ELEMENTARY PRESERVICE TEACHERS' VIEWS AND ENACTMENTS ON FOSTERING PERSEVERANCE

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The purpose of this study is to investigate three elementary preservice teachers' (PSTs') conceptions of and reflections on the role of perseverance in mathematics. This study presents findings regarding the PSTs' perseverance conceptions, student interactions, and the PSTs reflections from a two-week summer math camp in which they assisted in teaching elementary student campers. Additionally, this study used an analytic framework based on a corpus of literature to capture how the preservice teachers' conceptions, interactions, and reflections aligned with each other and with current and relevant research recommendations.

Keywords: Preservice Teacher Education; Affect, Emotion, Beliefs, and Attitudes

Research has connected student perseverance to many constructs, including productive struggle, self-efficacy, motivation, mindset, locus of control, and grit (Bettinger, Ludvigsen, Rege, Solli, & Yeager, 2018; Dweck, 2006; Pajares & Miller, 1994; Warshauer, 2015), all of these aid in students learning mathematics. How then can teachers help support students in their perseverance? How are practices of perseverance conceived, viewed, and developed for future teachers? More specifically, as elementary teachers "develop the foundation of mathematical understanding, beliefs, and attitudes among young learners that start children on their mathematical journeys" (Association of Mathematics Teacher Educators [AMTE], 2017, p. 48), how do they conceive, viewed, and develop the practice of perseverance?

Literature Background

Conceptions

Elementary preservice teachers (PSTs) often enter teaching programs with preconceived conceptions based on their own experiences as learners (Stohlmann et al., 2014). Thus, it is critical to understand PSTs' conceptions about teaching mathematics early in their educational program. These conceptions have been known to change through content courses that use artifacts of children's' mathematical thinking (Thanheiser et al., 2013) and are taught in ways that align with content standards for doing mathematics (Conference Board of Mathematical Sciences [CBMS], 2012). Thus, by changing PSTs conceptions to align with teaching standards, there is reason to believe that these newly-formed conceptions may influence teacher practice (Ambrose, Clement, Philipp, & Chauvot, 2004; Stohlmann et al., 2014; Thompson, 1984, 1992).

In order to better focus PSTs on the content, Philipp (2008) suggested centering the content around children's thinking. However, certain positive conceptions regarding teaching and mathematics should be maintained to optimize the benefits of this focus, as conceptions "play a significant role in shaping the teachers' characteristic patterns of instructional behavior." (Thompson, 1992) Therefore, mathematics teacher educators must understand what PSTs conceptions are, how they and other experiences influence their teachings, and how we as a mathematics community can help the PSTs develop.

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Perseverance

In this study, I follow Dweck's (2006) notion that perseverance is related to having a growth mindset. Students with a growth mindset pursue goals to attain a deeper understanding (Sun, 2018), see success as expanding their capabilities, thrive on challenges, and don't give up easily (Dweck, 2006). Bettinger and colleagues (2018), in agreement with many other researchers, note that "growth mindset interventions shape students' beliefs in their ability to learn and cause lasting improvements in school outcomes" (p. 2). Dweck (2006) maintained that confidence is not always needed to persevere in a task. In other words, confidence and perseverance do not automatically hold a bidirectional relationship, in that perseverance instills confidence but confidence is not necessarily needed for perseverance. In order for teachers to impart a growth mindset to their students, the teachers must take care that their praises are of the child's learning process and not ability, that mistakes are not met with anxiety or concern for the child's ability but should not be glossed over either, and teaching should be focused on understanding and not memorization of facts, rules, or procedures (Dweck, 2006). In addition, teachers should supplement textbook material with curricular tasks that incorporate opportunities for collaboration and sensitivities toward student autonomy (DiNapoli, 2016). Thus, "the aspects of classroom culture that seem to support student willingness to engage with challenging tasks are those related to the ways that the lessons are conducted and the expectations set for the students not only in terms of the mathematics but also the ways of learning it" (Sullivan, Aulert, Lehmann, Hislop, Shepherd, & Stubbs, 2013, p. 621).

Encompassed within the idea of a growth mindset, Russo, Downton, Hughes, Livy, McCormick, Sullivan, and Bobis (2020) note that further study on the topic has informed and altered Australian teachers views and beliefs about struggle. Moreover, "[i]n the United States…creating opportunities for students to persist in problem solving is a tenet of effective teaching that is often described as creating the condition for productive struggle." (Sengupta-Irving & Agarwal, 2017, pp. 115-116)

Productive struggle ensues when "students expend effort in order to make sense of mathematics, to figure out something that is not immediately apparent" (Hiebert and Grouws, 2007, p. 387). Warshauer, Starkey, Herrera, and Smith (2019) found that preservice teachers (PTs) in a mathematics content course, were unfamiliar with the ideas of productive struggle and generally saw struggle as something negative. Additionally, "PTs placed the responsibility of productive struggle on the student, not the teacher, when learning mathematics…and had not considered it as a teacher-driven educational tool for learning mathematics (Hiebert and Wearne,2003)" (Warshauer et al., 2019, p. 26). Although the semester was not long enough to fully develop "robust mathematical interpretations" of productive struggle, most PTs were able to indicate at least one teaching strategy notated from Warshauer (2015):

> (1k questions to help students focus on their thinking and identify the source of their struggle, (2) encourage students to reflect on their work, (3) give time and support for students to manage their struggles, and (4) acknowledge that struggle is an important part to learning and doing mathematics (Warshauer, 2015; Warshauer et al., 2019, p. 25).

3. Furthermore, there is evidence that shows mixed results regarding teachers' comfort with pedagogies that lead to students engaging with struggle, especially low-performing students (Russo et al., 2020). Although beliefs often differ from what is incorporated into practice, Russo and colleagues (2020) found that most teachers in their study (n=93) held positive beliefs about the value of struggle, citing "benefits of struggle were the

> opportunities it provided students to persist through challenge, take risks, build autonomy, build confidence, foster self-efficacy, learn through mistakes, and acquire a growth mindset" (Russo et al., 2020, p. 6), and only nine of the 93 teachers in this study held descriptive beliefs that contained neutral or negative ideas.

In an effort to illuminate teaching moves that could be made in the daily-classroom that help foster perseverance, Lewis and Özgün-Koca (2016) shared five categories of teacher moves to foster student perseverance in problems solving:

1) Selecting Mathematical tasks that require and support perseverance,
2) Talking about strategies for problem solving,
3) Demarcating phases in problem-solving process,
4) Naming feelings attendant to problem solving, and
5) Narrating internal processes.

Thus, research on these teacher moves is relatively new, and has not yet made its way into the teacher preparation work. Therefore, similar to other research about mathematical practices, Warshauer and colleagues recommend that teacher educators "introduce opportunities to connect PTs mathematical content knowledge to practices like understanding productive struggle in mathematics early in their teaching continuum" (Warshauer et al., 2019, p. 26).

Theoretical Orientation

This study operationalized a social constructivist perspective of collaborative learning (Vygotsky, 1978). In using this approach, the study conformed with the Vygotskian ideals of learning, meaning that people learn as they work to form understandings and create meaning through their shared experiences in any given situation. Therefore, the researcher acknowledges that the participants in this study were learning due to a multitude of factors from the social environment, such as their experiences from this study's interviews and the daily camp classroom and accompanying professional development. Thus, the study draws on these factors whilst operationalizing and tracking the categories from the study's perseverance framework.

Methods

Settings & Participants

This study, which is part of a larger project, follows three typical PSTs majoring in interdisciplinary studies, who had completed both content courses but had not yet completed elementary observations, and were participating as Fellows (teaching assistants) in a two-week research-based summer math camp program for elementary and middle school students and professional development. Specifically, the PSTs assisted in the classrooms focused on Integers & Algebraic Modelling (Grades 3-4). At the time of selection, one of the PSTs had two years of prior experience as a Fellow, two had one year experience, and one had none. The PSTs were chosen based on their applications, camp administrator recommendations, and selected degree plan. The PSTs are referred to typical PSTs in the sense that they could not explain why the basic algorithms of addition and subtraction of integers worked before learning the models used in this setting or from their content course. The PSTs mentioned that they were excited to work with students and hoped to learn how to teach math in helpful and engaging ways for their future students.

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Data collected consisted of pre-surveys, PST-student (individual/group) interaction recordings, stimulated-recall interviews, post-survey reflections, and clinical interviews. During the camp, whole class discussions were led by an experienced middle school teacher, but group and individual work were frequently facilitated by the PSTs. The PSTs helped answer questions, provided feedback, and assisted with other classroom needs and management. These PST-student interactions were recorded, with a select number used for the stimulated-recall interviews. Clinical interviews and post-survey reflections were conducted on the last day of the two-week camp.

Analytic Framework

The unit of analysis for the stimulated-recall interviews consisted of a daily interview. The interviews could not be separated into clear distinct segments since PSTs would often refer back to previously watched interaction recordings from the daily interview and blend their reflections regarding the different interactions together. Thus, this study looked at each interview holistically. However, the clinical interviews, which were task-based, were analyzed per task. Therefore, the framework was applied to the recorded interactions used in the stimulated-recall interviews and the clinical interviews. Moreover, the pre- and post-surveys supplied additional information, along with the clinical interviews as to the conceptions the PSTs held regarding perseverance. This allowed for a triangulation between what was observed by the researcher and how the PST reported and reflected upon their supports of perseverance.

In each of the selected interactions and reflections, instances of PST moves, or lack of moves, to support the students' mathematical perseverance were noted. These fell into one of three categories: (1) praise for unsuccessful efforts to answer a question (2) praise for the process, or (3) fosters perseverance. The first two categories stemmed from Dweck's (2006) growth mindset ideas, while the third category was based on Lewis and Özgün-Koca's (2016) ways of fostering perseverance. Based on Dweck's approach, the PST could have chosen two routes: (1) the route which can produce a fixed mindset and decreased perseverance by praising the student's unsuccessful effort or answer, or (2) they could have chosen the direction of a growth mindset and praised a productive process that yielded understanding. Adapting some of the teaching moves from Lewis and Özgün-Koca's whole class orchestration to a small group or individual conversations surrounding pre-determined problems, five moves similar to their five themes were established: (1) attending to students' emotional needs, (2) focusing the discussion on the strategy or different strategies, (3) changing the participation format of the conversation, (4) creating opportunities for students to reflect on their work, stuck points, or the language of the problem, and (5) creating an opportunity for the students to extend their knowledge beyond the problem.

After assigning the categories, I conducted individual member checks with two of the three participants about their conceptions and views to verify the accuracy of the coding and interpretations. Additionally, an external reliability check was made for a random 25% of the stimulated-recall interviews and 40% of the clinical interviews were checked. Resolution discussions were had and adjusted the framework, which was then reapplied to all remaining data.

Results

This section focuses on describing how the PSTs conceptualized, used, and reflected on supporting young students' mathematical perseverance. The PSTs will be known henceforth as Amy, Becky, and Linda. The conceptualizations were primarily based on the pre- and post-

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surveys, but also included information from the clinical interviews. Since the camp structure focused on using the word persistence, the surveys asked the participants to describe and define the teaching practice of fostering persistence, and to address what they thought to be valuable in the practice.

Amy

Amy was the most experienced PST in the study and had been a Fellow for the previous two summers. In conceptualizing perseverance, Amy was asked to think about persistence as this was the term used by the camp. She wrote the following:

> Allowing students to have enough wait time. Asking guiding questions instead of giving direct answers. The value is creating a growth mindset which gives students endurance to work on hard problems longer.

Amy's definition aligned with the ideas of *fostering perseverance* because it focused on providing time for the students to work on the problem, while focusing the student on the process and strategies through questioning during the problem-solving process. Additionally, Amy attributed the value of perseverance to the amount of time spent working on a problem and establishing a growth mindset. At the end of camp, when asked to reflect on what she had written about perseverance, Amy said, "I think like if I would add something, something that we talked about in seminar was asking purposeful questions and so I think that's more important than just like guiding questions, … purposeful questions would be like asking questions for understanding". This addition, although clarifying what type of questions Amy would use to foster perseverance, still did not add or alter Amy's conception of perseverance.

Throughout the camp, Amy was observed not only supporting the students both by *fostering perseverance*, and by *praising the process*. When reflecting on her interactions during the stimulated-recall interviews, Amy was able to point out some instances of both of these supports, noting her remarks about emotional states, focusing on the wording of the problem, changing the participation format, reflecting on the problem, extending the problem, and praising a productive process. Most of her reflective efforts regarding perseverance were spent toward ideas of *fostering perseverance*, with only one observed instance from the stimulated-recall interviews reflecting *praising the process*. The reflection seemed almost an oversight to Amy, who recalled more of her excitement for the student's discovery than her actual turn of praise by saying, "I got excited when she came to the conclusion that you had to do 7 minus 5 in the other problem. So I was like, "yeah you do, dang"…" In fact, Amy did not acknowledge her own efforts in the students' perseverance. Amy recognized when students persisted but did not attribute any of her own supports to students' persistence even if it meet her definition. She noted how the students persevered in the problem by responding to Amy's prompt to reflect and explain their process and answer.

> I really liked their responses, that they didn't give up. … And both [students], too, didn't go straight to thinking that they were wrong. So I liked that. That wasn't necessarily anything I put in them, but whoever they had in the past, teachers and stuff, they've given them that sense of confidence.

From this statement, it is clear that Amy attributes parts of perseverance and "endurance" to levels of confidence.

Linda

Linda had been a Fellow for the camp the previous summer and was about a semester behind Amy in their educational program coursework. When conceptualizing perseverance, she wrote the following:

> I would see this as an environment where students feel comfortable not getting the right answer on the first attempt. Instead its viewing problems as a journey that takes multiple attempts and you don't give up. These [sic] is extremely valuable when learning topics to truly understand the material.

Linda's definition aligned with the ideas of fostering perseverance because it focused on providing time for the students to work on the problem while focusing them on the process and strategies instead of the answer during the problem-solving process. Additionally, one can see that Linda attributed the value of perseverance to learning and understanding mathematical concepts. During the clinical interview, Linda noted that follow-up questions served to foster perseverance in that it made the student continue to think about a problem.

Like Amy, Linda although conceptualizing perseverance in terms of *fostering perseverance*, also supported and reflected on both *fostering perseverance* and *praising the processes*. During her stimulated-recall interviews, Linda reflected on her supports for perseverance by noting instances when she changed participation formats, praised students' processes, extended the problems, had students reflect on the problem, and prompted students to try different strategies. However, there was no observed instances of emotional supports for fostering perseverance, and Linda primarily worked in a one-on-one environment except on a rare occasion. Although her interactions were not typically group interactions, Linda frequently thought about wanting to have included other students in her conversations, noting that she thought this would have been beneficial. Linda also pondered the idea of using different strategies for the same problem. However, Linda reflected on the students using a model or strategy they were unsure of after they were already confident in their answer. Lastly, Linda made several moves to *praise processes* but did not always reflect on them. Linda would frequently and explicitly praise students during and after a productive conversation by saying things like, "Nice. Okay, so let's go look at our paper again", "Right. Awesome", or "Cool". Additionally, Linda would also show praise implicitly by becoming more excited and animated when a student began a productive argument using vocabulary and descriptive words. This was also evident in her reflection on the interaction when she noted, "I think it shows they're understanding when they start using that in their vocabulary… that's why I got excited when she said operator". This also highlights that Linda viewed student understanding as being tied to the student's processing and use of vocabulary words to describe their processes.

Becky

Becky was new to the camp program but was at a similar rank in her educational program to Linda. Although Becky's reflections and descriptions were often very detailed, they would sometimes double-back and re-examine things in a different way. Thus, her conceptions were constantly assimilating to her current experiences and she would often bring these ideas up during her reflections. When asked at the beginning to conceptualize perseverance, Becky wrote her definition from the prospective of the teacher by saying the following:

> Persistence to me is defined as the continuing to push through something with determination. So I believe fostering persistence would be able to grow/develop the ability to push through

negative behavior, confusion, and other obstacles that teachers may face in the classroom and turn it into positivity so the student can overall learn.

Becky's definition discussed how teachers persist in the classroom to help their students learn, but also spoke about helping the students persist. Becky added that you shouldn't just give up on students and that it is important for the students to persevere; however, she also stated that she wasn't sure how to "make them be persistent…if they don't want to". Thus, Becky spoke about pushing through negative behavior and confusion to foster perseverance.

In the clinical interview, Becky noted that the interviewers were persistent in asking the student questions, which encouraged the student to persevere in problem-solving. Becky said that by asking questions, the interviewers encouraged the student to keep thinking about the problem. Therefore, although unclear in her definition, Becky alluded to the importance of questioning related to fostering perseverance in students.

Becky's conceptualization of perseverance, in terms of *fostering perseverance*, was unclear as to how exactly she would support students, but when she reflected on her interactions, she noted supports for both *fostering perseverance* and *praising processes*. Becky attended to changing participation formats, emotional needs when students became frustrated, wanting to bring in multiple strategies, praising processes and productive efforts, problem extensions, and having students reflect on completed problems. Although a few of these interactions did successfully include a change in participation format and multiple strategies, these were two supports of fostering perseverance that Becky continually echoed wanting to include more. In fact, Becky noted that exploring ideas students are uncertain of after being confident in their answer would result in a more answer-driven process, whilst the opposite ordering, although less confident, would instill a sense of growth. Additionally, when trying to support students, especially those who were becoming frustrated, Becky would rely on praising their productive efforts. Becky notated this by saying, "I felt like I needed to give her some validation and that she was doing something right, she was in the right direction, she just kind of got confused on something or tripped up, … to help her not get so discouraged and still want to participate because she had turned her body away and gave her pencil away." Moreover, Becky was observed praising another student every step of the way by saying "good" or "good job", but didn't reflect on the praise she gave but reflected more on the questions she was using to have the student explain and reflect on their process.

Cross-Case Analysis

The conceptualization of perseverance seemed to be viewed uniquely across the PSTs, but all thought of it as a way to overcome struggles and confusion. Amy viewed perseverance as being synonymous with a growth mindset, which is similar to Linda's definition of "viewing the problems as a journey" and not being afraid of getting the wrong answer. Becky's definition was slightly different from Amy and Linda's, but this may be because Amy and Linda's perception of perseverance had been affected by the camp since persistence is a key component of the camp structure. Becky's ideas revolved around turning a negative situation into a positive one, and not giving up on the problem.

In addition, all of the PSTs have a conceptualization related to fostering perseverance, they all enacted moves to support this categorization and praising the students' processes. However, Linda was the only PST to not be observed supporting students' emotional needs for fostering perseverance but was more likely to praise the students' productive processes in a more

purposeful way. In contrast, Becky used praise as a form of student support when the student's emotional needs came to the forefront.

Discussion and Future Research

The PSTs mostly conceptualized perseverance in terms of productive struggle and a growth mindset but also included remarks that aligned with the five ways of fostering perseverance in the analytic framework that was adapted from Lewis and Özgün-Koca (2016). The PSTs intentions to define perseverance in terms of growth mindset (Dweck, 2006) and productive struggle (Hiebert & Grouws, 2007; Warshauer, 2015) was not surprising, given that their content courses and the camp were structured around these ideas. Moreover, this suggests that introduction to and involvement in such work altered the PSTs beliefs to include components of perseverance. This finding agrees with that of Russo and associates, who found that recent emphasis on growth mindset helped shift "teachers' willingness to embrace struggle and view it as a necessary aspect of learning mathematics" (Russo et al., 2020, p. 8). The PSTs views on perseverance fit mostly into Russo and colleagues (2020) ideas of conditionally positive responses, in that they held positive beliefs, but mentioned teacher involvement in the struggle. Linda's conception was the only one that fell into the perspective of a positive belief; however, in her reflections, she would often structure struggle with questions.

A possible explanation for why the PSTs viewed supporting perseverance in terms of fostering perseverance instead of both fostering perseverance and praising productive process could be influenced by how the PSTs perceived their role in students' perseverance. Evidence suggests that the PSTs saw perseverance as something that the students were responsible for, and the PSTs often had difficulty noticing their supports of perseverance as related to the practice. This was especially evident for Amy, who described students' confidence and willingness to explain ideas from previous problems as attributed to past teachers. This finding agreed with what Warshauer and colleagues (2019) found in their preservice teachers' understandings of productive struggle. Furthermore, the PSTs viewed perseverance as connected to confidence, however, it is unclear as to the direction of this relationship the PSTs imparted between the two ideas when research suggests a clear directional connection. Dweck (2000) noted that students with a growth mindset associated with their mathematical ability are more likely to have greater confidence that they will succeed; however, Amy noted that confidence allowed the student to persevere. This conceptualized connection to confidence and perseverance merits further study, to see if it aligns with current literature or if it disagrees, and if so, how.

Additionally, the PSTs noted that having students reflect on their work served the purpose of having students become confident in justifying answers. The thought of this as a move to foster perseverance seemed to be an afterthought or an accompanying outcome. Overall, the PSTs did not view perseverance as something supported by the teacher, but rather as something internal to the student. Although the PSTs' moves aligned with research, these moves were not all recognized as noteworthy, or for the purpose of supporting perseverance. Thus, consistent with the literature recommendations mentioned here (Lewis & Özgün-Koca, 2016; Warshauer et al., 2019), university coursework should include an awareness of perseverance and ways of fostering it. The PSTs in this study were already supporting perseverance to some extent, and when made more aware of perseverance and ways of fostering it, these existing supports could potentially be used more purposefully and provide better support for students' learning. Thus, future studies would benefit from providing PSTs with techniques to better support student perseverance and observing how these supports are taken up and could potentially alter the supports they use with

students. Implications from this study suggest a particular need for university coursework to emphasis ways of promoting student-to-student talk as a way to foster a productive and persevering learning environment. Similarly, reflective or metacognitive questions would be a beneficial addition to not only model but include as a topic for discussion in university coursework, as these skills not only foster perseverance but are valuable mathematical reasoning habits (NCTM, 2009).

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