PRODUCTIVE STRUGGLE LEADING TO COLLECTIVE MATHEMATICAL CREATIVITY

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In this article we show how students' productive struggle on a mathematical task can lead to collective mathematical creativity. We use observable (co)actions and interactions from a video record that features three Grade 6 students in a problem-solving session to document the emergence of collective creativity leading to a solution. We discuss some key features of the task and the learning environment and present implications for classroom practices aimed at helping students to capitalize on their mathematical struggles.

Keywords: Elementary school education, Problem solving, Number concepts and operations

In this paper we show how elementary students' productive struggle on a mathematics task can lead to collective mathematical creativity and what that process might look like in practice. We discuss some key features of the task and the learning environment and present implications for classroom practices aimed at helping students to capitalize on their mathematical struggles.

Literature Review

Productive Struggle

In the field of mathematics education, Boaler (2016) described a vision of mathematics learning where students are offered opportunities to engage in productive struggle, to thrive, and to become mathematical problem solvers. Lesh & Zawojewski (2007) noted that such a productive way of thinking involves iterative cycles of "expressing, testing, and revising mathematical interpretation—and of sorting out, integrating, modifying, revising or refining clusters of mathematical concepts from various topics within and beyond mathematics" (p. 782). There is an extensive literature discussing ways to support students in this kind of productive struggle in the mathematics classroom. The NCTM (2014) noted that effective teaching values productive struggle as a means to deepen conceptual understanding and "embraces a view of students' struggles as opportunities for delving more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas" (p. 48). In recent years, many authors (e.g., Townsend et al., 2018; Warshauer, 2015) have emphasized the socioemotional dimension of learning and have focused on the importance of building supports for, and valuing, struggle in the classroom. It is widely recognized that without appropriate supports students can spend a lot of time in unproductive struggle and that, for those students, timely intervention is key in nudging them forward from unproductive to productive struggle (Jonsson et al., 2014). Some studies, though, report that students are able to sustain productive struggle, given supports such as an appropriate task, successful strategy choice, and relevant tools. For example, in a study using GeoGebra, Granberg (2016) reported that the majority of the students were able to engage in productive struggle that enabled them to solve problems together. Successful students did this by observing knowledge gaps between their prior knowledge and the target knowledge, correcting incorrectly recalled information, and reconstructing partly forgotten knowledge.

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Creativity—Individual and Collective

While some see creativity as confined to special people, particular arts-based activities, or undisciplined play, scholars generally agree that creativity involves the combination of originality and task appropriateness or effectiveness (Beghetto & Kaufman, 2013; Runco & Jaeger, 2012). The word *creativity*, both in its origins and in most of its different uses, reflects a kind of newness, originality, or novelty; it indicates bringing something new and fruitful into being. Craft (2001) claimed that creativity in learning environments enables learners to generate and expand ideas, suggest hypotheses, apply imagination, and look for alternative, not-yet imagined approaches. In the field of mathematics education, Levenson (2011) characterized collective mathematical creativity using characteristics of individual creativity—namely, fluency, flexibility, and originality—and concluded that working as a collective may encourage students to persevere and try new ideas and that teachers can promote the emergence of creativity in their classrooms by encouraging diversity, supporting interactions, and allowing for a certain amount of instability (Levenson, 2014).

Theoretical Framing

Building on this scholarship, herein we draw on the first author's work on collective creativity in mathematics learning environments (Aljarrah, 2017, 2018, 2020; Aljarrah & Towers, 2019), and the work of the second author on the emergence of collective mathematical understanding (Martin & Towers, 2009, 2011; Martin et al., 2006). We bring these theoretical frameworks together to document and analyze the trajectory from productive struggle to the emergence of collective creativity.

Collectivity and Emergence

The second author and colleagues (Martin et al., 2006) laid the groundwork for the present study of collective creative acts in mathematics learning environments. They argued that doing and understanding mathematics are creative processes that should be considered at both the individual and the collective levels. Drawing on improvisational theory, Martin and Towers (2009) suggested that, when students are working together, acts of mathematical understanding "[can] not simply be located in the minds or actions of any one individual, but instead [emerge] from the *interplay* of the ideas of individuals, as these [become] woven together in shared action, as in an improvisational performance" (p. 2, emphasis in original). Martin et al. (2006) used the notion of coaction "to describe a particular kind of mathematical action, one that whilst obviously in execution is still being carried out by an individual, is also dependent and contingent upon the actions of the others in the group" (p. 156).

One of the most important ideas in the study of collectivity in learning settings is the notion of emergence. In our analysis of data later in this paper, we concentrate on three key features of collective emergence adopted from improvisational theory and already articulated in the mathematics education literature (e.g., Martin & Towers, 2009, 2011; Martin et al., 2006): (1) potential pathways, (2) collective structure and striking a groove, and (3) etiquette and the group mind. Noteworthy here is that the actions and interactions of a group working as a collective are usually prompted and constrained by a common purpose that guides the development of a collective structure. In referring to the development of such a collective structure, Martin and Towers (2011) adopted Berliner's (1994, 1997) expression *striking a groove*.

> Striking a groove involves 'the negotiation of a shared sense of the beat,' and is a subtle and fundamental process to allow the performance to develop to its fullest…. The 'groove' is the

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> underlying element of the structure that allows the improvisation to proceed in a coherent and productive way, and it is the responsibility of all the players to collectively maintain the groove. (Martin & Towers, 2011, p. 257)

Martin and Towers (2011) also borrowed the expression "etiquette" from Becker (2000) to refer to a number of conventions (group norms) that "govern the ways in which an improvisational performance develops and group flow[1] emerges" (Martin & Towers, 2011, p. 258). Based on the study of improvisational theater, Sawyer (2001) noticed that actors use guidelines (principles) to create better conversations. Three simple, yet overarching, principles were proposed by Sawyer (2001) as rules of improv: (1) Yes, and…, (2) Don't write the script in your head, and (3) Listen to the group mind. According to Sawyer (2001), the "Yes, and …" rule implies that every student should accept the material introduced by preceding student(s) and add something new to it. The second rule, "Don't write the script in your head," is intended to keep all improvisers, moment by moment, within the scene. It means do not plan in advance by foreshadowing or pre-determining where the problem-solving is going, for to do so shines the spotlight on oneself and results in "a lack of the necessary outward focus, toward the group creativity" (Sawyer, 2001, p. 17). Hence, an outward focus requires adherence to the third rule—listening to the group mind—being willing to abandon personal motivations to further the emerging collective structure.

Collective Creativity

Sawyer (2003) also asserted the improvised and the collective nature of group creativity. According to him, group creativity is: (1) unpredictable, in that each moment emerges from preceding flow of the performance, (2) collective, in that members of the group influence each other from moment to moment, and (3) emergent, in that the group demonstrates properties greater than the sum of its individuals. Based on the above ideas, and the first author's study of the nature of collective creativity in mathematics learning settings (Aljarrah, 2018), we define collective creative acts as particular kinds of "(co)actions and interactions of a group of curious learners while they are working collaboratively on an engaging problematic situation. Such acts, which may include (1) summing forces, (2) expanding possibilities, (3) divergent thinking, and (4) assembling things in new ways, trigger the new and the crucial to emerge and evolve" (p. 136). Below, we elaborate on the four metaphors for creativity, first proposed by Aljarrah (2018), that form core of our definition of collective creative acts:

Summing forces: This metaphor encompasses the ways in which learners coordinate their efforts to enable productive steering (Aljarrah, 2019) towards a mathematical understanding "that is not simply located in the actions of any one individual but in the collective engagement with the task posed" (Martin et al., 2006, p. 157).

Expanding possibilities: Expanding might be understood as broadening the learners' horizon by gaining new insights based on previous insights. It is a kind of stretching of the space of the possible as a result of the evolving and the growth of the learners' basic insights.

Divergent Thinking: Divergent thinking requires students to consider many potential pathways, look in many directions, journey outside a known content universe, go beyond the problem's clearly given conditions and information, and think outside-the-box (Aljarrah, 2019).

Assembling (things in new ways): This metaphor implies looking for associations and making connections. It is a vision of creativity based on an assumption that many educative things are with(in) the reach of learners in their learning environment.

In our analysis, we show how collective creativity emerges from productive struggle by detailing the students' pathways to collective creativity in pursuit of a solution to a mathematical problem.

Methods

The data described below are part of a broader, design-based research study exploring collective creativity in elementary mathematics learning environments (Aljarrah, 2018). Two mathematics teachers and 25 of their sixth-grade students in a Canadian school setting participated in the study. Students participated in problem-solving sessions in their regular mathematics classroom and in small groups under task-based interview conditions with the first author. Video-recordings of these group activities formed the core of the data.

The processes of analysis followed Pirie's (1996) advice to "sit, look, think, look again" (p. 556) supported by Powell et al.'s (2003) analytical model for studying the development of mathematical thinking, which consists of seven interacting, non-linear phases: (1) viewing the video data, (2) describing the video data, (3) identifying critical events, (4) transcribing, (5) coding, (6) constructing a storyline, and (7) composing a narrative (p. 413). Following Flanagan (1954), an event was considered to be critical if it was helpful in triggering and/or explaining the emergence of collective creativity in elementary mathematics learning environments. These events were transcribed and the key features of collaborative emergence (Martin & Towers, 2011) together with the first author's definition of collective creativity and metaphors for creativity as outlined in the previous section, were used to code the students' collaborative practices that were effective in the emergence of new and crucial ideas. For the purpose of this article, we selected one video excerpt that best displayed the way that productive struggle led to the emergence of collective mathematical creativity.

Findings

In order to explain how students' productive struggle on a mathematical task can lead to collective mathematical creativity, we use a video excerpt that features a group of sixth grade students, who were assigned the pseudonyms Maddie, Adam, and Frank, engaged in a problem-solving session with the first author. The first author introduced the following task to the group and asked them to work on it together: *What are the possible combinations to obtain a sum of one dollar using pennies, nickels, dimes, and quarters such that the four different types of coins are included in each combination?* Due to space limitations, we focus on describing three collective creative acts, namely, summing forces, expanding possibilities, and divergent thinking, which resulted from the group's productive struggle on the assigned mathematical task. (Note: In the transcript we use dashes to show an interruption of one speaker by another).

Productive Struggle Leading to Summing Forces

The presence of multiple potential pathways was evident at the beginning of the scene. The students started by negotiating the task, and a variety of ideas and suggestions were put forward as possible approaches to find all combinations to obtain a sum of one dollar. Quite quickly, one potential pathway garnered attention. Adam suggested getting "the basic ones [i.e., one penny, one nickel, one dime, and one quarter]." Maddie gave the sum of those basic ones: "Okay, there is forty-one—" and Frank suggested that they could "use all pennies" to make up the rest of the dollar (i.e., fifty-nine cents). He also started to pool the group's thoughts and ideas on their shared document. For example, he wrote down the expression 41¢ = 1 penny + 1 nickel + 1 dime + 1 quarter and labelled it as a fixed amount. He also wrote down 59¢ and under it he wrote 59 pennies as a first suggestion to make 59¢. Maddie noted that they "need at least one of each,

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though still." Frank responded by pointing to their shared document and explaining, "Yes, those are forty-one—" (he was trying to remind her that they already had one of each coin in their basic combination to a total of forty-one). Maddie agreed that they could "have all pennies," so Frank continued the discussion by wondering, "So, forty-one, um, that means there is, um, how many?" Adam responded "fifty-nine." Maddie was still doing the calculation in her head while she was whispering "fiftyyyyyy, um—" so Frank stressed Adam's answer by completing Maddie's whispering, "Nine, yes, fifty-nine." Frank summarized and rearticulated their initial thoughts by stating, "Okay, so fifty-nine left. Out of fifty-nine, how many can we make? So, one of them is fifty-nine pennies, um— [while he was looking to Adam and Maddie]." Our interpretation of Frank's pause and questioning look towards Adam and Maddie is that the space was open equally to all suggestions. As such, it was impossible to predict the direction of the group's unfolding interaction. None of the students seemed to be trying to force his/her ideas on the group, and none of them tried to convince the others to follow a specific strategy. To this point, what we found of particular noteworthiness was the group's collective engagement in "summing forces." They tried to understand the problem and to consider the conditions of it. Thus, decisions about where to start and how to proceed emerged from their interactions as a group. They listened respectfully to each other and responded thoughtfully to the wonderings and suggestions that emerged through the conversation. The respectful collaboration between the students set them on a pathway towards the mathematics that emerged.

Productive Struggle Leading to Expanding Possibilities

As the interaction continued, the task the students set for themselves shifted from finding all possible combinations to obtain a sum of one dollar to finding all possible combinations to obtain a sum of 59 cents. From here on, a collective structure started to evolve. This conceptual structure was located in, and stemmed from, the actions and doings of the group as a collective. Those acting and doings "determine[d] both the nature of the potential that [was] created, and also how the potential [was] then developed into a coherent performance" (Martin et al., 2006, pp. 159–160). Take as an example the occasion just mentioned above, where Frank initiated a space for a conversation to navigate potential pathways to proceed: "Okay, so fifty-nine left. Out of fifty-nine, how many can we make? So, one of them is fifty-nine pennies, um— [while he was looking to Adam and Maddie]." This opening prompted Adam to suggest making a table within which to arrange the group's choices, and, on their shared piece of paper, he drew an initial table with four columns and a few rows. Maddie pulled the paper toward her side of the table, labeled the columns of Adam's table (1¢, 5¢, 10¢, & 25¢), and started to suggest, with effective participation from Frank, some possible combinations to total fifty-nine cents (see Table 1). At this moment we see the students striking a groove (Berliner, 1994, 1997). Maddie and Frank needed no explanation of Adam's table, nor did Adam attempt to offer an explanation. Maddie didn't seek Adam's permission (and nor did he show any sign that such seeking was expected) to take control of the shared document containing Adam's blank table. Maddie added column headings, and these were not contested in any way. Maddie and Frank then began suggesting possible combinations of coins that would sum to 59. This kind of synchronous participation is characteristic of the coactions that are needed to sustain a collective structure. The metaphor of growth—of expanding possibilities—seems to characterize the students' participation in this episode as they built on and expanded the ideas, concepts, and approaches already developed.

Table 1: The Group's Initial Table Filled with Twelve Suggestions

1¢	5¢	10¢	25¢
59	0		
54	1		
49	2		
44	3		
39	4		
34	5		
29	6		
24	7		
19	8		
14	9		
9	10		
4	11		

Productive Struggle Leading to Divergent Thinking

While trying to lay out all possible combinations to a total of fifty-nine cents in their shared table, the students engaged in an interactional conversation to find an effective way to do this. Their interaction and conversation supported them in considering many potential pathways, looking in many directions. For example, Frank started to fill the table with some possible combinations while whispering, "Um, fifty-nine pennies—." But suddenly, a different potential pathway seemed to present itself to Adam who suggested trying to "get the total amounts [of combinations]; we can get the total amount for, like, if we change this (the fifty-nine) to fifty, and then we had some sort, like, the two combinations of nine (i.e., nine pennies; and four pennies and one nickel)—." Maddie, still wedded for the moment to the idea of finding combinations that made 59 not 50, tried to make sense of Adam's suggestion. She asked him to "wait, wait, wait," and then to "continue." Adam explained his suggestion by stating, "You could change the number to fifty [instead of fifty-nine], and then go from fifty, because it is easier to go from fifty and then multiply the answer by two." While Adam was explaining to Maddie "why [he would] multiply the answer (i.e., the number of combinations to a total of 50 cents) by two," Frank continued filling their existing shared table while whispering words like "forty-seven, um, forty-nine."

Two possible pathways were now in play and the group faced a choice about which pathway to follow. As the students negotiated their varied suggested strategies to proceed, Maddie pondered the options. The "Yes, and" rule, sometimes called the "Do not deny" rule, does not mean that you must agree with everything that comes from fellow learners, but it does mean that you have to listen to them thoughtfully, and fully respect, embrace, and respond to their contributions, which is what Maddie did when faced with the two potential pathways. Maddie made a commitment to their existing strategy, saying to Adam, "Okay, let us actually listen to him (i.e., to Frank)." Following Maddie's suggestion, the group suspended Adam's suggestion (to begin with 50 instead of 59) in favor of trying the strategy that Frank was still pursuing—to lay out all potential options to combine two or more types of coins, and then to find all possible combinations to obtain a sum of 59 cents under each option. They inferred that there were eleven options that were the basis of all possible combinations totaling 59 cents: pennies and nickels;

nickels and dimes; dimes and quarters; pennies and dimes; pennies and quarters; nickels and quarters; nickels, dimes, and pennies; nickels, dimes, and quarters; pennies, dimes, and quarters; quarters and dimes; and all (i.e., pennies, nickels, dimes, and quarters). For a while, all subsequent actions of the group were about developing a fast (or an effective) strategy to find all possible combinations that met these criteria (a collective goal). All the three students' contributions were critical in keeping the mathematics moving forward. Most speaking turns followed the "Yes, and…" rule, and listening to the group mind was evident throughout the whole problem-solving session with this group. Though Adam had interjected a new suggestion (to find combinations to sum to 50, then multiply by two), which could have destabilized the group process, the group collectively agreed to shelve Adam's idea for now and continue working on their present strategy. Although the group elected to listen to the group mind, they later returned to test Adam's suggestion but at the end of their second consideration of his mathematical idea, Adam was willing to abandon his personal motivations and to defer to the group mind (Martin & Towers, 2011) as the group returned once again to using 59 cents as their focus for generating combinations.

The metaphor of divergent thinking characterizes the students' collective process during this part of their problem-solving journey. Two competing solution paths emerged and were given consideration and one was agreed upon and pursued by the group. The group showed that it valued divergent thinking by re-considering the rejected proposal a second time, before ultimately letting it go.

Discussion

In the above extracts we can see that engaging in productive struggle, when viewed through the lens of improvisational concepts such as emergence of multiple pathways, collective structure and striking a groove, and etiquette and group mind, is an iterative process. The students began by considering multiple potential pathways and establishing an etiquette of working together and listening to group mind. At this stage of their problem solving the metaphor of summing forces can be used to describe their actions. At each point during the scene when the group faced or was confronted by a challenge, all the members of the group were eager to contribute their ideas and thoughts and to listen responsively to the others' contributions. The *momentum* that helped students to overcome such challenges and make remarkable progress should be attributed to the whole group as a result of the interaction between their ideas, thoughts, representations, metaphors, gestures, and words.

They gradually refined their problem-solving through striking a groove resulting in a collective structure of focus. Here, the metaphor of growth and of expanding possibilities characterizes their creative process. Students' creative acts were not just about finding their route around/through the problem. Even though they settled on an initial strategy, they still continued to generate alternative possible pathways. By continuing to explore (play with) ideas and thoughts, new spaces of possibility were opened. Learning was not just about zeroing in on a final end product or conclusion but about participating in a continuous process of growing (coming to understand). Later, although the collective structure could have been disrupted as they once again considered competing pathways to a solution, the metaphor of divergent thinking, which characterizes their creative process during this part of their collaboration, helps us to recognize the value of continually seeking out divergent views while still retaining the capacity as a group to defer to group mind to keep the collective moving towards a creative solution. In this data extract, we see students iteratively scope out multiple potential pathways to a solution, 'agree' (without ever discussing rules of engagement) on a way of working together (an etiquette) that allows them to defer to the group mind, develop a collective structure of engagement that affords insight into a credible route to solving the problem, create and reject further potential solution pathways, and again defer to the group mind to coalesce on a solution. This iterative process, we believe, is characteristic of the creative process, and we anticipate that it would be evident in other data extracts featuring collaborating groups who are able to sustain productive struggle in the pursuit of mathematically sound problem-solving.

Implications for Classroom Practice

The iterative process leading from productive struggle to collective creativity suggests a number of implications for classroom mathematics learning. We note that the task offered to this group of students was rich enough to allow for the possibility of multiple potential solution pathways to emerge. According to Martin and Towers (2011), although there exists the potential for many different directions for the 'performance' to take at any point of the scene, it is at the start that "the potential is unlimited…[and] it is here that the widest range of choices are open to the actors" (p. 256). However, for students to sustain productive struggle, the task also needs to afford the possibility of multiple potential pathways to emerge at many points in the solution so that the possibility of better alternative pathways can emerge during problem-solving.

In addition, the learning environment (and this includes structures such as resources offered to students) needs to afford the emergence of collective structure and striking a groove. As we have noted elsewhere (Martin et al., 2006), offering single piece of paper for students to share has proved fruitful in promoting the growth of collective mathematical understanding in that it becomes a place to 'pool' thinking. As we saw in the data presented here, the shared document enabled the emergence of the initial solution idea by providing a single focus for striking a groove based on which "a collectively created structure start[ed] to emerge" (Martin & Towers, 2011, p. 269). Finally, our data suggests that the kind of teaching that supports productive struggle is teaching that models and encourages the kind of etiquette and valuing of group mind that generate good improvisational performances. These students had learned such etiquette in a classroom that valued genuine collaboration, mathematical argumentation, and problem solving.

Conclusion

The students in this problem-solving session are good examples of *attentive and responsive listeners*. Their conversation was fundamentally creative; it required "trust among the group; the ability to listen and to respond to each other; the ability to work without a script or a director" (Sawyer, 2001, p. 196). Thus, they were able to struggle productively by listening to and watching what others were saying and doing and responding accordingly. No comment or gesture was ignored, i.e., mathematical ideas and actions stemming from any one of them became "taken up, built on, developed, reworked, and elaborated by others and thus emerge[d] as shared [structures] for and across the group, rather than remaining located within any one individual" (Martin et al., 2006, p. 157).

As VanLehn et al. (2019) concluded, though, it is not easy to create environments in which this kind of collaborative productive struggle can be sustained and in which there are opportunities for students to "work hard together to solve challenging, open-ended problems that afford many mathematical insights and discussions" (p. 8) and in which successful pedagogy "engages the students in mathematically meaningful, productive, collaborative behavior" (p. 8). Jardine et al. (2003) reminded us though that "children like to work hard—if that work is meaningful, engaging, and powerful" (p. 102). They used the expression "hard fun" to describe this kind of learning, which is rich in productive struggle, recognizing that it is the kind of learning that is called for to thrive in this rapidly changing and challenging world.

Note

[1] A property of the collective, where "everything seems to come naturally; the performers are in interactional synchrony" (Sawyer, 2003, p. 44). Sawyer (2003) suggested this expression based on Csikszentmihalyi's (1990) conception of flow. According to Sawyer (2003), "Csikszentmihalyi intended flow to represent a state of consciousness within the individual performer, whereas group flow is a property of the entire group as a collective unit" (p. 43).

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