IMAGE OF MATHEMATICS IN- AND OUT-OF-SCHOOL: A CASE STUDY OF TWO ORIGINAL PARTICIPANTS IN AN AFTERSCHOOL STEM CLUB- GIRLS EXCELLING IN MATH AND SCIENCE (GEMS)

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People often view mathematics as abstract, cold, and irrelevant to real-life, and their school experiences influence such views. In this case study, we investigated the mathematics learning experiences of two women who participated in an afterschool girls STEM club 26 years ago. We explored their experiences in and out of school and how such experiences informed their images of mathematics. Data were collected from a survey, focus group interviews, and individual interviews. Using qualitative analysis, we learned that their school mathematics experiences influenced the participants' images of mathematics. The findings also revealed the participants' continuous and discontinuous learning experiences between school and out-of-school mathematics. This study suggests creating spaces to develop curricula that bridge the gap between school and out-of-school learning experiences.

Keywords: Attitudes, Belief, Informal education, Integrated STEM

Introduction

People's experiences with learning mathematics in schools inform their images of mathematics (Sam & Ernest, 2000). The public often describes mathematics as difficult, cold, abstract, and primarily masculine (e.g., Darragh, 2018, Epstein et al., 2010, Ernest, 1996). In a *Journal of Research in Mathematics Education* commentary, Stephan et al. (2015) reported "grand challenges" for mathematics education, including "changing the public's perception about the role of mathematics in society," "achieving equity in mathematics education," and "changing perceptions about what it means to do mathematics" (p. 139). These challenges necessitate altering the public's image of mathematics. Sam and Ernest (2000) conceptualized the image of mathematics as "a mental representation or view of mathematics, presumably constructed as a result of social experiences, mediated through school, parents, peers or mass media" (p. 195). Researchers suggested that the widespread and narrow public image of mathematics may have resulted from instruction in mathematics education that portrays the subject as isolated from out-of-school experiences (e.g., Darragh, 2018; Sam & Ernest, 2000).

Reconstructing the public image of mathematics requires widespread support from many stakeholders. Specifically, the mathematics education community is responsible for supporting the public to see mathematics as "normal and ordinary but at the same time important and useful" (Darragh, 2018, p. 203). Researchers have found that in contrast to the image of school mathematics; out-of-school mathematics, including everyday mathematics and mathematics learning in designed informal environments, is often viewed as useful and real (Civil, 2007; Cooper, 2011; Nunes, 1999; Pattison et al., 2017). Exploring out-of-school mathematics could be a way to disseminate alternative images of mathematics (Nemirovsky et al., 2017). Therefore,

informal mathematics learning has the potential to address the described grand challenges to change the public image of mathematics. However, limited extant literature has reported continuities and discontinuities between people's mathematical experiences in in- and out-of-school contexts. Conducting such a study has the potential to inform school mathematics teaching and broaden the public image of mathematics.

We investigated two women's in-school and out-of-school mathematics learning experiences to: (a) examine how these experiences informed their images of mathematics, and (b) identify continuities and discontinuities across their experiences. The women had participated in an afterschool STEM program, *Girls Excelling in Math and Science (GEMS*), in fifth and sixth grades. The two participants had different mathematics learning experiences as well as different views of mathematics. The following research questions guided this study:

- How did the participants' mathematics experiences inform their images of mathematics?
- How did the participants' descriptions of mathematics reflect continuities and discontinuities between their in-school and out-of-school experiences?

Literature Review and Theoretical Perspectives

School contexts typically have highly regulated learning environments. Below, Bronkhorst and Akkerman (2016) summarized key characteristics at school:

(a) learning is intended; (b) students and teachers as main actors, with teachers as knowledgeable others; (c) what and how one learns is formalized in a curriculum; (d) validation of learning by assessment; (e) cumulative qualification; (f) school building; (g) mandatory attendance. (p. 22)

Unlike in-school learning, out-of-school learning often has more flexibility regarding time and space and is not constrained by the school schedule, national or state standards, and standardized tests. Informal learning is usually voluntary and allows students to bring in their cultural knowledge and personal experiences (Copper, 2011). In this section, we synthesize literature on in- and out-of-school mathematics learning as well as continuities and discontinuities between school and out-of-school contexts.

In- and Out-of-School Mathematics Learning

Out-of-school learning refers to curricular and non-curricular learning experiences that are provided for students outside of the school environment (Resnick, 1987). Resnick mentioned several discontinuities between in- and out-of-school learning. For example, school environments often focus on individual performance, independent thinking, symbolic representations, and generalized skills and knowledge. In contrast, learning out of school typically involves collaborative engagement, tools, and is situated in authentic environments. School mathematics learning frequently focuses on preparing students for standardized tests, and providing isolated instruction with limited opportunities to make connections between mathematics and daily life experiences (Copper, 2011). Students often view school learning as completing assignments required by teachers, which at times diminishes their motivation and interest in learning mathematics (Nunes et al., 1993).

Learning in school is not necessarily disconnected from out-of-school learning; in particular, some intended continuity efforts strengthen school learning by bridging learning between school and out-of-school spaces (Bronkhorst & Akkerman, 2016). The out-of-school contexts provide a rich environment for authentic and experiential learning (Nielsen et al., 2009). Authentic

mathematics is introduced in the classroom to remedy the common disengagement in school mathematics, wherein students are expected to develop formal mathematics by mathematicizing their informal mathematical activities (Bonotto, 2005). In doing so, school activities further engage students and provide opportunities for them to consolidate knowledge and develop deeper understandings (Nielsen, 2009). Some researchers have suggested fostering informal learning as a supplemental formal learning method (e.g., Xiao & Carroll, 2007). Other researchers have proposed that the schooling system should incorporate informal curriculum to bridge learning in formal and informal contexts (e.g., Hung et al., 2012).

The term "informal learning environment" is often used as a general reference for a learning setting which is different from school. Though there is not an agreed-upon definition of informal learning environment in the literature, it often refers to everyday activities such as family discussions, pursuing one's hobbies, or daily conversations and designed environments such as museums, science centers, or afterschool programs (e.g., Civil, 2007, National Research Council, 2009). Nemirovsky et al. (2017) stated that designed informal mathematics learning environments are "intentionally designed to support mathematics learning, whether because they are structured through programs with regular schedules and assigned educators or because they host technologies, tools, or exhibits designed to engage the user with mathematics" (p. 970). In this study, we use the terms *informal mathematics* and *out-of-school mathematics* interchangeably to refer to mathematics practices in everyday life, in professions, and in designed informal learning environments.

Continuities and Discontinuities between In-School and Out-of-School Settings

Researchers have found that people use flexible strategies in diverse settings to solve mathematical problems outside of school, which are significantly different from those taught in school (Lave, 1988; Nunes et al., 1993; Saxe, 1988). Nunes (1999) suggested that school mathematics, concepts, methods, and procedures are the goals for instruction, solving problems for the teacher's sake. As such, "informal mathematics education is an emerging field of learning with a unique potential to disseminate alternative images about the nature of mathematics and to realize the potential for everyone to engage with mathematics in creative and diverse ways" (Nemirovsky et al., 2017, p. 975).

Bronkhorst and Akkerman (2016) synthesized 186 empirical studies to investigate continuity and discontinuity in students' learning across school and out-of-school contexts. They found that both continuity and discontinuity can result from different educational intentions, but it also occurs as a given. Due to the fundamental role in students' learning, the school is responsible for establishing the continuity for students' learning across contexts. There are possibilities and challenges for schools to develop this continuity. First, introducing out-of-school practices in school might be a challenge for teachers who usually have limited expertise in out-of-school teaching practices, while on the other hand this can be an opportunity for teachers to engage in a new practice. Second, a challenge for teachers adopting informal practices is to ensure that all required content and standards are met. Third, in order to extend schools' influence, some informal learning contexts provide opportunities to supplement school education, such as a tutoring center, that intentional continuity limits students' experiences differing from school.

Akkerman and Bakker (2011) claimed that discontinuities result from boundaries, which can be seen as socio-cultural differences between different contexts. As learners engage in different practices, learning is not necessarily bounded in a particular stable domain. Rather, learning involves crossing boundaries between multiple practices, in which a learner should be approached as a whole person who participates in school and many other places. Our theoretical

perspective is informed by learning as crossing boundaries between in-school and out-of-school contexts (e.g., Akkerman & Bakker, 2011; Bronkhorst & Akkerman, 2016), which provides a lens to explore each participant as a whole person with interconnected identities when they engage in different practices.

Methods

In this study, the designed informal learning environment particularly refers to an afterschool STEM program—*GEMS*. *GEMS* was initiated in 1994 by a mother who aimed to help her daughter and other girls develop positive dispositions towards mathematics and science. The first *GEMS* club was started at an elementary school in Virginia; at that time, most participants were white and from working class families. The mother and her daughter's fifth-grade classroom teacher co-led the club; fifth and sixth grade girls were enrolled by teacher's recommendation, parental request, and/or voluntarily.

This study draws its data from a larger study of the original *GEMS* participants in 1994-1995. In the current study, we used a collective case study approach (Yin, 2017) to understand two participants' descriptions of their mathematics learning experiences in formal and informal settings and to explore their images of mathematics. We aimed to explore across the two cases to draw case-specific characteristics. As such, the goal of this case study was not to generalize the image of mathematics from the two cases. Instead, our goal was to identify the possible impact of mathematics experiences on the participants' images of mathematics (Simons, 2009).

Data Collection

Based on information-oriented sampling (Yin, 2017), Kate and Stella (pseudonyms) were selected from the larger study because of their different experiences and views on mathematics. Kate expressed her positive experiences with school mathematics; she identified herself as "naturally good in mathematics." In contrast, Stella, consistently reported struggling with mathematics in and out of school, saying, "I had always been awful at math."

Reflecting on Yin's (2017) emphasis on the role of theory in guiding case study research, we developed data collection protocols based on relevant literature. For instance, building on the literature on mathematical identity (Boaler, 2002), we designed an interview question, which asked the participants to describe themselves as math learners. We also built from Sam and Ernest's (2000) work that proposed that adults' images of mathematics are influenced by their mathematics teachers. The data sources included surveys, focus group interviews, and individual interviews.

Data Analysis

Data analysis involved a review of the three data sources to derive relevant themes across cases regarding mathematics experiences and conceptions of mathematics in- and out-of-school. Drawing on Akkerman and Bakker (2011) and Bronkhorst and Akkerman's (2006) foundational work on continuities and discontinuities crossing boundaries, we conceptualized continuities when individuals make connections between their participation in various contexts (Bronkhorst & Akkerman, 2006) in which individuals might change roles across contexts. Discontinuities refer to an individual's experiences, interests, and perspective in one context that conflict with his/her experiences in another context. We followed a qualitative content analysis protocol, which allowed coding to be both data driven and theory driven (e.g., Schreier, 2012).

The first author reviewed all data and developed the primary codes. Using the primary codes, the two members of the research team then independently coded each participant's survey, focus group interview, and individual interviews. The researchers also wrote research memos to record

the emerging new codes and suggestions for revising the codebook. The third researcher oversaw the coding process and tracked the discrepancies between the first two researchers' codes. Then the authors discussed and resolved these divergences. Any unresolved divergences, confusions, ambiguities were brought to the whole research team and discussed in the team meeting until a consensus was reached.

Findings

In this section, we report our findings to highlight Kate and Stella's mathematics learning experiences, how they influenced their views of mathematics, and continuities and discontinuities within their mathematical learning experiences across settings.

Kate's Description of Mathematics

Math is binary. In Kate's descriptions, mathematics is objective and a natural aptitude. She explicitly mentioned her positive learning experiences in elementary and secondary mathematics classrooms, saying "when I was growing up, I knew I could do the math and science as well as the boys....I was interested in math and science at an early age. It just came naturally to me." Kate perceived mathematics as a neutral and objective subject (i.e., right and wrong answers). The objective aspect of math gave her autonomy when she interacted with math. She stated:

I liked knowing there was an answer, and that you just had to go through the steps to get it right or wrong. And you know why you got the grade you did in math and science. I knew that if I studied, I would get it right. I could and can control it.

Kate compared math and science with other subjects in which there is no right or wrong answer; the learners then need to rely on evaluation from authorities, which from her perspective is subjective. She stated that "in English, instead [of a right or wrong answer], you could write an essay and the teacher may or may not like it depending on what day she reads it."

Even though Kate enjoyed the binary right and wrong aspect of mathematics and considered herself a natural mathematics learner, she was also hindered by this aspect of mathematics. She was hesitant to share her answers in the class as she did not want to present wrong answers. She said, "but you completely second guess about yourself. I do not want to raise my hands just because if I am wrong that's gonna feel really bad in front of everyone."

Discontinuities between school and GEMS broaden the image of mathematics. Kate described the nature of mathematics learned in a formal space as "rigid of structure" and objective ("there is a right answer") while the mathematics learned in the informal learning space was flexible and more subjective. The discontinuities of mathematics within GEMS and the classroom broadened her image of mathematics.

Kate connected her mathematics learning experiences with her daily life activities and experiences. Meanwhile, she acknowledged that she was not able to perceive the continuity in formal classroom mathematics and her daily-life activities in her childhood, "I remember we got our first computer. We're going to have fun games. They're so much fun. It was always learning without making it feel like you're learning. It was always the problem-solving games. This is super fun!"

Kate acknowledged that she engaged in those games without realizing she was learning mathematics (i.e., logic and reasoning). She distinguished between her mathematics learning experiences in the mathematics classroom and in *GEMS*. She mentioned:

[in a mathematics class], it's gonna look really bad, if I get this wrong in front of everybody. I think one thing that helped in GEMS [that] it was okay if you were not super confident, like yes, I know this. And then they [leaders] were like you're close, but you're not quite there.

This indicated that her learning experiences in these two spaces were discontinuities in terms of being open to share her answers for two reasons. First, the mathematics content presented in the *GEMS* was different from the school mathematics content; it was more subjective, and it did not necessarily depend on "Yes or No type of answers." She stated,

[Math] was just fun and group kind of thing and, what if add this in and how does that affect? It was not obviously a rigid structure. It's not like a question/answer. So, that definitely helped in a way that builds confidence.

Second, Kate described that the learning environment in the *GEMS* was safer ("it's okay not to get the right answer") and less judgmental ("everybody here they like the same thing as you do") than the classroom learning environment. Kate mentioned that *GEMS* made her realize that mathematics can be learned in a fun way, which aligned with her childhood experiences of learning mathematics with computer games. She stated that she never felt she was learning "rigid structure[s]" and "just question/ answer" in *GEMS*, but it was like building something as a group and having a product at the end. Specifically, she mentioned:

There was not that big of a, I guess, a fear of that as much as you have in class, it was just a smaller group of people and everybody kind of understood we all like this and we all love doing this, and it's okay to not get it right.

Kate enjoyed the discontinuities of mathematics between school and *GEMS*; in fact, she intentionally sought out these discontinuities. When she was in high school, she joined an after-school math club and expected to do activities similar to *GEMS*. But when she got there, she found that "it was just sitting there doing mathematics problems. I don't need another math class after school, no thanks."

Stella's Description of Mathematics

Math is multifaceted. Stella struggled a lot in basic mathematics, in particular, numbers, but she perceived mathematics as multifaceted, including logic, process, language, statistics, and spatial design. She described her early school years, saying "I always sucked with numbers. I was awful at the times table. I was a straight A student except for any math class that I had; I was constantly getting Cs [in math]." Stella said she was traumatized by mathematics:

I had always been awful at math, especially after third grade where I always failed the multiplication table timed quizzes and tests because I didn't understand the concepts. I don't think I passed a single one of those tests besides the 5s, maybe the 9s because of logic. I was traumatized by math.

In elementary and middle school, people surrounding her did not understand her difficulties, which exacerbated her struggles in mathematics. She said,

I really struggled in elementary school. And my mother did not get it at all because I was a straight A student in any of the language arts and I really did well in writing. But I just could not understand math and they didn't have, I guess, it wasn't something that was really talked about or knew. So, nobody really thought to be like, hey, this girl has a learning disability because I was doing so well everywhere else. I really grew to hate math because of it.

In high school, Stella was often quiet in math classes. In spite of her hard work on algebra, Stella mentioned that she performed poorly. She said that it was incomprehensible for her at the time to get "such a bad grade." However, she performed well in geometry and got an A. Because she did not want to take extra mathematics classes, she opted to take a programming course instead of trigonometry or calculus. For Stella, programming was not related to mathematics, just logic and languages, "even if it is like math in that it is a logical process, I did not have to do any of the calculation and numbers, that was really hard for me."

Supports discontinuities in mathematics learning. Stella portrayed herself early on as a person who struggled with mathematics. To avoid being called on to do numerical problems in the class, she would make sure to raise her hand and be overly participating in any process- and logic-related questions. The discontinuities between mathematical results and process provided opportunities for her to enhance her strength (i.e., logical thinking and conceptual understanding). Stella mentioned that in GEMS, her teacher helped her address these discontinuities.

Unlike in the mathematics classroom, Stella was not nervous in *GEMS*, "[The teacher] was so good and motivated everyone involved. You can ask questions, there was definitely no stupid question." *GEMS* did not focus on memorizing rather promotes application of math which released Stella's struggles, she said "*GEMS* made me think that that math wasn't as scary. I learned that I understood applied math in a way that I didn't get from memorizing times tables." Even though she was traumatized by numbers and did not like math, she developed confidence in math, "[in *GEMS*] I learned that I even though I was bad at writing down equations on paper, I was decent at applied math. I ended up doing computer programming in high school because I felt comfortable with applied math." At home, Stella's father also supported her conceptual understanding by encouraging Stella to use tools, she recalled "he would even let me use my fingers and he gave me an Abacus so I could use that. It really helped because it took the numbers out of the math and I could understand."

The discontinuities between learning mathematics that focus on getting correct results and understanding concepts supported Stella to appreciate different foci of mathematics. Rather than hindering her development, the discontinuities redeemed her struggles in mathematics and provided room for her to develop her confidence and interests, which impacted her course selection and later career decision. Looking back, she laughs, "even with all my math issues, I ended up going into STEM."

Kate and Stella's Explorations of Continuities In and Out-of-School

Even though Kate experienced discontinuities in her mathematics learning between formal and informal learning environments, Kate's learning experiences in informal settings and everyday life indicated that her descriptions of the nature of mathematics were somewhat continuous as she described mathematics presented in informal classrooms and everyday life as "fun-type" learning. Kate wants her daughter to engage with more mathematical toys because she wants her daughter to choose a STEM-related career: "She has blocks we are building, for her it's just let's stack them down, and right now she accesses the construction all that comes in and knocks it over, but you do not realize this in a way you are engineering but something you are building blocks." Kate also mentioned that her experiences in *GEMS* are parallel with this because in *GEMS* she "never thought [they] were learning, [they] tried fun experiments" without realizing they were "actually learning bigger concepts."

Stella consistently experiences mathematics struggles across different settings. She said, "I still really struggle with the [math], when I'm on the phone with somebody and I need to read off

a credit card number, I always mess it up" she added, "I still cannot do basic math like a *normal* person." However, she accepts these continuities and uses resources to overcome the challenges. She depends on Excel and other software when she is doing finances in her personal and professional life. In *GEMS*, continuities of mathematics struggles did not hinder her development. Being with friends and gaining support from her teacher, Stella felt a sense of belonging and being involved, "it was no stupid questions for me [in *GEMS*]. If she [the teacher] could see that you were not understanding it, she would explain it a different way. She had like a million different ways to explain something until you understood it."

Discussion

In this study, we sought to answer two research questions: How did the participants' descriptions of mathematics experiences inform their images of mathematics? How did the participants' descriptions of mathematics reflect continuities and discontinuities between their inschool and out-of-school experiences? For the first research question, we learned that participants' school experiences influenced Kate and Stella's images of mathematics. Kate reported liking mathematics and referred to happy memories of doing mathematics across her experiences at school, home, and her professional life. Stella, who explicitly reported not liking mathematics, referred to school experiences wherein she remembers struggling or feeling defeated. These findings are aligned with existing literature that reports that schools and mathematics teachers greatly influence people's image of mathematics (Sam & Ernest, 2000). Though Kate and Stella hold different attitudes toward mathematics, their views of mathematics are aligned with the public image of mathematics. School mathematics for the two participants was perceived as difficult, cold, abstract, and masculine (e.g., Darragh, 2018, Ernest, 1996, Epstein et al., 2010). Kate and Stella perceived that people either can do school mathematics naturally or not. Kate said that she liked math since she was young while Stella described how she "was awful" in math. Kate's mathematics experiences were pleasant, she enjoyed doing mathematics at school, with her family, and at GEMS. Yet, for Stella, school mathematics, especially elementary mathematics, was difficult and unpleasant. In contrast, Stella enjoyed doing mathematics that was related to logic and application, such as her experiences doing mathematics at GEMS or with her family.

Concerning findings related to the second research question, we identified continuities and discontinuities in both participants' in-school and out-of-school experiences. A continuity we identified is that when participants reported having uplifting experiences with mathematics in school, the confidence continues in other learning contexts. We identified a discontinuity when Stella reported not liking school mathematics, but doing mathematics in her daily life. The two participants' descriptions of their professional mathematics activities were uplifting and reflected confidence. It appears that even though school experiences had a profound influence on the participants' images of mathematics, their experiences in other learning contexts, particular positive learning experiences, also impact on their construction on alternative images of mathematics.

Future studies could design school mathematics interventions and study their influences on students' images of mathematics. We learned from the participants that doing mathematics in their professions is doable and enjoyable, which prompted us to think that designing activities that resemble those done by professionals might be a way to bridge in-school and out-of-school mathematics, which could broaden students' images of mathematics. Participants reported using

mathematical practices and concepts in their daily lives. Curriculum designers might use those practices and concepts to develop K-12 curricula.

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