# DIFFERENTIAL BACKWARD TRANSFER EFFECTS FOR STUDENTS WITH DIFFERENT LEVELS OF LINEAR FUNCTION REASONING ABILITIES

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*Backward transfer is when learning about a new topic influences an individual's prior ways of reasoning about a topic they previously learned about. This study looked at how quadratic functions instruction differentially influenced students' prior ways of reasoning about linear functions. Specifically, we compared students at three levels of reasoning about linear functions, low-, mid-, and high-level, using a pre/posttest design that bracketed a two-week quadratic functions math program. Results showed that students at different reasoning levels experienced different backward transfer effects, that particular mathematical reasoning processes were most involved in the effects, and that the effects spanned two dimensions of productiveness of mathematical reasoning. Results from this study are significant for better understanding the construct of backward transfer, and have implications for teaching quadratic functions.*

Keywords: algebra and algebraic thinking; cognition; learning theory

The study reported in this article integrates two ideas that thus far have not yet been intentionally studied together. The first idea is that when individuals learn about a new concept ($C_2$), that learning may have the unintended side-effect of influencing the individuals' ways of reasoning about a previously-encountered concept ($C_1$) (i.e., a concept they previously learned about and already developed ways of reasoning about). We call this effect *backward transfer* (BT) (Hohensee, 2014). A number of studies have reported a variety of BT effects (e.g., Bagley et al., 2015; Hohensee, 2014; Melhuish & Fagan, 2018; Van Dooren, 2004). Importantly, these studies have also shown there can be different BT effects for different students.

The second idea is that students develop ways of reasoning that are more or less productive. Greeno (1989) characterized *productive ways of reasoning* about problem situations as when reasoning is deeply embedded in a problem situation and when that reasoning accounts for the essential properties and relations of that problem situation.

Our study examined the interplay between these two ideas. To explain what we mean, imagine two students with different pre-established ways of reasoning about $C_1$, who participate in the same learning experiences about $C_2$. How the BT effects on those students' ways of reasoning about $C_1$ might compare is an open question. Our study set out to make these kinds of comparisons.

Insights these comparisons reveal would be consequential for the development of BT theory because research thus far has only looked at *what* effects are produced (e.g., Hohensee, Gartland, Willoughby, & Melville, 2021), without trying to account for *how* those effects are different across students. Insights would also be consequential for future research on BT because, once more is known about differences in BT effects across students, future research can examine how

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to address the differences (e.g., find ways to promote backward transfer that enhances productiveness of ways reasoning about $C_1$).

## Theoretical Perspective

Our theoretical perspective has three parts: part one is about mathematical reasoning in general, part two is about what productive mathematical reasoning is (and is not), and part three is about BT effects on mathematical reasoning. It is the relationship between productiveness of mathematical reasoning and BT effects on reasoning that is the topic of our study.

### Mathematical Reasoning

Our theoretical perspective on mathematical reasoning aligns with Jeannotte and Kieran's (2017) view that all mathematical reasoning is made up of thought and communicational elements that are organized on two interrelated dimensions, a process dimension and a structural dimension.[1] The process dimension refers to the steps taken by thought or communicational elements to reach an intended mathematical goal. Jeannotte and Kieran specify nine such processes: generalizing, classifying, comparing, identifying a pattern, validating, justifying, proving, formal proving, and exemplifying. Of the nine processes, classifying, comparing, and identifying a pattern were most central to our study.

One type of reasoning important for our study was *quantitative reasoning*. As we conceive it, quantitative reasoning requires the processes of *classifying* and *comparing*. Classifying is defined as the process of inferring "a class of objects based on mathematical properties and definitions" (Jeannotte & Kieran, 2017, p. 11). For an example of a falling rock, two quantities that could be classified are the distance and time the rock falls. Additionally to measure a particular quantity, comparisons must be made between amounts of a quantity and a standard of measure for that quantity (e.g., compare a meter stick to the distance a rock falls).

A second type of reasoning important for our study was *covariational reasoning*. As we conceive it, covariational reasoning requires the process of *comparing*. Comparing is defined as "the search for similarities and differences [to infer a] narrative about mathematical objects or relations" (Jeannotte & Kieran, 2017, p. 11; parenthetical added). During covariational reasoning, what is being compared are the ways "two varying quantities . . . change in relation to each other" (Carlson et al., 2002). For the falling rock example, corresponding differences in distance and time could be compared. Note that while quantitative and covariational reasoning are tied to classifying and/or comparing, other process likely also play a role (e.g., making *generalizations* during quantitative reasoning, justifying one's covariational reasoning, etc.).

### Productiveness of Mathematical Reasoning

Our theoretical perspective on productiveness of mathematical reasoning aligns with Greeno (1989), who characterizes productiveness on four dimensions, two of which are the following: (a) the degree to which reasoning is deeply embedded in the problem situation, and (b) the degree to which reasoning accounts for essential properties and relations in a problem situation.[2] According to our interpretation of these dimensions, when comparing students, those who engage in *more* of a particular kind of reasoning (e.g., more of the same kind of classifying or comparing) in ways that are relevant to a particular problem situation, are more deeply embedded in the problem situation. Similarly, we interpret those students who engage in the kinds of classifying and/or comparing that is more relevant to the problem situation, as better accounting for the essential properties and relations of the problem situation. Thus, these are two ways students' reasoning can be categorized in terms of its productiveness.

### Backward Transfer Influences on Mathematical Reasoning

Finally, our theoretical perspective on how BT influences mathematical reasoning is based on Lobato's (2012) perspective on forward transfer, which is that transfer is "the influence of a learner's prior activities on her activity in novel situations" (p. 233). BT, which is "the influence that constructing and subsequently generalizing new knowledge has on one's ways of reasoning about related mathematical concepts that one has encountered previously" (Hohensee, 2014, p. 136), aligns with Lobato's perspective on forward transfer because we too were interested in studying *all* influences. However, our definition of BT also departs from Lobato's definition of forward transfer because we were interested in influences on reasoning about previously-encountered concepts by new knowledge, rather than in the opposite direction.

A number of other mathematics education researchers have observed this phenomenon (e.g., Bagley et al., 2015; Lima & Tall, 2008; Melhuish & Fagan, 2018; Van Dooren et al., 2004). However, to our knowledge no studies have specifically examined the relationship between BT effects and productivity of mathematical reasoning. In this study, we were driven by the following research question: *In what ways are BT effects similar and/or different for students whose prior ways of reasoning are more productive (e.g., deeply embedded in a problem situation) compared to students whose prior ways of reasoning are less productive (e.g., not grasping as essential properties or relations of a problem situation)?*

## Methods

### Setting and Participants

Our study took place during a summer mathematics program in the Mid-Atlantic region of the United States. Participants were recruited from an organization that helps students of color enhance their college readiness. The students' grade-level ranged from 9th to 11th grade. Our study was centered around a two-week summer math program on quadratic functions. The program took place at a local university and was taught by the primary investigator. Students had two 60-minute lessons per day. This study focused on data from four students whose reasoning about linear functions represented varying levels of productiveness.

### Procedure

The study began on the first day of the math program with a linear functions paper-and-pencil pre-assessment. The students had previously learned about linear functions, and as the assessment showed, came in with varying levels of productiveness in reasoning about linear functions. Students were also interviewed about their solution methods on the assessment. Next, students participated in 16 lessons about quadratic functions that focused on covariational reasoning (i.e., the math program). At the end of the program, students took a linear functions paper-and pencil post-assessment and were interviewed again about their solution methods.

**Assessments.** The assessments assessed the students' abilities to reason about various linear function problems. There were three main problems on the assessment each containing several sub-questions. The first problem made use of graphical representations, the second made use of tabular representations, and the third made use of pictorial representations. Two versions, A and B, of the linear functions assessment were developed. The versions varied in context and in numerical values, but not in structure or in mathematical intent. Students were randomly assigned to one version for their pre-assessment and the other version for their post-assessment.

**Math program instructional pattern.** The math program was designed as a two-week course on quadratic relationships. The principle investigator, a university professor who was previously a high school mathematics teacher, was the instructor for the course. The focus of the

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program was to develop students' abilities to reason covariationally with quadratic functions. An inquiry-oriented instructional approach was utilized and quadratic functions were represented with tables and with SimCalc dynamic software.

**Data Set**

Our data set consisted written responses to the pre- and post-assessments, and video/audio recordings and transcripts from semi-structured interviews.

**Data Analysis**

We began by reviewing the assessments and the interviews in order to identify four students, of varying levels of productiveness in reasoning about linear functions. We looked for at least one student from each of the following categories: higher-, mid- and lower-level linear function reasoners. We also looked for students who appeared to exhibit changes in ways of reasoning from pre- to post-assessment. We ended up choosing one high-level, one low-level, and two mid-level linear function reasoners.

During analysis, each member of the research team analyzed one student's data, taking a grounded theory approach (Strauss & Corbin, 1998). During open coding, each research team member went sub-question by sub-question through the written and interview responses for their student, looking for changes in ways of reasoning from pre- and post-assessment. Each new change in reasoning became a new code. When each student had been analyzed, we compared the codes and consolidated those that were similar. For each student, a second member of the research team reviewed the coded changes in ways of reasoning f to triangulate the data. During axial and selective coding, each team member identified associations between categories of changes in ways of reasoning and organized and integrated the categories into a story for each student and presented the story to the group for feedback. Finally, the team interpreted each change in reasoning in terms of Jeannotte and Kieran's (2017) mathematical reasoning processes and Greeno's (1989) dimensions of productive reasoning.

## Results

In this section, we present each student's changes in ways of reasoning, starting with Rashana, the higher-level linear function reasoner, followed by Layla, the lower-level linear function reasoner, followed by Yolanda and Damien, the mid-level linear function reasoners. For each student, we state the core category and several subcategories of how their reasoning changed from pre- to post-assessment. Then, we illustrate the core category and one subcategory.

**Rashana**

Rashana, the highest-level linear function reasoner of the four students, changed some of her ways of reasoning linear functions from the pre- to post-assessment. There was a core category we called *improved quality of the responses*, and four subcategories we called (a) *expansion of covariational reasoning*, (b) *more quantities notice*, (c) *exploration of relationship between quantities*, and (d) *different representations used*. Each subcategory represents a dimension on which Rashana's reasoning became more productive from pre- to post-assessment.

**Core category: Improved quality of responses.** From pre- to post-assessment, Rashana improved the quality of several responses. Interestingly, however, on the six problems that we coded her response as having improved from pre- to post-assessment, the correctness of her answers did not change. For example on problem 3(a) of the pre-assessment, Rashana correctly solved a problem about a plant growing at a constant rate by first finding the equation $y = 1.6(x+1) + (-.2)x$. This equation was technically correct. However, Rashana indicated she was uncertain about why the $(–.2)x$ was needed, other than that the equation did not work without

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adding that expression. In contrast, on problem 3(a) of the post-assessment, Rashana correctly solved a similar problem by representing the data set with a table, without any uncertainties.

**Subcategory: More quantities noticed.** We subcategorized some changes in Rashana's ways of reasoning as *more quantities noticed*. For example, on 3(d) of the pre-assessment Rashana noticed the following quantities for a plant growing at a constant rate: the changes in the day, and changes in the height. In contrast, on problem 3(d) of the post-assessment, Rashana again noticed the changes in the day and the changes in height, but also changes in the changes in the day and the changes in the changes in height.

Applying Jeannotte and Kieran's (2017) conceptualization of mathematical reasoning to this subcategory, we interpreted the change to notice more quantities as an increase in the process of *classifying* (i.e., classifying more quantities). Also, applying Greeno's (1989) conceptualization of productivity of reasoning to this subcategory, we interpreted this a productive change in favor of becoming more deeply embedded in the problem on the post-assessment problem.

In sum, Rashana, who represented a high-level linear function reasoner, nevertheless exhibited BT changes in her ways of reasoning that considering correctness alone did not reveal.

**Layla**

Layla, the lowest-level linear function reasoner of the four students, also changed some of her ways of reasoning about linear functions from the pre- to post-assessment. There was a core category we called *mixed changes in quantitative reasoning*, and three subcategories we called (a) *new ways of reasoning with quantities* (b) *new ways of reasoning with changes in quantities*, and (c) *new ways of finding and reasoning about rates of change.* In contrast to Rashana, Layla's responses in several instances became less productive from pre- to post-assessment, although at times there were also more productive aspects.

**Core category: Mixed changes of quantitative reasoning.** From pre- to post-assessment, Layla's ways of reasoning changed on six responses. Moreover, four of the six were less correct from pre- to post-assessment, and the other two stayed at similar levels of correctness. However, we did observe some productive development in her ability to reason quantitatively. For example, on problem 1(a) of the pre-assessment, Layla *correctly* applied the slope formula to a linear graph representing gas left in a car's gas tank vs the distance the car traveled. However, her explanation lacked evidence of quantitative reasoning: "So basically, I started out by finding the total amount. So I did the slope equation for these two first and then I found out that that was the total number of gas use between point A and point C."

On problem 1(a) of the post-assessment, Layla *incorrectly* divided corresponding values of gallons used by distance driven. However, her explanation had more reasoning with quantities:

> So, I said the gas in Car 1 is decreasing as the miles driven increases. The gas in Car 1 has decreased drastically by point C. So, basically, I did the distance driven over the gallons left in the tank . . . Those were the changes. The changes in the - oh my gosh - the changes in the um, we were just talking about this! The changes in, um, I would say the changes in gallons.

We interpreted this excerpt as evidence of Layla trying to reason with several quantities, distance driven, gallons of gas in the tank and changes in the gallons. Most changes in Layla's ways of reasoning similarly reflected increased attempts at quantitative reasoning.

**Subcategory: New ways of reasoning with changes in quantities.** We subcategorized some changes in Layla's ways of reasoning as *new ways of reasoning with changes in quantities*. For example, on problem 2(a) of the pre-assessment, Layla used the slope formula to correctly determine that for a table displaying the additional cost for extra megabytes of data used on a cell phone plan "it increased by .75 cents, each time you used a megabyte." In this response, reasoning with the changes in quantities involved multiplicatively comparing changes in one quantity and changes in the other quantity (i.e., by dividing).

On problem 2(a) of the post-assessment, Layla found changes in additional megabytes used and changes in additional cost between rows of the table to "find the constant rate of additional, that used. But that wasn't really working for me." Thus, Layla went from multiplicatively comparing changes in quantities on the pre-assessment, to looking for additive patterns in the changes in each quantity separately. Altogether, changes in how Layla reasoned with changes in quantities were observed on five problems.

Applying Jeannotte and Kieran's (2017) conceptualization of mathematical reasoning to this subcategory, we interpreted changes in reasoning about changes in quantities as primarily a change in the process of *comparing* (i.e., going from multiplicatively comparing to additively comparing). Also, applying Greeno's (1989) conceptualization of productivity of reasoning to this subcategory, we interpreted this an unproductive change toward grasping *less* of the essential properties and relations for the post-assessment problem than the pre-assessment problem.

In sum, Layla, who represented a low-level linear function reasoner, exhibited BT changes in her ways of reasoning that were mostly not productive but did reflect increased attempts at quantitative reasoning..

**Yolanda**

Yolanda, one of the mid-level linear function reasoners, also changed her ways of reasoning from pre- to post-assessment. There was a core category we called *greater focus on changes in quantities*, and three subcategories we called (a) *more changes in quantities found*, (b) *more changes in quantities represented*, and (c) *changes in reasoning about changes in quantities.* Like Rashana, each subcategory represents a dimension on which Yolanda's reasoning became more productive from pre- to post-assessment.

**Core category: Greater focus on changes in quantities.** From pre- to post-assessment, Yolanda's reasoning changed in favor of a *greater focus on changes in quantities*. For example, on problem 3(d) of the pre-assessment, Yolanda focused only on the changes in the height for the growing plant, recording magnitudes of each change in height, and adding brackets to indicate where each change in height applied. On problem 3(d) of the post-assessment, Yolanda again focused on the plant's height, adding brackets to indicate where the changes in height applied. She also focused on the changes in days and the changes in changes in the days. We found evidence of this increased focus on changes in quantities on five problems.

**Subcategory: Changes in reasoning about changes in quantities.** We subcategorized some changes in Yolanda's ways of reasoning as *changes in reasoning about changes in quantities.* For example, on problem 1(b) of the pre-assessment, Yolanda compared changes in the gallons left in the tank from points D to E and from points E to F (see Figure 6), saying:

> Car 2 does not use the same gas at the same rate between D and E as it does between E and F due to the reason that D to E takes up 1.50 gallons while E to F took up only .75 gallons.

In contrast, on 1(b) of the post-assessment, Yolanda tried to iterate a difference in one quantity to go from one data point to the other:

> So one way I found out, well made me confident, was I just did the pattern again and again on the whiteboard I had. And since I just did 42 times like 42 times 9, 42 times 8 to try to get to 408 but I didn't come to that number.

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Thus, Yolanda went from comparing two changes for the same quantity (i.e., between points D and E and points E and F), to iterating a difference in one quantity to go from one value of the quantity to another (i.e., iterating 42 miles to go from 42 to 408 miles). Changes in how Yolanda reasoned about changes in quantities were observed on three problems.

Applying Jeannotte and Kieran's (2017) conceptualization of mathematical reasoning to this subcategory, we interpreted changes in reasoning about changes in quantities as a change in the process of *comparing* (i.e., how Yolanda compared changes in quantities). Also, applying Greeno's (1989) conceptualization of productivity of reasoning to this subcategory, we interpreted this a productive change in favor of grasping more of the essential properties and relations for the post-assessment problem. We claim this because iterating a change in one quantity repeatedly is more consistent with a constant rate of change than comparing static changes in a particular quantity. In sum, Yolanda, who represented a mid-level linear function reasoner, exhibited productive changes in her ways from pre- to post-assessment but that, like Rashana, did not impact correctness.

**Damien**

Finally, Damien, one of the mid-level linear function reasoners, also changed his ways of reasoning from pre- to post-assessment. There was a core category we called *improved covariational reasoning* and three subcategories we called (a) a *change in reasoning about different quantities*, (b) *better understanding of rates of change*, and (c) *change in the stability of correct application of the slope formula*. Overall, Damien's reasoning appeared to change in favor of an increased ability to reason covariationally in a more productive manner.

**Core category: Improved covariational reasoning.** From pre- to post-assessment, Damien improved his ability to reason covariationally. In particular, on each of the five problems we coded as having changed responses from pre- to post-assessment, despite not all responses becoming more correct, Damien provided evidence of improved covariational reasoning. For example, on problem 1(a) of the pre-assessment, which was about the graph of the gas remaining in the tank of the car and the distance driven, Damien wrote down the correct slope formula, but incorrectly calculated the slope by dividing $\Delta x$ by $\Delta y$ instead of vice versa. Trying again, he subtracted $\Delta x$ from $\Delta y$ rather than dividing $\Delta y$ by $\Delta x$. When asked what his calculation meant, Damien struggled to reason covariationally, replying, "for each, um, mile driven, 30 gallons are wasted." This incorrect response suggested he did not have a clear understanding of the meaning of slope. Understanding slope is an important aspect of reasoning covariationally.

On problem 1(a) of the post-assessment, Damien used the slope formula to correctly calculate that the slope between points A and B and between points B and C was -0.031, and correctly wrote "per mile driven 0.031 gallons of gas are used." In the interview, Damien confirmed, by interpreting the slope, that there had been somewhat of a productive change in his covariational reasoning, saying "It's negative 0.031 because that's how much is decreasing by." This response suggests Damien was reasoning more covariationally.

**Subcategory: Better understanding of rates of change.** We subcategorized some changes in Damien's ways of reasoning as indicating a *better understanding of rates of change*. For example, on problem 2(a) of the pre-assessment, Damien was asked to consider the cell phone data table. Damien correctly applied the slope formula but was unclear about why that worked: "I don't know how to describe it, but, um . . . when I was in slope intercept in eighth grade and I just remember doing this for every question that I would get that would be like this." In contrast, on problem 2(a) of the post-assessment, Damien correctly found and correctly interpreted the rate of change:

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> I found out that the one megabyte of data costs 0.75 cents. So they said that they wanted to know how much, um, an additional 51 MB of data would cost. So I've taken 0.75 and multiply . . . Since one megabit of data is 75, well .75, I want it to multiply that by 51 times.

This excerpt suggested Damien had a better understanding of the rate of change.

Applying Jeannotte and Kieran's (2017) conceptualization of mathematical reasoning to this subcategory, we interpreted changes in reasoning in favor of better understandings of rates of change as a change in the process of *comparing* (i.e., rates of change are multiplicative comparisons between changes in one quantity and the corresponding changes for a related quantity). Also, applying Greeno's (1989) conceptualization of productivity of reasoning to this subcategory, we interpreted this a productive change in favor of better grasping the essential properties and relations of a problem situation.

In sum, Damien, who like Yolanda was a mid-level linear function reasoner, exhibited productive changes in his ways of reasoning from pre- to post-assessment. However, in contrast to Yolanda, whose reasoning changed primarily in favor of a greater focus on changes in quantities, Damien's reasoning changed primarily in favor of improved covariational reasoning.

## Discussion

The results from this study can be summed up with the following five points. First, three of four students' level of correctness remained stable from pre- to post-assessment, while one student's level of correctness dropped. Second, all students, including the student whose level of correctness dropped, showed at least some productive changes in reasoning from pre to post (i.e., most BT effects were productive). Third, productiveness was impacted on both of Greeno's (1989) productiveness dimensions. Fourth, BT effects largely involved changes in quantitative reasoning and somewhat involved covariational reasoning. Fifth, the reasoning process that appeared most involved in BT was the process of comparing.

**Significance.** This study is significant because it provides new insights into how BT influences mathematical reasoning processes and productiveness, as well as into how the reasoning of students at different levels is influenced by BT. With respect to reasoning processes, this study is significant because it showed that particular reasoning processes (i.e., *classifying* and *comparing*) can be influenced by BT. Moreover, it showed that BT can influence the amount that reasoning processes are used (e.g., classifying more quantities) and the ways reasoning processes are used (e.g., comparing different quantities).

With respect to mathematical reasoning productiveness, this study is significant because it showed how productiveness can be influenced by BT. Although other studies have reported productive and unproductive BT effects on mathematical reasoning (e.g., Hohensee, 2014), this study was the first to show that these effects can manifest themselves on two of Greeno's (1989) dimensions of productiveness.

Finally, with respect to students who represent different reasoning levels, this study is significant because it showed that BT can influence the reasoning of students at all levels. This finding challenges our previous theory about BT (Hohensee, 2014), that BT primarily affects mid-level reasoners, and that high-level reasoners know too much and low-level reasoners too little to be influenced by BT. It is also significant that our study unpacks ways that students at different levels are influenced by BT. To our knowledge, our study is the first to do so.

**Implications.** We mention two implications for practice. An implication from our finding that our lower-level linear function reasoner, whose reasoning became less correct from pre to post but who also showed some new quantitative reasoning, is that perhaps it could be useful for this level of reasoner, if teachers revisited an old topic after covering the new topic. By reasoning more quantitatively, these learners may be more ready to further their thinking of the old topic.

A final implication is that emphasizing quantitative and covariational reasoning during quadratic functions instruction should be promoted. Our results suggest that this emphasis productively influences most students' ways of reasoning about linear functions.

## Notes

[1] The structural dimension, which is about whether the mathematical reasoning is deductive, inductive, or abductive, was not examined.

[2] Creativity and flexibility, the other two dimensions of productiveness of reasoning, were not examined.

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