POSITIONING OF NOVICE ELEMENTARY TEACHER LEADERS IN ADVICE AND INFORMATION NETWORKS FOR MATHEMATICS

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In this paper we examine how teachers who are pursuing their Elementary Mathematics Specialist certification—Elementary Mathematics Specialists in Training (EMSTs)—are positioned in their advice and information networks for mathematics. We analyzed the instructional networks of six elementary schools in one Midwestern school district. Our analysis suggests that EMSTs did occupy central positions in their networks. EMSTs were sought out by more individuals compared to other teachers, and when sought out by others, provided advice and information at a greater frequency than formal leaders. We also considered the school's informal and formal structure, finding that EMSTs' positioning was related to the broader school's information seeking behavior and whether there is a math-specific formal leader.

Keywords: leadership; elementary mathematics specialists; social network analysis

Teacher leaders have the potential to play an important role in supporting instructional improvement. While some teacher leaders occupy formal, full-time positions (e.g., as coaches), many continue as full-time classroom teachers. Though they are not afforded dedicated time for leadership, teacher leaders with full-time classroom responsibilities may be more likely to engage with other teachers about classroom instruction and viewed as more credible sources than formal leaders (Spillane & Kim, 2012). Therefore, a potentially productive type of informal leadership that teacher leaders can enact is offering advice and information about mathematics teaching and learning. The extent to which teacher leaders can engage in such leadership, however, depends on the context of their schools. A collegial and collaborative school culture, for example, supports teacher leadership, while hierarchical and formal designations can increase distance between teachers (York-Barr & Duke, 2004). In this paper, we examine how novice elementary mathematics teacher leaders who maintain full-time classroom responsibilities are positioned in the instructional and advice networks for mathematics, and how their positioning might be related to the school's informal and formal leadership structures.

Theoretical Framings & Related Literature

In line with research that takes a distributed perspective (Spillane et al., 2004), we recognize leadership as extending to those with no formally designated position, and as the product of *interactions* between leaders, followers, and their situation. The situation shapes teacher leaders' interactions with others and includes, for example, school norms, structures and routines (e.g., grade-level teams), and formal positions (e.g., presence of an instructional coach) (Diamond & Spillane, 2016). Thus, teacher leaders' social influence interactions—providing advice and information about mathematics teaching and learning—constitute a form of leadership. We focus on advice- and information-giving because such leadership activities improve mathematics teaching through a variety of professional supports, including increasing teachers' knowledge about the learning and teaching of mathematics (Gigante & Firestone, 2008). Social network analysis allows us to examine social influence interactions while simultaneously attending to the school's formal structure and how it constitutes said interactions (Moolenaar & Daly, 2012).

Previous research on advice and information networks in mathematics suggests that, while principals do not figure prominently in their school's networks, formal leaders with subject-specific positions are the most central, with teacher leaders offering advice and information to more people than other teachers (Spillane & Kim, 2012; Spillane & Hopkins, 2013). This research has primarily focused on *centrality*—the extent to which an individual is connected to others—and less so on the nature of those interactions. A notable exception is the study by Coburn and Russell (2008) which investigated how district policy shapes teachers' social networks, including the frequency of interactions (*strength*), the substance of those interactions (*depth*), and the extent to which those interactions span different functional areas (*span*).

Research has identified factors of the situation that support and constrain teacher leadership. In general, the literature suggests that inadequate time for collaboration and traditional top-down structures can inhibit teacher leadership, while cultural norms of openness and trust, positive working relationships, and support from school administration support it (Wenner & Campbell, 2017; York-Barr & Duke, 2004). Because leadership, particularly exercised through social influence interactions, is grounded in authority and legitimacy (Diamond & Spillane, 2016), the positioning of teacher leaders as knowledgeable and expert is crucial. Therefore, our study builds on this literature by investigating how novice teacher leaders are positioned—in terms of centrality, span, strength, and depth—in their networks for mathematics instruction, and how their positioning might be related to their situation. Specifically, the research questions that guided our investigation are: 1) How are novice teacher leaders positioned in their advice and information networks for mathematics, especially compared to teachers and formal leaders? 2) How is their positioning related to school level factors?

Methods

Study Context

The data analyzed for this paper is part of a larger project in which 24 teachers in a Midwestern state received funding to complete Elementary Mathematics Specialists (EMS) certification and serve as informal leaders in their schools. Data was collected in Fall 2019, the first year of teachers' participation in their EMS programs. Because teachers were not formal leaders, nor necessarily identified by school administration or colleagues as experts, we consider them novice teacher leaders, or Elementary Mathematics Specialists in Training (EMSTs). In this paper, we focus on survey data from six elementary schools (Briar, Palm, Reed, Rowan, Thorn, Woods) in one participating district. Thirteen EMSTs worked together in school-based teams, ranging in size from 1-3 EMSTs in each of the six schools. As part of their graduate coursework, each team was asked to distribute a survey to the teachers in their school, analyze the results, and use the results to inform a plan for improving support for mathematics instruction at their school.

Data

The survey included items related to advice- and information-seeking interactions in mathematics, which were based on those developed and validated in other studies (Pitts & Spillane, 2009). In particular, we asked "During this past school year, is there a person in your building or district you have turned to for advice or information about teaching mathematics?" (Middle School Mathematics and the Institutional Setting of Teaching, n.d.). Respondents listed up to three individuals, and for each of those individuals, were also asked "how often do you seek advice or information from this person" and "what type(s) of advice or information do you seek from this person? Please check all options that apply." The options for these questions are described in the analysis section, which we turn to next.

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Analysis

For each individual that responded or was named, using the school and district websites, we collected data for the individual's role (e.g., leader, teacher), associated site (e.g., school, central office), and, if applicable, grade level. Using the social network data, we calculated centrality, and span, strength, and depth of relationships (ties) of each individual. *Degree centrality* measures how well connected an actor is in a network (Freeman, 1979), and can be broken into in-degree—the number of people who sought out that actor for advice and information—and out-degree—the number of people that actor sought out. *Betweenness centrality* measures brokering and the extent an actor connects two other actors in the network (Freeman, 1979). Specifically, betweenness measures the number of shortest paths between two other actors that go through a given actor.

In addition to centrality, we also calculated measures to describe ties actors had with others. For each, we considered whether the tie *spanned* outside the actor's grade level (1 = yes, 0 = no). For ties with teachers that taught multiple grade levels, if the two teachers had at least one overlapping grade, we considered this as not spanning grade levels. For *strength*, we considered the frequency of the interactions, with four options: a few times a year (1), once or twice per month (2), once or twice per week (3), and daily or almost daily (4). For *depth*, we based our definitions on those of Coburn and Russell (2008), with three options: low (1), medium (2), and high (3) (see Table 1). Because respondents were able to select multiple options, we calculated an average depth, in addition to whether or not the interaction included at least one high-depth activity (1 = yes, 0 = no). For any relation between two actors, there are two possible ties, one from actor A to actor B, and the other from actor B to actor A. For example, if actor A responded that she asked actor B for advice daily, then the strength of actor A's out-tie with actor B and strength of Actor B's in-tie with actor A would be 4. For individuals that were named but did not respond to the survey (e.g., formal leaders), we only computed measures related for in-ties, including in-degree centrality and associated strength and depth; span was not relevant since respondents (teachers), by definition of role, were outside leaders' functional area.

Table 1: Depth of Interactions

Depth	Types of Advice and Information
Low	• Discussing pacing • Sharing materials or activities • After a lesson, sharing whether students "got it" • Updating one another on a student or students' progress in mathematics
Medium	• Discussing what materials to use for a lesson • Analyzing student work to see if students "got it" • Discussing why some students didn't learn as expected in a lesson in order to plan for future success • Doing mathematics problems together with discussions of different solution strategies
High	• Discussing different ways students are likely to solve tasks • Analyzing examples of student work to understand the different ways that students solve problems • Analyzing examples of student work in order to adjust instruction • Discussing how to make use of student solution strategies in whole class mathematical discussions

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To examine the positioning of EMSTs in their networks, for the first research question, we compared the measures previously described (centrality, span, strength, depth) for EMSTs, teachers, and formal leaders and tested differences for significance using analysis of variances with permutation tests. Because social network data are not independent, we used UCINET software (Borgatti et al., 2002) to conduct a random replication procedure with 5000 permutations (Carrington et al., 2005; Spillane & Kim, 2013). Because respondents had ties with those outside their school, including district leaders, we did not limit networks to those of the school.

For the second research question, to examine the relation between EMSTs' positioning and school factors, we limited networks to those of the school. First, we explored to see if there were between-school differences in how EMSTs were positioned. To account for the size of the network, we normalized centrality by expressing it as a percentage of the maximum possible centrality an individual could have had. The school factors that we investigated included the school's formal (e.g., whether there was a mathematics-specific formal leader) and informal (e.g., advice- and information-seeking behavior) structure. Regarding the latter, we calculated network *density* for each school. Network density is the total number of ties divided by the total number of possible ties. In addition, we compared the average span, strength, and depth of ties between schools and tested differences for significance using analysis of variances with 5000 permutations. To illustrate our findings, we share network diagrams of three schools, selected based on contextual variation. Some of this variation included the size of schools, the nature and density of school networks, and whether there was a mathematics-specific formal leader.

Findings

First, we describe the positioning of EMSTs in their advice and information networks for mathematics, especially compared to other teachers and formal leaders. Then, we turn our attention to school networks and how EMSTs' positioning might be related to school factors.

EMSTs' Positioning in District Network

Overall, the EMSTs in our study occupied central positions in their advice and information networks for mathematics (see Table 2). Specifically, EMSTs were sought out by more individuals than other teachers (in-degree, $p < 0.01$), and were more often positioned as brokers for advice or information (betweenness, $p < 0.001$). All of the EMSTs had at least one tie, while 19.35% of teachers had no ties. There were no significant differences in advice-seeking behavior (out-degree), nor differences in span, strength, or depth of ties.

Only three formal leaders were named as individuals whom teachers sought out for advice and information, and none of those included school principals. The three formal leaders named were the district mathematics coordinator, an instructional mentor in the district special education department, and the Title I Math teacher at Woods (Title I is a United States government program in which schools with high levels of low-income students receive federal funding which can be used to hire additional teachers or instructional aides (United States Department of Education, n.d.). While these three formal leaders did have the highest average in-degree, because we did not ask formal leaders to complete the survey, we were not able to compare centralization between formal leaders and EMSTs. We were, however, able to compare the strength and depth of the ties that were reported. When sought out by others, EMSTs provided advice and information at a greater frequency than formal leaders (strength, $p < 0.05$), but the depth—average and whether or not the interaction included at least one high-depth activity—did not differ significantly.

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Table 2: Means and Standard Deviations of Centrality and Tie Dimensions by Position

	EMSTs	Teachers	Formal Leaders
N	13	124	3
Betweenness	5.231 (10.892)	0.548 (1.876)	
In:			
Degree	2.077 (1.979)	0.694 (0.785)	7.333 (4.643)
Tie Span	0.193 (0.267)	0.086 (0.273)	
Tie Strength	2.686 (0.682)	2.924 (0.620)	1.389 (0.550)
Tie Depth (Avg)	1.902 (0.215)	1.808 (0.269)	1.675 (0.139)
Tie Depth (High)	0.765 (0.377)	0.737 (0.413)	0.398 (0.308)
Out:			
Degree	1.385 (1.003)	0.944 (1.117)	
Tie Span	0.533 (0.420)	0.290 (0.413)	
Tie Strength	2.518 (1.011)	2.522 (0.913)	
Tie Depth (Avg)	2.008 (0.273)	1.825 (0.368)	
Tie Depth (High)	0.900 (0.200)	0.674 (0.434)	

Note: As a reminder to the reader, tie span refers to whether it extended beyond a teacher's grade level, strength refers to frequency, and depth refers to the substance of an interaction.

EMSTs' Positioning in School Networks and Related Factors

There was great variation in the centrality among our EMSTs. We found that, while the normalized in-degree averaged 0.098, it ranged from 0 (EMST was not named as a provider of advice or information) to 0.333. Similarly, normalized out-degree averaged 0.045 and ranged from 0 to 0.136, and normalized betweenness averaged 0.095 and ranged from 0 to 1.183. Because of this variation, we wondered how differences might be related to school-level factors. In particular, we looked at the school's advice- and information-seeking behavior (see Table 3) and the formal structure—whether there was a formal mathematics leader. Regarding the former, we found significant between-school differences for span ($p < 0.05$) and strength ($p < 0.01$) of in-ties, and span ($p < 0.01$) and strength ($p = 0.08$) of out-ties, suggesting that some schools had more frequent sharing of information, particularly across grade levels. The only school in our sample with a formal mathematics leader was Woods. To illustrate our findings, we focus on and share network diagrams for three schools: Briar, Rowan, Woods (see Figure 1). Individuals were labeled by role and grade, with those teaching multiple grades labeled as "Other."

Briar was one of the larger schools in our sample but had the lowest network density. A significant number of teachers (31%) had no relationships with others, though the ties that were present were quite frequent (second highest strength). The Briar network also had more substantive interactions (an above average depth rating), but they were only within grade levels. We see these school-level patterns repeated in the ties EMSTs had with colleagues at their schools. Only one EMST sought out colleagues for advice and information, with the other two being sought out by others. And, all EMST ties were with peers teaching at the same grade.

By contrast, the network at Rowan was the densest (i.e., had the most total ties relative to possible ties). However, teachers' ties were not as frequent or deep, though this might be because of the higher proportion of interactions that spanned grade levels (often teachers seeking EMSTs for advice). In addition to being sought out, EMSTs at Rowan also went to colleagues for advice and information. Because of this, the EMSTs at Rowan connected and brokered advice and information about mathematics across the first and 3-5 grade levels. Though the kindergarten and

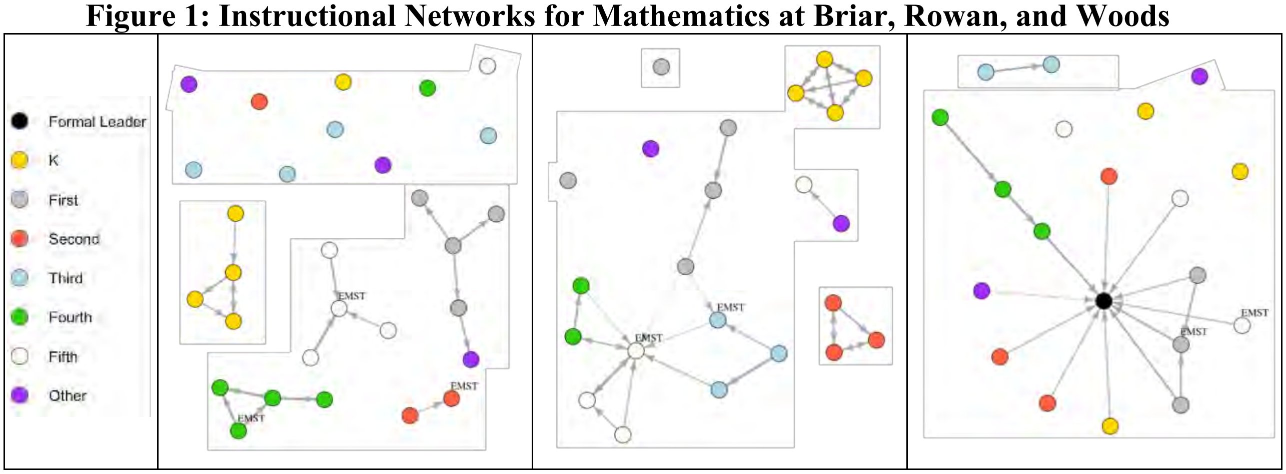
Table 3: Density and Means and Standard Deviations of Tie Dimensions by School

	Briar	Palm	Reed	Rowan	Thorn	Woods
N	29	32	23	23	10	20
Density	0.021	0.029	0.043	0.065	0.044	0.045
In Ties:						
Tie Span	0	0.015	0.028	0.214	0.333	0.200
		(0.071)	(0.118)	(0.385)	(0.471)	(0.447)
Strength	3.083	3.121	2.694	2.702	1.500	3.033
	(0.633)	(0.517)	(0.518)	(0.717)	(0.707)	(0.650)
Depth (Avg)	1.992	1.825	1.843	1.754	1.667	1.694
	(0.186)	(0.216)	(0.308)	(0.224)	(0.923)	(0.232)
Depth (High)	0.875	0.894	0.639	0.659	0.500	0.607
	(0.311)	(0.255)	(0.479)	(0.439)	(0.707)	(0.487)
Out Ties:						
Span	0	0.036	0.083	0.287	0.500	0.750
		(0.133)	(0.289)	(0.399)	(0.577)	(0.380)
Strength	2.917	3.000	2.778	2.546	1.750	2.392
	(0.793)	(0.784)	(0.641)	(0.885)	(1.500)	(0.738)
Depth (Avg)	1.941	1.883	1.907	1.815	2.000	1.774
	(0.250)	(0.328)	(0.326)	(0.347)	(0.816)	(0.365)
Depth (High)	0.833	0.857	0.667	0.667	0.750	0.607
	(0.389)	(0.363)	(0.449)	(0.424)	(0.500)	(0.487)

second grade teachers were isolated from those in other grade levels, they had fairly reciprocal relationships as teachers reported ties with one another.

Woods was the only school with a mathematics-specific formal leader. Though the school had an average network density, a large majority of ties were to the formal leader. Ties spanned outside the grade level, though this was, again, only to the formal leader. So, similar to the ties at Briar, teachers were isolated from those outside their grade level, and sometimes, even from those in the same grade. Ties were somewhat frequent but were less substantive (relatively low depth). Both EMSTs only sought the formal leader, and only one had others seeking her for advice and information.

Figure 1: Instructional Networks for Mathematics at Briar, Rowan, and Woods



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Discussion

There is little research on the positioning of (novice) teacher leaders with full-time classroom responsibilities, and this research primarily considers centrality. Our findings align with this literature—that EMSTs were more central than teachers, but not as central as formally designated leaders with subject-specific positions—and also adds to it by examining the nature of social influence interactions. We found that, when sought out by others, EMSTs in our study provided advice and information at a greater frequency than formal leaders. This is important because interactions with greater frequency facilitate the learning of complex knowledge (Coburn & Russell, 2008), which teachers need to improve their instruction. For example, researchers have found that interactions with colleagues who have developed more ambitious instructional visions can support improvements in teachers' own visions, particularly in cases where interactions were more frequent (Munter & Wilhelm, 2020).

Findings from our study also add detail regarding how teacher leaders' positioning is related to school structures. First, EMSTs' interactions with their colleagues were similar to the overall school advice- and information-seeking behavior (e.g., density and if interactions spanned grade levels). Second, similar to prior research that identified subject-specific formal leaders as the most central actors in school networks (Spillane & Kim, 2012; Spillane & Hopkins, 2013), at Woods, the majority of interactions, including those of the EMSTs, went to the formal mathematics leader. One interpretation of these findings is that school norms of collaboration and views of expertise shape teachers' advice- and information-seeking behavior, particularly who they turn to (Wenner & Campbell, 2017; York-Barr & Duke, 2004). And the extent teachers at a school interact and communicate regularly, particularly with those outside their grade level, influences whether and how they seek information from teacher leaders. Also significant is whether colleagues perceive teacher leaders as knowledgeable. For schools with a mathematics-specific formal leader, like Woods, EMSTs' expertise might be undervalued by their colleagues.

Implications

One of the implications of our findings is related to the division and coordination of leadership between formal leaders and teachers who exercise leadership through informal means. Because effective professional development typically includes sustained learning opportunities over time and sensitivity to local contexts (Sztajn, Borko, & Smith, 2018), it seems that there are opportunities for formal leaders to enlist novice leaders with mathematical expertise in change efforts. That is, teacher leaders like the EMSTs in our study could serve as brokers for efforts initiated at the district level, as well as sources of information with regard to teachers' perspectives and impressions of these efforts. Formal leaders could explicitly position teacher leaders as resources for ongoing conversations about mathematics teaching and learning, including serving as leaders of professional learning teams, book studies, video clubs, etc.

Our findings also highlight the limited nature of some of the information networks that exist in schools, limitations that could be explicitly attended to by school leadership. For schools with only grade-level connections, like Briar, it might be helpful to leverage teacher leaders as agents for promoting across grade-level collaborations. The presence of a formal mathematics specialist at Woods seemed to promote advice-seeking, but such interactions were dominated by the formal leader. Positioning teacher leaders with expertise and authority could support more collaboration among teachers and teacher leaders. And, for schools with a robust network of within and across grade-level connections like Rowan, teacher leaders can be mobilized to support bottom-up change across a school by, for example, creating additional opportunities for teachers to share their practice, visit classrooms, and talk with colleagues teaching at different grade levels.

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