruction (Stockero et al., 2020). To better understand and improve teachers' ability to engage in complex practices, Grossman and her colleagues (2009) suggested that practices be decomposed into their "constituent parts" (p. 2069) for the purpose of helping teachers develop these practices. We have previously conceptualized the teaching practice of *building on student mathematical thinking* and the four elements that comprise it (see Van Zoest et al., 2016). In this paper we begin to further decompose this complex practice by looking closely at its first element, *establish*.

Theoretical Framework

Before describing the teaching practice of building, we first introduce the type of instances of student thinking that this practice is intended to take advantage of. As we have described elsewhere in greater detail (Leatham et al. 2015), MOSTs (**M**athematically Significant Pedagogical **O**pportunities to Build on **S**tudent **T**hinking) occur at the intersection of three critical characteristics of classroom instances: student mathematical thinking, significant mathematics, and pedagogical opportunity. Particularly relevant to this paper, MOSTs are observable instances of student mathematical thinking that provide sufficient evidence to “make reasonable inferences about student mathematics” (Leatham et al., 2015, p. 92).

When we say *building on student mathematical thinking* we mean the teaching practice that takes advantage of the opportunity that a MOST provides (Van Zoest et al., 2016). More specifically, we define *building on a MOST* (hereafter referred to as *building*) as making a MOST “the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea” (Van Zoest et al., 2017, p. 36). As we unpacked that definition in the context of our collective experience with analyzing teaching (our own and that of others), we theorized that building is comprised of four elements: (1) *establish* the student mathematics of the MOST as the object to be discussed; (2) *grapple toss* that object in a way that positions the class to make sense of it; (3) *conduct* a whole-class discussion that supports the students in making sense of the student mathematics of the MOST; and (4) *make explicit* the important mathematical idea from the discussion.

As part of our ongoing research, we have been investigating these elements. The current paper focuses on the first element and addresses this research question: What are necessary components of the *establish* element of building as revealed through teachers’ attempts to enact the practice?

Literature Review

Productively using student thinking during whole-class discussion involves teachers capturing the essence and relevant details of student contributions (a central purpose of the *establish* element of building). Thus, research on productive whole-class discussions sheds some light on this important facet of teaching, although it has seldom been the direct focus of studies. For example, van Zee and Minstrell (1997) described a *reflective toss*, which is a teacher response that “elicit[s] further thinking by *catching the meaning of the student’s prior utterance* and throwing responsibility for thinking back to the students” (p. 241, italics added). Another example comes from the work of Webb et al. (2014), who identified teacher moves that facilitate students “referencing *the details of another student’s idea*” (p. 88, italics added) as an important aspect of promoting students’ productive engagement with their peers’ mathematical thinking. Implicit in these findings is the need for the meaning and details of student contributions to be available for reference. Knowing more about capturing the essence and relevant details of student contributions (and thus about aspects of *establish*) is critical to understanding productive use of student mathematical thinking during whole-class discussion.

One significant contribution to understanding this preliminary facet of productively using student thinking is Staples’ (2007) model of a teacher’s role in supporting collaborative inquiry. A key component of this model, which was conceptualized through her longitudinal study of one high school teacher, is the work a teacher needs to do to *establish and monitor a common ground*. Staples identified a variety of instructional strategies that a teacher may use to establish student ideas as the common ground. One strategy was repeating student contributions and using multiple modes of communication (e.g., verbal, written) to provide students with a variety of opportunities to access one another’s ideas. Another strategy involved publicly recording ideas in a structured way on the board to provide some permanence of student contributions and to facilitate students’ development of an idea throughout an inquiry. Later she further elaborated by indicating that the goal of such practice is “not perfect use of vocabulary or formal sentences, but

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rather clear enough expression of ideas so that both the teacher and other students can consider the contribution" (Staples & King, 2017, p. 40). Our study builds on Staples' work in this area by investigating the *establish* element of building with multiple teachers who were conducting whole-class discussions around the same tasks and often the same student thinking. Broadening the pool of teachers and simultaneously focusing on comparable situations across them provided a rich data set that allowed us to more fully identify necessary aspects of *establish* and the subtleties that are involved in teachers accomplishing it.

Methods

In order to study our theorized practice of building we enlisted 12 teacher researchers—practicing secondary mathematics teachers who desired to more productively use their students' mathematical thinking. These teachers enacted the building practice in their classrooms using four mini tasks (see Figure 1) that were designed to elicit particular MOSTs, resulting in 27 building enactments. We compared these enactments to our initial conceptualization of building by coding transcripts of the enactments for actions that seemed to either facilitate or hinder the overall practice of building. Analysis of these coded data led to refinement of the four elements of building, including identifying necessary aspects of each and a variety of associated subtleties. With respect to the focus of this paper—*establish*—our analysis revealed both aspects of this practice and actions teachers might take to effectively position student contributions to become the object of discussion.

(a) Percent Discount The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?	**(c) Points on a Line** Is it possible to select a point B on the y-axis so that the line $x + y = 6$ goes through both points A and B? Explain why or why not. 
(b) Variables Which is larger, x or $x + x$? Explain your reasoning.	**(d) Bike Ride** On Blake's morning bike ride, he averaged 3 miles per hour (mph) riding a trail up a hill and 15 mph returning back down that same trail. What was his average speed for his whole ride?

Figure 1: The Four Mini Tasks Used in Creating Instantiations of Building

Results

We describe here two aspects of *establish*: *establish precision* and *establish an object*. (A third identified aspect, *establish intellectual need*, is beyond the scope of this paper.) In order to effectively position a student contribution (a MOST) to become the object of discussion teachers must establish (a) precision—the student contribution must be clear, complete, and concise so that the class can focus on making sense of that contribution, and (b) an object—the contribution must take on a measure of permanence and identity so that it can clearly be referred to during the remainder of building. In the following sections we elaborate on these two aspects, describing actions teachers take in association with each aspect. Note that although students might spontaneously take actions that contribute to making the contribution precise or an object, we

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focus here on the actions teachers take to ensure that these aspects are satisfied. Those teacher actions initiate the work, even though the actor could be students or the teacher.

Establish Precision

The first aspect of *establish* requires that the teacher ensure that the student contribution is clear, complete, and concise. Precision is important because making imprecise thinking the object of consideration is likely to hamper building (Peterson et al., 2020). By carrying out this aspect, the teacher establishes what it is the class is going to make sense of during the *conduct* element of building.

Of course, not all student contributions are imprecise; some are stated precisely to begin with. Precise contributions, however, were more the exception than the rule in our data. Analysis of the building enactments revealed a number of ways that student contributions were not precise enough for students to engage in making sense of them. In the following sections, we consider three actions that might be needed to establish the precision of a student contribution: clarifying, expanding, and honing.

Clarifying. Clarifying is about making clear WHAT the student has said. We discuss here two types of situations where clarifying actions may be needed. First, student contributions often need to be clarified because students use informal language or pronouns with vague referents (Peterson et al., 2020). For example, during a Percent Discount (Figure 1a) enactment, as a student was sharing their solution, they said, "Like you're subtracting it." The teacher followed up with, "Okay, subtracting what?" and the student replied, "50 percent." We see here that the teacher's question helped to make clearer this part of the student contribution by making the referent explicit. This type of clarifying also occurs when details that are naturally left out of a contribution due to conversational conventions, such as the prompt the student was responding to, are added in.

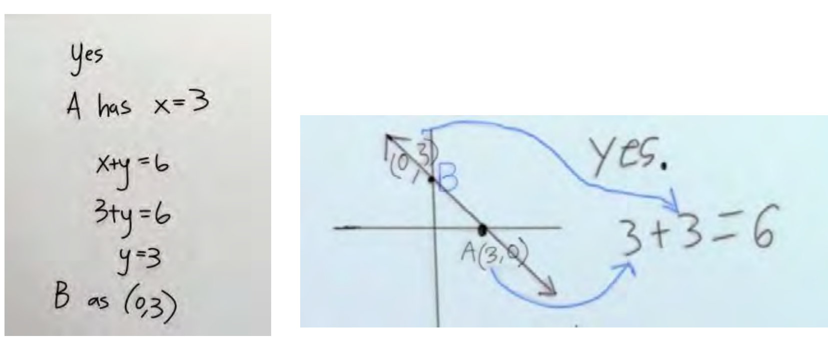
Second, student contributions need to be clarified when students share the substance of their reasoning, but the logical structure of those ideas are not clearly articulated. For example, consider this student contribution during a Variables (Figure 1b) enactment: "I believe that x plus x is larger because if x is just one value, x plus x would be double the value, which in this case makes it larger," and the teacher's response, "You were thinking x plus x is larger than x, because when you add the values, it makes it double, so it's larger?" Without changing the logic of the student's contribution, the teacher clarified the logical structure. By confirming with the student that the clarification was accurate, the teacher has clarified the contribution for other students in the class and kept the focus on the student contribution.

Expanding. Expanding is about making the contribution complete and involves adding something to the contribution that is needed to position the class to engage in making sense of it. The most common expansion situation that we saw in the enactments we analyzed was when a student provided an answer without reasoning. Student contributions that are merely an answer need to be expanded because the class will not be able to fully make sense of the contribution without the underlying reasoning behind that answer. For example, in response to Variables (Figure 1b), a student initially simply stated, "x + x is greater than x." Although the teacher knew the student had reasoning for their answer from monitoring students' work, without expanding the student's contribution to include the reasoning, the class would be left guessing about what exactly they were to make sense of. When asked to share their explanation for their answer, the student elaborated, "So x plus x will be 2x, and x will be just 1x." This expansion provided the necessary fodder for a sense-making discussion. This teacher expanded further when they responded, "So you're saying 2x is bigger than 1x, is that what you're saying?" This response seems to be important as it makes explicit the critical reasoning that was missing. As mentioned above, confirming that an inference is accurate keeps the focus on the student's contribution.

Student contributions also need to be expanded when reasoning is present, but a piece of information needed to make the contribution complete is missing. For example, during a discussion about Bike Ride (Figure 1d), a student wrote (a+b)/c on the board and explained that c represented the number of speeds. Although it is possible to infer what a and b represented, asking the student to explicitly define these variables made the contribution more complete. There was nothing unclear about what the student said, but their explanation did not provide all the information the class would need to make sense of it. In both these latter examples, rather than counting on students to guess the missing information or to infer the implicit information, the expansion made that information explicitly available, and resulted in a more complete contribution—one the class was better positioned to collaboratively make sense of.

Honing. Honing is about making the student contribution concise and involves reducing it to its essence. Sometimes a student contribution contains extra verbiage or extraneous information that is unnecessary for, and may even interfere with, making sense of the student contribution. Making these contributions concise requires removing unnecessary information that might distract students from the main sense-making opportunity. For example, during a Variables (Figure 1b) enactment, a student explained part of their reasoning as, "Because they have the same shirt, so they can be added together, so x plus x will be 2x." The teacher response honed the student contribution by taking up the "so x plus x will be 2x" piece of the student contribution and omitting the part about "the same shirt."

More often, honing is a matter of capturing ideas within a student contribution more succinctly by using symbols or other shorthand. For example, in a Points on a Line (Figure 1c) enactment, a student explained, "I put 'yes' because A has the point, like, its x equals 3," to which the teacher commented, "All right, so, 'Yes, A has x equals 3,'" as they wrote that same information on the board (Figure 2a, lines 1 and 2). The student continued their explanation, "And then the equation is x plus y equals 6, so then I just plugged in the x, which is 3, plus y, equals 6 and figured out y, it needs to be 3, and then just put point B as (0,3)." The teacher listened and carefully captured what the student was saying on the board (Figure 2a). In this case, the teacher made the student contribution more concise by the use of mathematical symbols. In a different Points on a Line enactment, part of a student explanation included, "3 plus 3 equals 6 where the first 3… come[s] from A and the second 3 comes from B." In Figure 2b, we see how, rather than writing down what the student said in words, the teacher concisely captured their contribution by drawing lines from the 3s in each of the two points on the line to the corresponding 3 in the equation. All of these honing actions contribute to making the student contribution more precise by making it appropriately concise.



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(a) (b)

Figure 2: Two Illustrations of Teachers Establishing Precision through Honing

Establish an Object

Beyond establishing what the student contribution is (just described in Establish Precision), *establish* also entails the work of ensuring that the student contribution is established as an object, as a "thing" that can be considered. The initial goal of establishing a contribution as an object is to support making the *grapple toss* as efficient and effective as possible; it is much easier to toss an object—and for students to then engage with it during the *conduct* element—when the object is well-defined. We have come to see objects as well-defined when they have a high degree of both permanence and identity.

Our analysis of building enactments revealed a number of teacher actions that have the potential to contribute to establishing the student contribution as an object. In the following sections we describe two main sets of teacher actions that seem to contribute to this aspect of *establish*: re-presenting the object, which makes it more permanent, and referring to the object, which contributes to the identity of the object.

Re-presenting the Object. Re-presenting happened most frequently in the enactments we analyzed when student contributions were first made public orally (as opposed to students initially sharing their work at the board or on a document camera). In order to set these contributions apart from the ongoing conversation, teachers can re-present them. Re-presentation acts serve to demarcate a student contribution from the ongoing discussion and thus give it a degree of permanence, a staying power that often does not exist with the numerous passing comments of classroom discourse. These re-presentation acts signal a pause in the ongoing dialogue and begin to create space for a new kind of activity—one that will make the student contribution the object of consideration.

One option is for the teacher to re-present the oral student contribution orally. Two common forms of re-presentation occurred in our data: repeating and revoicing. Consistent with the definitions of others (e.g., Chapin et al., 2009; Forman et al., 1998), repeating is when the entire object is restated with no replacement in language and revoicing is when the student contribution is paraphrased without changing its meaning. One benefit of re-presenting through revoicing is that the re-presentation may be a more precise object than the original. One risk of re-presenting through revoicing is that a poorly executed revoicing may result in an object that is less precise.

Another way to re-present an oral student contribution is to switch to a written presentation (as the teachers did in Figure 2). Creating a public record of an oral student contribution by, for example, inviting the student to write what they said on the board or by acting as scribe themselves, is a way for the teacher to take the somewhat ephemeral spoken word and make it more tangible. That is, the student contribution becomes something the teacher and students can hold on to, can refer to, can operate on. It creates, in essence, a physical object that can be referred to in the *grapple toss* and pursuant discussion. Creating a public record sets the MOST apart from other verbal contributions during a whole-class discussion, giving it a permanence that is difficult to achieve otherwise.

Referring to the Object. Another way that teachers establish the student contribution as an object involves referring to the thinking AS an object. In other words, treating the student contribution as an object makes it more of an object. Such referring creates a sense of identity for the object. But student contributions are complex entities (often several sentences in length). We have found a number of ways that teachers refer to student contributions, some of which have more potential than others to contribute to making the student contribution an object.

One way a teacher may refer to a student contribution as an object is to use a pronoun (i.e., that, this, it) for which everyone would likely know that the referent is the student contribution. For example, during a Percent Discount (Figure 1a) enactment, a student contributed, "Because you're adding fifty and then you're taking away fifty percent," and the teacher responded, "Say it again, what you just said." Members of the classroom would likely recognize that the "it" in "say it again" was referring to the entirety of the students' contribution, which helps to make the contribution an object. Furthermore, students would likely recognize that the phrase "what you just said" was also a somewhat generic way of referring to the student contribution.

A second way of referring to the object is to name it. We have seen teachers name student contributions by characterizing the nature of the thinking (e.g., this claim, this reasoning), attributing that thinking to the student by name (e.g., Tray's thinking)—and sometimes by doing both (e.g., Jaden's claim). Naming, a form of *metatalk* (Leinhardt and Steele, 2005), is a way of marking the student contribution so the class can access it again when the name is used.

A third way of referring to the object is for a teacher to point to or make a gesture toward a public record of the student contribution. The action of pointing at the board contributes to the student contribution being the object that the class is to focus on as the discussion continues.

The aforementioned ways of referring to a student contribution (pronouns, generic terms, naming, and pointing) vary in their potential to contribute to making a student contribution an object. Referring to the object by name seems to have the most potential for making the student contribution an object because a) the name reduces the potential for ambiguity in the referring, and b) because the name makes the student contribution easily identifiable for future reference.

Although we are not claiming that any particular subset of objectifying actions is necessary for "sufficiently" establishing the student contribution as an object, our analysis of teaching enactments suggests that re-presenting the contribution by creating a precise public record of it and referring to it by name (based on the nature and/or the contributor) provide a strong foundation for the *grapple toss*. Given the difficulties students have in focusing on making sense of a specific contribution (Franke et al., 2015; Webb et al., 2014), the more scaffolding we can provide in that regard, the more likely they are to maintain this focus.

Discussion & Conclusion

Establish is comprised of three aspects (see Figure 3), two of which were discussed in this paper: establish precision and establish an object. The goal of establish precision is to ensure that the student contribution is clear, complete, and concise, accomplished respectively by clarifying, expanding, and honing actions. The goal of establish an object is to ensure that the student contribution achieves a measure of permanence and identity, accomplished respectively by re-presenting and referring actions. In other words, we want the class to know exactly what the student contribution is and also position that contribution as an object that can easily be referred to and acted upon throughout the remaining elements of building.

Element	Establish						
Aspect	Precision			An Object		Intellectual Need	
Action	Clarifying	Expanding	Honing	Re-presenting	Referring		

Figure 3: Establish Broken Down by Aspects and Associated Actions

We conclude with several observations about these aspects of *establish*. First, whereas the work of establishing precision operates on the pieces of the student contribution in order to create a clear, complete, and concise object, the work of establishing an object operates on the object as an entity, re-presenting and referring to the entire object in order to make it more of an object. For instance, when expanding, one adds a piece to the object, and when honing, one removes a piece from or replaces a piece of the object. In contrast, when re-presenting through revoicing, one paraphrases the entire object, and when referring through naming, one names the entire object.

Second, the *establish* element of building is a teacher practice—it is always the teacher's responsibility to *ensure* that the student contribution is a precise object. The teacher does not always need to be the one who makes a student contribution a precise object, but they DO always need to consider WHETHER the contribution is precise and a well-defined object and take action if it is not. That said, there are many different ways that both the teacher and the contributing student carry out *establish* actions. For example, although the desired action might be "clarify," the teacher might invite the contributing student to clarify or they themselves might provide the clarification with a confirmation from the contributing student. As we have discussed elsewhere (Van Zoest et al., 2021), it is helpful to disentangle the actor from the action in order to unpack critical nuances of teacher responses to student mathematical thinking.

Third, although we have discussed these aspects and associated actions of *establish* discretely, generally they do not occur as such in practice. That is, teachers often accomplish multiple aspects of *establish* simultaneously. We see this in Figure 2a, where the teacher is engaged in honing (as discussed), as well as re-presenting by creating a public record of the student's oral contribution and clarifying the reasoning of the contribution by placing each piece of the logic on a separate line.

Finally, the individual actions we have identified are not new—they have been discussed to some degree in the literature. Furthermore, others have observed relationships between these actions and broader teacher practices, noting that it is valuable to consider actions (e.g., clarifying) with respect to "the purpose that those techniques are serving" (Boerst et al., 2011, p. 2854). Our work here illustrates the importance of coordinating a collection of actions in order to accomplish a particular purpose, in this case to *establish* a student contribution as part of the broader teaching practice of building on that contribution.

Unpacking *establish* has allowed us to better understand the complexity and craft of this critical element of building, better positioning us to work with teachers to develop their abilities to productively use student mathematical thinking.

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