PROSPECTIVE SECONDARY MATHEMATICS TEACHER RESPONSES AND THE STRUCTURE OF APPROXIMATIONS OF PRACTICE

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To support productive struggle, prospective secondary mathematics teachers (PTs) need to elicit and respond to students' mathematical ideas in ways that focus on those ideas and that position students to build on those ideas. Using the Teacher Response Coding framework (Van Zoest et al., 2021) we analyzed PTs' responses in three rehearsals of instruction. We identified significant differences in the natures of responses in one rehearsal compared to the other two. Using a Levels of Constraint framework based on findings of Kavanagh et al. (2020) we compared structures of the rehearsals and developed hypotheses regarding which aspects of structure might account for differences in PTs' responses.

Keywords: Classroom Discourse; Instructional Activities and Practices; Preservice Teacher Education

Over multiple decades, mathematics education researchers have identified mathematics teaching practices that aim to use and build on student thinking. Particularly, practices that invite students to share and elaborate their thinking (Franke et al., 2009) and that build on student thinking by asking students to connect their thinking to others' ideas (Smith & Stein, 2018) have been shown to support student learning (Webb et al., 2014). One goal of teacher education is to support prospective teachers (PTs) in developing such practices. Yet there is often dissonance between what prospective teachers learn about teaching in their professional coursework and the teaching that they have opportunities to enact and witness in field placements (Grossman et al., 2009). In response, MTEs have developed approximations of practice (Grossman et al., 2009) in which PTs to interact with simulated or authentic students (e.g., Arbaugh et al., 2019; Lampert et al., 2013). Our use of *approximations of practice,* in this context, is meant to describe events that occur in a context that is similar to authentic classroom teaching but less complex, less authentic, and more controlled. In theory, such opportunities provide contexts for PTs to engage in teaching practices, such as eliciting and responding to student thinking, in settings that are less complex than those in which teaching typically occurs. However, approximations of practice differ with regard to multiple features, including how students are embodied and by whom, and the duration of the experience. The purpose of this study is to examine how differences in the structure of approximations of practice relate to differences in how PTs engaged in teaching within those approximations of practice.

We report our empirical investigation into the following research questions:

1. How do the ways that PTs respond to student thinking within approximations compare across approximations of practice that differ in structure?

2. What aspects of structure of approximations of practice might account for differences in the ways that PTs respond to student thinking?

Theoretical Perspectives

Our study is situated at the intersection of research and scholarship on how teachers respond to instances of student mathematical thinking and research and scholarship related to the use of approximations of practice in mathematics teacher education. In the next two sections we share the perspectives informing this study.

Perspective on Teacher Responses

In a thorough review of literature on researchers' ways of examining teacher responses, Van Zoest et al. (2021) identified three core aspects that describe productive teacher responses: the "who, what, and how" of a response. In other words, *who* in the classroom is asked to publicly interact with a student's mathematical contribution (SMC)? *What* actions do classroom member(s) get to take with respect to the SMC? And *how* does the teacher's response relate to the substance of the SMC? For students to engage with each others' SMCs, it is important for teachers to attend to SMCs in their responses and for teachers to position students to take action on the mathematical idea contained in the SMC (Bishop et al. 2020; Robertson et al. 2016).

Perspective on Approximations of Practice

Grossman et al. (2009) described the nature of professional learning in human improvement professions, including teaching, in terms of three constructs: Representations of practice, decompositions of practice, and approximations of practice. We define an approximation of practice as an activity that is similar to, but not identical to, typical activities of a professional teacher, such as participating in a simulation of an interaction with students around mathematics content. Engaging in approximations of practice provides PTs with opportunities to act as teachers in a context designed to resemble an instructional interaction. Approximations of practice aim to support PTs to learn in and from practice (Lampert, 2010). Based on the positions taken by others who have worked in this area, we argue that the use of approximations of practice in mathematics methods courses is guided by three principles. First, PTs need repeated opportunities to engage in practices that are challenging for novices (Grossman et al., 2009), such as responding to student thinking. Second, rehearsals must increase in complexity across consecutive rehearsals by involving more authentic classroom interactions, broader components of instruction, and less intervention from the teacher educator (Boerst et al., 2011; Grossman, 2009). The third principle is that a teacher educator (TE) must mediate PTs' experiences in an approximation. This includes representing teaching in terms of significant practices (e.g., Grossman et al., 2009) to support the PTs' "development of situationally appropriate knowledge and skill" (Lampert et al., 2013). Mediation is significant because it can shape what teachers attend to within an approximation and how they respond (Kavanagh et al., 2020).

Methods

Context of The Study

We report the results of an analysis of prospective teachers' responses to SMCs within three approximations of practice conducted across a single semester in a secondary mathematics methods course at a large public university in the mid-Atlantic region of the United States. Each approximation occurred within a learning cycle (Lampert et al., 2013) in which PTs engaged in a mathematical activity, examined representations of teaching the activity, planned to teach an activity, enacted that teaching within the methods course, and reflected on their experiences (see Arbaugh et al., 2019). These activities were all focused on a set of communication moves (Freeburn & Arbaugh, 2017) that served as a decomposition of eliciting and responding to student thinking. We focus our study on the planning and enactment within each cycle in which PTs focused on developing those communication moves. We refer to a *rehearsal* as an approximation of practice in which PTs, in the role of teacher, interact with one or more individuals in the roles of students, whom we call Enacted Student(s) (ES). We designed each rehearsal to align with the three guiding principles described in the theoretical perspectives. Figure 1 provides a summary of the intended variations in structure across the rehearsals.

Element	**Rehearsal 1**	**Rehearsal 2**	**Rehearsal 3**
Ratio Teachers: Students	2:1	1:2	1:7
Intended Duration	6 minutes	3 minutes	15 minutes
Enacted Student(s)	Elif or Lewis	Elif and Lewis	Small Group of Peers
Student Behavior	The ES is partially finished with the task at the start of the rehearsal	The ESs believe they are finished with the task at the start of the rehearsal	ESs begin the task at the start of the rehearsal
Student Work	Student work constructed prior to and during the rehearsal	Student work constructed prior to and during the rehearsal	Student work constructed during the rehearsal
Mathematical Task	Textbook task focused on linear functional relationships	Construct an argument for a given number theoretic claim	Textbook task focused on exponential functional relationships or statistics

Figure 1: Intended Variations Across Rehearsals

Prior to each rehearsal PTs received a copy of the task that would frame their conversation with one or more ESs. PTs did the task and then planned in small groups to enact the task in their rehearsals. In RH2, PTs received copies of partially-complete student work before the rehearsal. We did not provide partially-complete work prior to RH1 or RH3. In Rehearsal 1 (RH1) and Rehearsal 2 (RH2), PTs interacted with Elif or Lewis, who were graduate students portraying ESs based on protocols that we designed to guide the representations and student misconceptions that the graduate students would enact during the rehearsal. In Rehearsal 3 (RH3), peers were the ESs that were engaging with the mathematical task for the first time.

Data Collection and Analysis

We analyzed transcripts of video-recorded rehearsals for three pairs of PTs. Each pair participated in RH1 together, and then in RH2 and RH3 as individuals, which resulted in a total of 5 rehearsals per pair and 15 rehearsal videos as data for the study. We also analyzed artifacts of class sessions (e.g., boardwork, handouts, and other such artifacts) as well as statements made verbally or in writing by the research team during their planning sessions.

We analyzed data in three phases. In Phase 1 we analyzed recordings of our planning sessions, video-recordings and artifacts from the class meetings preceding each rehearsal, and video records of rehearsals using a framework based on the findings from Kavanagh et al. (2020) which delineated four elements of rehearsals that can vary in degrees of scaffolding:

- *disciplinary content* (e.g., task, learning objective),
- rehearsing teacher's *instructional routine,*
- *texts/tasks* used in the rehearsal,
- and TE *facilitation moves.*

Kavanagh et al. further delineated each element along a dimension of constraint ranging from *loosely constrained* to *tightly constrained,* depending on the extent to which decisions related to that element are made by the TE or left open for the PT to decide. We analyzed the design and enactment of each rehearsal to characterize levels of constraint with respect to each of the elements from Kavanagh et al. (2020). Our analysis led us to extend to two additional elements, which we describe in the findings.

In Phase 2 we applied a subset of the Teacher Response Coding Scheme (TRC; Van Zoest et al., 2021) to video transcripts to analyze PTs' responses to each ES mathematical contribution. The subset of the TRC is composed of three categories of codes that align with the *who*, *what,* and *how* facets of teacher responses described in our perspectives section. We coded a total of 304 PT responses across the three rehearsals and compiled contingency tables that reported the frequency of each code in each category of the TRC across rehearsals.

Category Description	Code Descriptions
Actor (Who?): The person publicly given the opportunity to consider the instance of SMC.	**teacher, same student(s), other student(s), whole class**
Action (What?): What the actor is doing or being asked to do with respect to the instance of SMC.	**check-in** (elicits self-assessment or understanding of a SMC), **clarify** (asks an actor to make an SMC more precise), **collect** (asks an actor to contribute a new or alternate SMC), **connect** (asks an actor to connect an SMC to a previously introduced idea), **develop** (asks an actor to build on an SMC)
Student Ideas (How?): The extent to which the student who contributed the instance of SMC is likely to recognize their contribution in the teacher response.	**core** (the response explicitly references the SMC), **peripheral** (the SMC is implicit but recognizable in the response), **other** (the SMC is not recognizable in the response), **not applicable** (SMC can not be inferred or the teacher response is too vague)

Figure 2: Subset of TRC Categories and Codes

In Phase 3 we compared differences in proportions of codes in the three categories of Actor, Student Idea, and Actions pairwise between the rehearsals. Because our goal was to examine the development of practices that engage students in sharing ideas and in building on their own and others' ideas, we grouped codes for Actor into student-centered codes (e.g., student, other student(s), and whole group) and teacher codes, and grouped codes for Student Idea into two

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groups: Core/Peripheral versus Other/Not Applicable. We used Chi-Square Tests of Independence (p=0.05) to test for significance of differences related to Student Idea and Actor. Because there are 15 distinct codes for Actions in the TRC, the Chi-Square test becomes inappropriate (28 degrees of freedom with many small expected values. Therefore, we identified those Actions that accounted for 10% or more of the codes in each Rehearsal and compared the results across Rehearsals.

Results

Finding 1: RH2 Was Constrained Differently From RH1 and RH3

Our analysis revealed that RH2 was constrained at a different level from RH1 and RH3 in five elements: *Disciplinary Content, Text/Tasks, Student Work, Instructional Routine,* and *Pedagogical Learning Objectives*. We found the *Facilitation Moves* element had the same level of constraint across all three approximations. We elaborate in the next few paragraphs.

Disciplinary Content: Learning Objectives. In RH1 and RH3, PTs constructed the mathematical goals for student learning based on mathematical content and practices that the PTs identified while working through their respective mathematical tasks. In contrast, for RH2 the TE identified and communicated the mathematical learning goal to the PTs during the planning class session prior to the rehearsal.

Texts/Tasks: Mathematics of the Tasks. The TE scaffolded PTs' engagement with the mathematics of the tasks differently in RH2 than in RH1 or RH3. In preparing for RH1 and RH3 the TE did not constrain PTs' activities to focus on the mathematical content or practices involved in the task. Mathematical ideas in the tasks surfaced when PTs asked questions during their planning. For example, one group shared the difficulty they were having with creating questions in preparation for RH1 because they were uncertain of student approaches. The TE responded, "So, one way you may approach those is to think about what the mathematical goal is. Then, go back to your assessing questions and examine how well these questions help me to understand where the student is with respect to that goal." However, in preparing for RH2, the TE pushed PTs to stipulate criteria for acceptable student responses and to explicate multiple ways that students might engage in the task. The TE also engaged PTs in constructing arguments and critiquing arguments.

In addition, there was a difference in the nature of the mathematical tasks for the rehearsals. The mathematical tasks in RH1 and RH3 were both selected from units in the *Connected Mathematics Project* (Lappan et al., 2002) that presented questions about a realistic scenario in order to develop the mathematical content, such as linear versus exponential growth or measures of center in data sets (see Figure 1a for one example). While mathematical content was certainly involved in the RH2 argumentation tasks (Figure 1b), the nature of the task was oriented more towards mathematical practices than content.

a.cerpt from *Moving Straight Ahead Task* (Lappan et al., 2002b, pp. 24–25) from RH1.

Cycling time (hours)	Distance (miles)		
	José	Mario	Melanie
0	0	0	0
1	5	7	9
2	10	14	18
3	15	21	27

1.

b.) **Perfect Squares Task from RH2.**

Write an argument for the claim, "the product of perfect squares is a perfect square".

1. a.) How fast did each person travel for the first 3 hours? b.) Assume that each person continued at this rate. Find the distance each person traveled in 7 hours.	

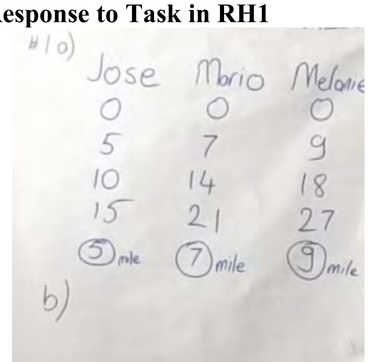
Figure 3: RH1 and RH2 Tasks

Instructional Routine: Discussion Structure. In RH1 and RH3 the TE did not stipulate a prescribed discussion structure for PTs' interactions with ESs. TE instructed PTs to ask questions to elicit ESs' thinking and to support their progress towards the mathematical goals of the activity, but otherwise left it to PTs to decide the structure of their interactions with ESs. However, in the planning session for RH2, the TE directed PTs specifically to facilitate a conversation between two ESs to support their progress in constructing a valid argument. The TE directed,

> You are to work with these two students to get them to talk to each other in a way that takes the arguments they've crafted and help them make progress . . . in writing a valid argument by connecting the representations together. . . . Questions you are asking are not just for you to get information . . . but also to get them to give information to each other . . . . The arguments don't have to be the same [by the end of the rehearsal] but they need to be coordinated.

Student Work: Availability and Amount. RH2 was more constrained than RH1 or RH3 with respect to 1) how PTs accessed student work and 2) the extent to which student work represented a "finished" product. In RH2 the TE provided PTs with ESs' arguments the day before the rehearsal as a resource for their planning (see Figure 2a). The work was presented as each ESs' "finished"'’rgument for a given claim. In RH1 and RH3, however, PTs' first encounters with ESs' work was during the rehearsals, and the presented work was that of a student(s) still in the process of completing the task (see Figure 2b).

a.**if's Partial Response to Task in RH1**



2.

b.) **Lewis's Argument in RH2**

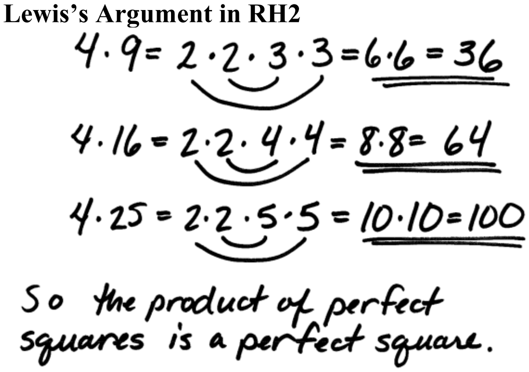


Figure 4: Samples of Student Work from RH1 and RH2

Finding 2: PTs Positioned Actors Differently in RH2 than in RH1 or RH3

The contingency table shown in Table 2 presents the number of PT responses that we coded from each rehearsal in the Student Idea category and in the Actor category. The extent to which responses explicitly incorporated student ideas does not appear to depend on whether the

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responses occurred in RH1, RH2, or RH3 (X^2 (2, N = 304) = 2.42, p = .30). However, the proportion of PTs' responses that positioned students as actors was significantly different across rehearsals (X^2 (2, N = 304) = 25.9, p <0.05). Closer examination revealed that the proportion was significantly different in RH2 than in either RH1 (X^2 (1, N = 111) = 12.15, p < 0.05) or RH3 (X^2 (1, N = 247) = 29.44, p <0.05), but not between RH1 and RH3 (X^2 (1, N = 250) = 4.365, p = 0.113). We interpret this as evidence that some aspects of RH2 must have supported a different kind of response pattern, not with respect to student ideas, but in terms of the extent to which students were invited to consider and to build on their own and others' ideas.

Table 1: Contingency Tables of Codes in Student Ideas and Actor Across Rehearsals

Coding of Response for Student Ideas	RH1 (n=57)	RH2 (n=54)	RH3 (n=193)
Student Idea: Core or Peripheral	44	37	128
Student Idea: Not applicable or other	13	17	65
Coding of Responses for Actor			
Teacher	26	10	111
Student(s)	31	44	82

Finding 3: Responses Involve A Broader Set of Actions In RH2 than in RH1 or RH3.

As shown in Table 3, PTs' responses in RH2 had the greatest variety of *actions* with relative frequencies of 10% or above. We found this somewhat surprising, given that PTs had the least amount of time (three minutes) to interact with Elif and Lewis in RH2 than with students in RH1 or RH3 (6 minutes and 15 minutes respectively). The *develop* action was the most frequent action across all three rehearsals. These actions were ones in which the PT expanded or requested ES(s) to expand on an ES's contribution.

Table 2: Most Frequent Actions Across Rehearsals

	Check-in	Clarify	Collect	Connect	Develop
RH1 (n=57)	**	8 (14.04%)	**	**	26 (45.61%)
RH2 (n=54)	6 (11.11%)	7 (12.96%)	9 (16.67%)	9 (16.67%)	12 (22.22%)
RH3 (n=193)	**	**	25 (12.95%)	**	55 (28.50%)

Discussion

If we characterize PTs' capacity to build on student thinking in terms of their observed patterns of response via the TRC scheme, our findings suggest that progressive increase in complexity and authenticity across approximations is insufficient to account for differences in the patterns of their responses within the approximations --in fact, variations in constraints across other elements of the approximations seem to better explain those differences. We found that

PTs' responses in RH2 were directed more at students and involved a greater repertoire of actions than in either of RH1 or RH3. When we examine the structures of approximations through the lens of levels of constraints with respect to some elements (Kavanagh et al., 2020), we find ways that RH2 was more constrained than RH1 and RH3--namely, in terms of *disciplinary content, texts/tasks, instructional routine,* and *student work*. Our findings further support the findings of Kavanagh et al. (2020) which suggested that more tightly constrained rehearsals might explain differences in how PTs engage in those rehearsals.

Grossman et al. (2009) used the metaphor of "learning how to kayak in calm waters" (p. 2076) to describe the notion of learning to engage in complex practices in environments of decreased complexity. Kavanagh et al. (2020) used this metaphor to describe how various constraints serve to calm the waters for novices by constraining the complexity of the context in which they engage in approximation of practice. We also find this metaphor helpful for thinking about how some of the constraints may be related to differences in the PTs' responses.

Truly learning to kayak involves learning to navigate toward a destination while considering and choosing among multiple possible routes. The constraints in *disciplinary content, texts/tasks, and student work* that we identified in RH2 may have served to support the PTs by making explicit the destination (the learning goals and the criteria for evaluating students' arguments) and anticipated routes (potential ways that students might approach the task, and the specific discursive structure to use in the rehearsal) that they might experience within the rehearsal. For *disciplinary content*, the MTE established a well-defined and common mathematical destination for the PTs to focus their students towards in RH2. Additionally, they had more scaffolded experiences working with mathematics related to the *texts/tasks* and *student work* of RH2 than in RH1 and RH3. In preparing for RH2, PTs were able to become more familiar with the mathematics of the rehearsal tasks and the ways students might approach the task. Having a greater understanding of the mathematical goal and of the multiple routes students may take may have freed PTs to invoke a broader repertoire of actions to help copilot students in ways that positioned the students as the ones to consider each other's thinking.

The constraints in *instructional routine* related to the PTs' discussion structure in the RH2 may also have contributed to calming the waters in the rehearsal. In preparation for RH2, the MTE and PTs discussed how a teacher may support students in connecting their thinking and analyzed the way a teacher in a narrative case positioned the students to consider each other's thinking. Understanding how a teacher is able to enact this discussion structure seems to have supported the PTs with the repertoire of *actions* in our findings that one would use to get students to notice and respond to each other's contributions during a classroom discussion. Additionally, the discussion structure itself in RH2 naturally oriented the PTs to position the students (*actor* category) as the ones who needed to make sense of each other's work.

Preparing PTs involves supporting them to develop knowledge about teaching alongside their emergent skills as teachers. Lampert et al. (2013) and others suggest that the exchanges that TEs have with PTs during rehearsals affect the opportunities for PTs to develop knowledge of concepts and ideas related to teaching. Our study extends that understanding by illustrating how differences in rehearsal design, along with TEs' pre-rehearsal actions, also may shape PTs' opportunities to engage in core practices within rehearsals. Our study reinforces the findings of Kavanagh et al. (2020) on teacher responses to student thinking in rehearsals and extends that work by characterizing responsiveness in ways that allow for clear examination of differences across rehearsals.

Although our study contributes to understanding how differences in structure may relate to PTs' enactment of core practices within approximations of practice, our rehearsals share important features with each other (e.g., use of semi-scripted ESs in a live interaction) that are not necessarily characteristic of all approximations of practice. Further research will be needed to explore other aspects of design and enactment of approximations of practice and their relationships to the ways that PTs engage in core practices within those approximations.

References

Arbaugh, F., Graysay, D., Konuk, N., & Freeburn, B. (2019). The Three-Minute-Rehearsal Cycle of Enactment and Investigation: Preservice secondary mathematics teachers learning to elicit and use evidence of student thinking. *The Mathematics Teacher Educator*, *8*(1), 22–48. https://doi.org/10.5951/mathteaceduc.8.1.0022s

Boerst, T., Sleep, L., Ball, D., & Bass, H. (2011). Preparing teachers to lead mathematics discussions. *Teachers College Record*, *113*(12), 2844–2877.

Franke, M. L., Webb, N. M., Chan, A. G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education*, *60*(4), 380-392. https://doi.org/10.1177%2F0022487109339906

Freeburn, B., & Arbaugh, F. Supporting productive struggle with communication moves. *Mathematics Teacher, 111*(3), 176–181. https://www.nctm.org/Publications/Mathematics-Teacher

Grossman, P., Hammerness, K., & McDonald, M. (2009). Redefining teaching, re-imagining teacher education. *Teachers and Teaching: Theory and Practice*, *15*(2), 273–289. https://doi.org/10.1080/13540600902875340

Kavanagh, S. S., Metz, M., Hauser, M., Fogo, B., Taylor, M. W., & Carlson, J. (2020). Practicing responsiveness: Using approximations of teaching to develop teachers' responsiveness to students' ideas. *Journal of Teacher Education, 71*(1), 94–107. https://doi.org/10.1177/0022487119841884

Lampert, M. (2010). Learning teaching in, from, and for practice: What do we mean? *Journal of Teacher Education, 61*(1–2), 21–34. https://doi.org/10.1177/0022487109347321

Lampert, M., Franke, M. L., Kazemi, E., Ghousseini, H., Turrou, A. C., Beasley, H., Cunard, A., & Crowe, K. (2013). Keeping it complex: Using rehearsals to support novice teacher learning of ambitious teaching. *Journal of Teacher Education*, *64*(3), 226–243. https://doi.org/10.1177/0022487112473837

Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (2002a). *Connected Mathematics 2. Growing, Growing, Growing: Exponential Relationships*. Prentice Hall.

Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (2002b). *Connected Mathematics 2. Moving Straight Ahead : Linear Relationships*. Prentice Hall.

Smith, M. S., & Stein, M. K. (2018). *5 Practices for Orchestrating Productive Mathematics Discussions* (2nd ed.). NCTM.

Van Zoest, L. R., Peterson, B. E., Rougée, A. O. T., Stockero, S. L., Leatham, K. R., & Freeburn, B. (2021). Conceptualizing important facets of teacher responses to student mathematical thinking. *International Journal of Mathematical Education in Science and Technology.* https://doi.org/10.1080/0020739X.2021.1895341

Webb, N. M., Franke, M. L., Ing, M., Wong, J., Fernandez, C. H., Shin, N., & Turrou, A. C. (2014). Engaging with others' mathematical ideas: Interrelationships among student participation, teachers' instructional practices, and learning. *International Journal of Educational Research, 64*, 79–93. https://doi.org/10.1016/j.ijer.2013.02.001

Webel, C., Conner, K., Zhao, W., & Kuntze, S. (2018). Simulations as a tool for practicing questioning. In O. Buchbinder (Ed.), *Mathematics Teachers Engaging with Representations of Practice* (pp. 95–112). Springer International.