EXPLORING CONNECTIONS BETWEEN PROSPECTIVE TEACHERS' VIEWS OF AUTHORITY AND EXPERIENCES IN JUSTIFICATION

Brenda Rosencrans
Portland State University
ros8@pdx.edu

David Brown
Portland State University
drb6@pdx.edu

Diana Salter
Portland State University
dsalter@pdx.edu

Eva Thanheiser
Portland State University
evat@pdx.edu

The purpose of this project was to understand how implicit views of authority support or limit prospective elementary teachers' (PTs) mathematical activity of justifying and to understand how the experience of justifying might support a development of an internal source of authority. In this case study of 18 PTs, we coordinate an analysis of 1) their responses to two justification tasks and 2) interview transcripts in which they discuss their experiences in learning to justify. Preliminary results indicate ways in which their views of authority limited their reasoning about mathematics by not recognizing their own sense-making and supported a sense-making exploration of mathematics that was freeing and empowering. These results provide mathematics teacher educators with insight to help them identify and address limiting views of authority and leverage productive views of authority.

Keywords: Preservice Teacher Education, Reasoning and Proof, Teacher Beliefs

Purpose of the Study

Attending to this year's theme of "persevering through challenges", we address a familiar challenge encountered in mathematics teacher education. When prospective elementary teachers (PTs) enter content courses, they typically hold limiting views of mathematics as memorizing procedures. They often are not aware that mathematics makes sense and that procedures can be justified (Ball, 1990; Feiman-Nemser, 2001; Ma, 1999; Spitzer et al., 2010; Thanheiser, 2009), nor do they generally view themselves as a source of authority for reasoning about mathematics as mathematical sense-makers (Cady et al., 2006; Perry, 1970; Povey, 1997). As mathematics teacher educators (MTEs), we want our students to take ownership of their mathematical sense-making. Explaining and justifying one's thinking are activities that support the vision of sense-making and argumentation described in national documents: "By developing ideas… justifying results, and using mathematical conjectures… at all grade levels, students should see and expect that mathematics makes sense" (NCTM, 2000, p. 56).

When student generated mathematical contributions are validated through collaborative reasoning, students are supported in developing the skills of explaining and justifying their thinking along with assessing the validity of that thinking (Gresalfi & Cobb, 2006; Reinholz, 2012). Such support is necessary for students to develop an internalized authority that is based on sense-making through their own reasoning, rather than relying on an external source of authority represented by experts such as the teacher or textbook (Boaler & Selling, 2017; Engle & Conant, 2002; Lampert, 2003; Reinholz, 2012; Schoenfeld, 1994). To better understand how to support students in taking ownership of their sense-making, we use the context of a justification-feedback-revision cycle to explore the connections between PTs' justifications, their process of justification, and their views of authority and sense-making in the mathematics classroom.

Olanoff, D., Johnson, K., & Spitzer, S. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

Understanding how PTs view authority to reason about mathematics in the classroom provides valuable insight into how MTEs can uncover these views, support PTs in interrogating their views, and then help PTs learn how to use their authority to contribute ideas and evaluate the reasonableness of contributed ideas. In this proposal we share how one class of 18 PTs engaged in a justification-revision cycle and argue that attending to PTs' views of authority provides insight into (1) potential barriers to learning to justify and (2) productive views of authority that support learning to justify.

To support this argument, we seek to answer the following questions:

1. How do PTs' descriptions of their justification process provide insight into their views of authority?
2. How do PTs' views of authority support or limit their justification?

Perspective/Theoretical Framework

As justification and authority are central to our study, we unpack these two constructs and explain how the relationship between them frame our work (see Figure 1).

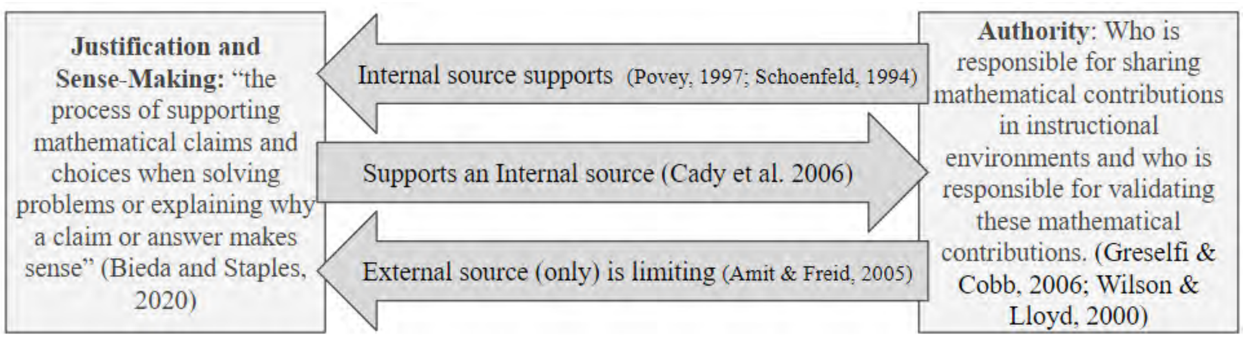


Figure 1: Framework that connects authority and justification

Authority

We define authority as who (or what) is responsible for sharing mathematical contributions in instructional environments and who (or what) is responsible for validating these mathematical contributions (Gresalfi & Cobb, 2006; Wilson & Lloyd, 2000). In instructional environments, students engage with a web of authority that includes instructors, their peers, themselves, textbooks, and other authorities in their life (Amit & Fried, 2005). The development of an internalized source of mathematics authority in which students view themselves as an authority is associated with the development of mathematical sense-making abilities (Povey, 1997; Schoenfeld, 1994). This view of the self as an authority supports sense-making because it carries the expectation that the students take on responsibility for reasoning about what makes sense – first through sharing their own ideas, and then through providing an explanation that justifies their thinking and solution to their peers and instructor. This view is in contrast to the expectation that the teacher or other external sources is responsible for telling them what makes sense and what is correct, a view that limits students' sense-making.

Viewing mathematics authority as residing in the teacher negatively impacts students' conceptual mathematical thinking "by turning always from one figure to another, and never to themselves, the students …fail to develop their own mathematical thinking," (Amit & Fried,

Olanoff, D., Johnson, K., & Spitzer, S. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

2005, p. 165). In addition, they position "themselves as outsiders with respect to mathematical discourse" (ibid). Undergraduate students develop an internal source of authority as they learn to think critically and develop persuasive arguments (Povey, 1997). A shift in authority from external to internal becomes apparent when students "support their opinions with reason and logic" (Cady et al., 2006, p. 296), in other words, when they justify.

Justification

Students are sense makers (Ernest, 2000) and justification is essential to sense-making (Bieda & Staples, 2020). We adopt Bieda and Staples' (2020) definition of justification as "the process of supporting mathematical claims and choices when solving problems or explaining why a claim or answer makes sense," (p. 103). (See left box in Figure 1). We think of justification as a way of communicating understanding (Jaffe, 1997), and as distinct from mathematical proof, which is a final product, in that a justification does not have to be logically complete (Melhuish et al., 2020). A justification should seek to convey structure and generality if applicable. For example, for their first justification task, PTs were asked to determine if the sum of two odd numbers is always odd, always even, or sometimes odd and sometimes even, and then justify their thinking. While we did not expect them to use a number theoretic approach to the justification, they were encouraged to attend to the structure of an odd number as a collection of groups of two and one left over, or as two groups of the same size with one left over and use one of these definitions of odd to argue a general case (see Table 1 in the next section for examples).

PTs experience challenges in learning to justify and in supporting children in learning to justify. (G. J. Stylianides et al., 2013). PTs often conflate justification with providing/checking multiple examples rather than viewing justification as a general argument based on mathematical properties and definitions of terms (Harel & Sowder, 2007). Teachers (including PTs) need to develop a common language and understanding of justification so they can understand what justification and proving look like in an elementary classroom and can support their students in this activity (Harel & Sowder, 2007; Staples & Lesseig, 2020; A. J. Stylianides, 2007). We add to the literature by building on our understanding of PTs' justifications. We coordinate this understanding with their views of authority through examining the limiting/supporting relationship between views of authority and justification (see Figure 1).

Methods

Participants

The 18 PTs participating in this study were enrolled in their first mathematics content course for prospective elementary teachers. The study was conducted at an urban public university in the Pacific Northwest of the United States. PTs at this university were required to complete a sequence of 3 mathematics content courses to enter their teacher education program. This course was taught by one of the authors.

Context

This first course in the sequence was inquiry-based with the goal to develop students' mathematical knowledge for teaching (Ball et al., 2008; Hill et al., 2008) and the expectation that students share their reasoning as a learning community. The main topics of the course were number and operation with an emphasis on sense-making through justifying, representing ideas in multiple ways, and making connections between these multiple representations.

For the present study, the course was taught asynchronously online via an online learning platform and shared Google slides. Emphasis was placed on the value of reviewing and reflecting on previous work and providing feedback to their classmates with the intention of positioning PTs as sense-makers and as sharing responsibility for theirs and their classmates' learning as a part of a community. To support PTs in sense-making and justification, they were asked to complete multiple cycles of (a) sharing a rough draft (Jansen, 2020) of a justification, (b) reviewing and providing feedback on other PTs' justifications, and (c) revising their initial draft based on the feedback they received from classmates and the instructor. In this study, we focused on their first two justification tasks, in which they completed the following statements and then justified why the statements were true:

- The sum of any two odd numbers is [always odd, always even, sometimes odd/even].
- The sum of three consecutive numbers is [always, sometimes, never] divisible by three.

Data Collection

Data includes (a) PTS' written rough draft and revised justifications provided via Google slides, and (b) transcripts of an hour-long interview with PTs conducted via Zoom. Interviews were conducted by the lead author during week six of a ten-week term after PTs had completed both justification-revision cycles. The semi-structured interviews included questions asking PTs to describe the process they went through when creating their justification, how confident they were that their response was a valid justification, and how (and to what extent) they utilized classmates' 'ork and the instructor's and classmates' feedback.

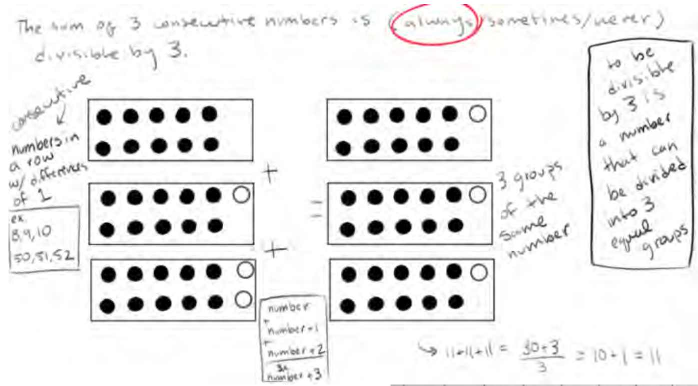
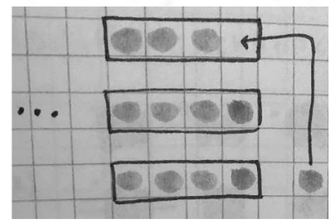
Data Analysis

To analyze our data, we used an inductive approach (Thomas, 2006). We started with reading through the PTs' responses to the justification tasks (provided in our shared Google slides) and their interview transcripts and recording observations about (1) how PTS justified, (2) key moments in the interviews in which PTs described their experience in justifying, and (3) statements from the transcripts that provided insight into their views of authority. After an initial pass through our data, we then developed categories as described below.

Justification Data. We analyzed PTs' draft (D) and revised (RIesponses to the two justification prompts (see above). Since the PTs were introduced to the construct of convincing yourself, convincing a friend, and convincing a skeptic to characterize PTs' levels of justifications in the class (Mason et al., 1982), we leveraged these categories in our analysis to code their justifications (see Table 1). Note that the code *Misinterpreted* was used to code PTs' justifications that misinterpreted the prompt. Two of the authors coded each of the justifications individually and met to resolve disagreements through debate.

Table 1: Coding Scheme for Justifications

	Description	Example for: The sum of three consecutive numbers is [always, sometimes, never] divisible by three.
Self	PTs relied solely on specific examples	Example: 4+5+6 = 15 Justification: 15/3 = 5
Friend	PTs described general structure and didn't yet use this description of structure to argue for the general case,	If you add any 3 same groups of #s together, that large sum can always be separated again back into 3 equal parts. Because consecutive numbers are 3 of the same number plus an additional 3, that additional 3 that is added into the numbers to make them consecutive, can therefore be separated into 3 equal parts. (PT 1)

	or their argument was unclear	
Friend/Skeptic	PTs written justification is a general argument based on structure and the visual representation uses specific examples	
Skeptic	PTs written justification and visual representation that build a general argument based on structure.	The sum of three consecutive numbers is ***ALWAYS*** divisible by 3 because when you add 3 consecutive numbers, it is equal to adding the first number in the sequence times 3 plus 3. 3 consecutive numbers added together can be grouped into 3 equal groups. 

Interview Data

To create an initial analysis of the interview data, the first author wrote brief descriptions of each interview summarizing each PTs' general approach to justification and how their discussion of their justification process provided insight into their view of authority. To further this analysis, we reviewed the interview transcripts and identified moments the PTs talked about sources of authority they looked to while writing their justifications (self, peers, instructor, previous experience, etc.) and how they determined that their justification was valid. For example, in describing their experience with justification, PT 1 said she did not understand "what you guys wanted" and that she didn't "know what you need." This indicated to us that the student was seeking verification from an external source of authority. In contrast, PT 2 shared, "For some reason, especially math, when I learned something that I can connect something on my own. I feel so much more accomplished!" This indicated that this student was starting to view this responsible for their own sense-making, indicating an internal source of authority. After all PTs' responses were read with such moments indicated, we categorized the PTs as having a primarily external source of authority, internal source of authority, or mixed. The mixed source of authority emerged when a student alternated between describing internal and external sources of authority. It is important to note that these interviews are snapshots of one moment during a 10-week term. Thus, their discussions of justification give us insight into their views of authority at the moment of their interview, but do not yet provide a comprehensive description of their overall orientation toward authority.

Results

Justification

Results from our analysis of the PTs' justifications are shown in Table 2. In general, the PTs did attend to the mathematical structure of the concepts they were justifying, as shown by the prevalence of Friend and Friend/Skeptic justifications. However, their justifications demonstrated mixed success with developing logic to link the structure to the concept. For example, for the first justification, 9/18 students' justifications were Friend level with missing or unclear logic preventing the justification from being coded Friend/Skeptic. Including a generalized visual representation of their justification proved to be the challenge that limited many of the justifications to Friend/Skeptic rather than reaching the level of Skeptic. While students demonstrated some success with written generalizations of their justifications, with few exceptions, visuals were limited to "dot drawings" of one or two examples.

Table 2: Results of Justification Coding

Justification	Misunderstood	Self	Friend	Friend/Skeptic	Skeptic
Sum of 2 Odds (D)	2	3	9	4	0
Sum of 2 Odds (R)	0	2	9	7	0
3 Consecutive #s (D)	5	5	3	5	0
3 Consecutive #s (R)	1	3	4	8	2

Authority

In characterizing PTs' views of authority (as seen during their week 6 interview) we found that of the 18 PTs, 8 provided evidence of viewing themselves as an authority (internal), 6 PTs primarily looked to an external source of authority, and 4 provided evidence of a mixed view of authority. From the analysis of interview transcripts, we selected two PTs that represent contrasting examples of views of authority. The two examples are contrasting because one PT's descriptions indicated more external views of authority, whereas the other was more internal. We used the following figure (Figure 2) to represent the distribution of how we categorized PTs' views of authority and how each of 18 PTs' justifications were coded in our analysis. We then trace PT 1's and PT 2's level of justification.

Olanoff, D., Johnson, K., & Spitzer, S. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

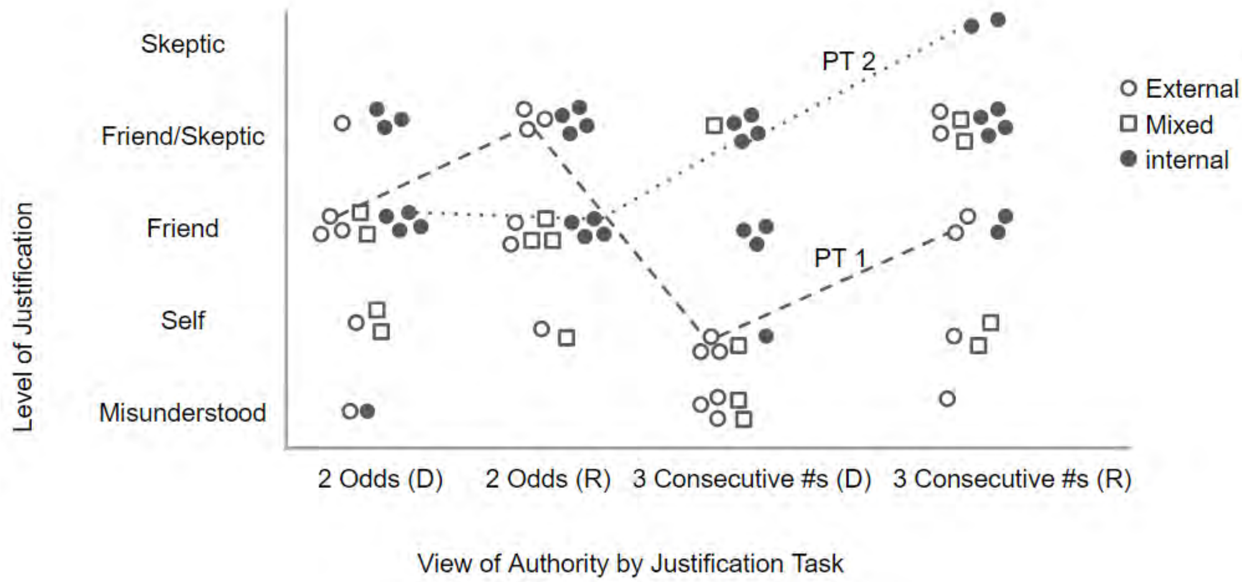


Figure 2: PTs Views of Authority and Level of Justification

PT 1. We categorized PT 1 as primarily looking to external sources of authority. Overall, in terms of justifying, PT 1 struggled with providing a clear argument. For example, for the first justification PT 1 revised her justification from the level of Friend to a Friend/Skeptic, but in the second justification, her justification started at Self and improved to the level of Friend after revision. In her interview she shared a desire to explain the math behind a concept while also wanting to know what "we," as the instructors, wanted for a solution. She explained that she did not understand "what you guys wanted" and that she did not "know what you need." She also shared that she found this difficult because the instructor did not provide an outline of an expected solution. At times, PT 1 did say that her explanation made sense to her: "I'm just like, I think this works. I hope it works. I don't know if it works, but it makes sense to me." This suggests that PT 1 is experiencing tension between viewing herself as responsible for determining what consists of a valid justification and wanting to meet the external standards of what the instructor wants for a justification to be considered valid.

PT 2. In contrast, we categorized PT 2 as having an internal source of authority. PT 2 clearly articulated coming to understand mathematics through her own sense-making. Throughout the interview, she mentioned first wanting to remember "math I was taught" but then recognizing that she could create her justification based on her own understanding, saying "for some reason, especially math, when I learned something… or like I can connect something on my own… I feel so much more accomplished!" In addition to viewing herself as an authority, she also mentioned looking to the instructor as a source of authority in sharing her uncertainty about "what [the instructor] was looking for" and looking to her classmates' slides as a source of authority for contributing mathematical ideas. She stated that she used her classmates' slides to help her know what to do on the first justification. While PT 2 viewed herself as a primary source of authority, she also recognized there is value in seeking help from the instructor, and her classmates. This illustrates how PTs with an internal source of authority incorporate, but do not replace, other external sources of authority with their own internal source of authority.

Discussion

PTs in this study struggled to provide justifications that were general both verbally and/or visually and the prevalence of the Friend and Friend/Skeptic codes indicate that PTs struggled with providing generic examples through a visual representation. This is consistent with current literature in our field (Lo et al., 2008; Martin & Harel, 1989; Rø & Arnesen, 2020). This analysis gives us context as we examine PTs' views of authority. The interview analysis indicated that nearly half (8 of the 18) of the PTs viewed themselves as primary sources of authority, while the others (10 of the 18) looked primarily to external authorities or had mixed views of authority. The purpose of this study is to explore potential limiting and supportive views of authority as evidenced through their discussion of their justification process. We turn now to a comparison of their justifications and their views of authority.

To address our first research question, "How do PTs descriptions of their justification process provide insight into their views of authority?" we analyzed PTs' interview transcripts as described in the results section above, finding that 7 of 18 PTs' first justifications were at the level of friend/skeptic and 10 of 18 PTs' second justification were at or above the level of friend/skeptic. We use this analysis to now address our second research question, "How do PTs' views of authority support or limit their justification?" and look back to PT 1 and PT 2 as illustrative examples, comparing their justifications with our findings about their views of authority.

PT 1

During the interview, PT 1 explained that she primarily looked to external sources of authority for validation. If we look at PT 1's justifications, we see that she did not develop a robust understanding of justification with her second justification remaining at the level of friend (see Table 1). However, several times in her interview she explained that "it makes sense to me, I don't know why." This is a productive place to start – recognizing the need for mathematics to make sense – to develop a robust understanding of justification. However, the need to "do what the instructor wants" appeared to limit her reasoning about each justification task. Instead of reasoning about the examples she had tried out, she expressed that she felt lost because she did not have an outline to follow, saying "Like there's no outline so I'm like, I don't know what you need." PT 1's description of this tension in determining whether or not she had a valid justification indicates that her need for external validation limited her efforts to produce a valid justification by way of making sense.

PT 2

In her interview, PT 2 described a process of discovery as she reflected on her experience in learning to justify. Her growing awareness that she can make sense of mathematics and does not need to rely on rules that she was taught, supported her exploration and reasoning. Several times in her interview she expressed that "When I figured it out, I was so glad!" and "It was like, just a cool connection to make!". PT 2 developed an understanding of justification that aligns with the course goals, i.e., she describes justification as a process of making sense, and we see the result of this in the improvements that were observed across justifications 1 and 2. Her work for her initial justification 1 and revision were coded at the same level of Friend/Skeptic and her justification 2 was coded as Skeptic (see Table 1). PT 2's view of herself as someone who could make sense of mathematics supported her experience in learning to justify.

Conclusion

In this study we observed PTs developing sense-making and mathematical reasoning skills through justification. We saw how students whose ideas about learning mathematics were focused on remembering what they had learned or trying to "do what the instructor wanted" limited their exploration of these tasks and contributed to their sense of frustration. PT 1's story illustrates this experience. In contrast, we saw students excited about their growing awareness that they can reason about mathematics for themselves, that they could contribute ideas in the instructional space through our shared Google slides, and could learn from their classmates' work. PT 2's experience illustrates this freedom in exploring mathematical ideas. Our PTs' descriptions of their experiences in learning to justify provides insight into views of authority PTs hold and provide evidence that what we see in their justifications do not tell the entire story. On the surface, reviewing their justifications does not explain the reasons for their incomplete justifications. Understanding how their views of authority set up barriers to reasoning about mathematics informs our work as mathematics teacher educators (MTEs). For example, noticing when PTs may view justification as writing "what the teacher wants" helps us identify this barrier and then address it through dialogue about a) the value of sharing one's initial understanding of a task and b) how to build on this understanding to reason about mathematics and "justify *your thinking"*. Furthermore, identifying moments when PTs use their authority to reason about mathematics helps us to support and leverage these moments and to alleviate uncertainty PTs may experience about sharing their reasoning. It is when we identify these barriers and address them, and when we identify productive views of authority and encourage these views, that MTEs will be better able to support PTs in justifying their reasoning and make sense of mathematics. Future studies can build upon this understanding and examine the impact of different teaching practices and tasks that are designed to support PTs in developing their internal source of authority.

References

Amit, M., & Fried, M. N. (2005). Authority and Authority Relations in Mathematics Education: A View from an 8th Grade Classroom. *Educational Studies in Mathematics*, *58*(2), 145–168. https://doi.org/10.1007/s10649-005-3618-2

Ball, D. L. (1990). The Mathematical Understandings That Prospective Teachers Bring to Teacher Education. *The Elementary School Journal*, *90*(4), 449–466. https://doi.org/10.1086/461626

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389–407. https://doi.org/10.1177/0022487108324554

Bieda, K. N., & Staples, M. (2020). Justification as an Equity Practice. *Mathematics Teacher: Learning and Teaching PK-12*, *113*(2), 102–108. https://doi.org/10.5951/MTLT.2019.0148

Boaler, J., & Selling, S. K. (2017). Psychological Imprisonment or Intellectual Freedom? A Longitudinal Study of Contrasting School Mathematics Approaches and Their Impact on Adults' Lives. *Journal for Research in Mathematics Education*, *48*(1), 78–105. JSTOR. https://doi.org/10.5951/jresematheduc.48.1.0078

Cady, J., Meier, S. L., & Lubinski, C. A. (2006). Developing Mathematics Teachers: The Transition From Preservice to Experienced Teacher. *The Journal of Educational Research*, *99*(5), 295–306. https://doi.org/10.3200/JOER.99.5.295-306

Engle, R. A., & Conant, F. R. (2002). Guiding Principles for Fostering Productive Disciplinary Engagement: Explaining an Emergent Argument in a Community of Learners Classroom. *Cognition and Instruction*, *20*(4), 399–483. https://doi.org/10.1207/S1532690XCI2004_1

Ernest, P. (2000). Why teach mathematics? In S. Bramall & J. White (Eds.), *Why Learn Maths?* (pp. 1–14). Bedford Way Papers.

Feiman-Nemser, S. (2001). *From Preparation to Practice: Designing a Continuum to Strengthen and Sustain Teaching*. http://bir.brandeis.edu/handle/10192/33196

Gresalfi, M. S., & Cobb, P. (2006). Cultivating students' discipline-specific dispositions as a critical goal for pedagogy and equity. *Pedagogies*, *1*(1), 49–57.
Harel, G., & Sowder, L. (2007). Toward comprehensive perspectives on the learning and teaching of proof. *Second Handbook of Research on Mathematics Teaching and Learning*, *2*, 805–842.
Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking Pedagogical Content Knowledge: Conceptualizing and Measuring Teachers' Topic-Specific Knowledge of Students. *Journal for Research in Mathematics Education*, *39*(4), 372–400.
Jaffe, A. (1997). Proof and the Evolution of Mathematics. *Synthese*, *111*(2), 133–146.
Lampert, M. (2003). *Teaching problems and the problems of teaching*. Yale University Press.
Lo, J.-J., Grant, T. J., & Flowers, J. (2008). Challenges in deepening prospective teachers' understanding of multiplication through justification. *Journal of Mathematics Teacher Education*, *11*(1), 5–22. https://doi.org/10.1007/s10857-007-9056-6
Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Erlbaum.
Martin, W. G., & Harel, G. (1989). Proof Frames of Preservice Elementary Teachers. *Journal for Research in Mathematics Education*, *20*(1), 41–51. https://doi.org/10.5951/jresematheduc.20.1.0041
Mason, J., Burton, L., & Stacey, K. (1982). *Thinking mathematically*. Addison-Wesley.
Melhuish, K., Thanheiser, E., & Guyot, L. (2020). Elementary school teachers' noticing of essential mathematical reasoning forms: Justification and generalization. *Journal of Mathematics Teacher Education*, *23*(1), 35–67. https://doi.org/10.1007/s10857-018-9408-4
NCTM. (2000). *Principles and Standards for School Mathematics*. http://www.nctm.org/flipbooks/standards/pssm/index.html
Perry, W. (1970). *Forms of intellectual and ethical development in the college years: A scheme*. Holt, Rinehart and Winston.
Povey, H. (1997). Beginning mathematics teachers' ways of knowing: The link with working for emancipatory change. *Curriculum Studies*, *5*(3), 329–343. https://doi.org/10.1080/14681369700200016
Reinholz, D. L. (2012). Reinholz, D. (2012). Becoming a mathematical authority: The solution lies in the solution. In L. R. V. Zoest, J.-J. Lo, & J. Kratky (Eds.), *Proceedings of the 34th [a]nnual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Western Michigan University.
Rø, K., & Arnesen, K. K. (2020). The opaque nature of generic examples: The structure of student teachers' arguments in multiplicative reasoning. *The Journal of Mathematical Behavior*, *58*, 100755. https://doi.org/10.1016/j.jmathb.2019.100755
Schoenfeld, A. H. (1994). Reflections on doing and teaching mathematics. In A. H. Schoenfeld (Ed.), *Mathematical thinking and problem solving* (pp. 53–70). Erlbaum.
Spitzer, S. M., Phelps, C. M., Beyers, J. E. R., Johnson, D. Y., & Sieminski, E. M. (2010). Developing prospective elementary teachers' abilities to identify evidence of student mathematical achievement. *Journal of Mathematics Teacher Education*, *14*(1), 67–87. https://doi.org/10.1007/s10857-010-9141-0
Staples, M., & Lesseig, K. (2020). Advancing a Teacher-Centered Perspective on Support-for-Claims Terminology. *For the Learning of Mathematics*, *40*(1), 28–35.
Stylianides, A. J. (2007). The Notion of Proof in the Context of Elementary School Mathematics. *Educational Studies in Mathematics*, *65*(1), 1–20. https://doi.org/10.1007/s10649-006-9038-0
Stylianides, G. J., Stylianides, A. J., & Shilling-Traina, L. N. (2013). Prospective Teachers' Challenges in Teaching Reasoning-and-Proving. *International Journal of Science and Mathematics Education*, *11*(6), 1463–1490. https://doi.org/10.1007/s10763-013-9409-9
Thanheiser, E. (2009). Preservice Elementary School Teachers' Conceptions of Multidigit Whole Numbers. *Journal for Research in Mathematics Education*, *40*(3), 251–281. eric.
Wilson, M. (Skip), & Lloyd, G. M. (2000). Sharing Mathematical Authority with Students: The Challenge for High School Teachers. *Journal of Curriculum and Supervision*, *15*(2), 146–169.