PROFILES OF TEACHERS' EXPERTISE IN PROFESSIONAL NOTICING OF CHILDREN'S MATHEMATICAL THINKING

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Noticing children's mathematical thinking is foundational to teaching that is responsive to children's thinking. To better understand the range of noticing expertise for teachers engaged in multiyear professional development, we assessed the noticing of 72 upper elementary school teachers using three instructional scenarios involving fraction problem solving. Through a latent class analysis, we identified three subgroups of teachers that reflected different profiles of noticing expertise. Consideration was given to the noticing component skills of attending to children's strategy details, interpreting children's understandings, and deciding how to respond on the basis of children's understandings. We share theoretical and practical implications for not only the three profiles but also our choice to explore separately two versions of deciding how to respond (deciding on follow-up questions and deciding on next problems).

Keywords: Teacher Noticing, Professional Development, Elementary School Education

Our work is aligned with a vision of teaching that is responsive to children's mathematical thinking. In this vision, teachers attend to and pursue the substance of children's ideas and important mathematical connections within those ideas (Richards & Robertson, 2016). This type of responsive teaching builds on research on children's mathematical thinking and connects to numerous policy recommendations, but has proven challenging to achieve (Cai, 2017; National Council of Teachers of Mathematics, 2014; National Research Council, 2001).

Efforts to support teachers in achieving this vision have included attention to core practices of teaching (Grossman, 2018; Grossman et al, 2009; McDonald et al., 2013). We focus on one of the core practices—teacher noticing—that has been researched extensively in mathematics education (for compilations, see Schack et al., 2017; Sherin et al., 2011). Although multiple conceptions exist, *teacher noticing* fundamentally refers to how teachers focus their attention and make sense of what children say and do so that teachers' instructional responses are productive.

We chose to focus on teacher noticing of children's mathematical thinking, with an awareness that this type of noticing is only one of many that teachers must use to be successful. Examples of noticing research with different foci include curricular noticing (Amador et al., 2017), racial noticing (Shah & Coles, 2020), and noticing of participation and status (Kalinec-Craig, 2017; Wager, 2014). We view these different types of noticing as potentially mutually supportive in that using one focus as a starting point can provide entry into other types of noticing. In this study, we foreground noticing children's mathematical thinking as foundational for teaching that is responsive to children's thinking—one can only be responsive to what one has noticed. Further, research has shown that teachers usually do not gain this expertise solely from teaching experience (Copur-Gencturk & Rodrigues, 2021), but it can be learned (see, e.g., Casey & Amidon, 2020; Lee, 2019; Roth McDuffie et al., 2014; Schack et al. 2013; Simpson & Haltiwanger, 2017; van Es & Sherin, 2008).

Our conception of teacher noticing comes from our earlier work on professional noticing of children's mathematical thinking in which we identified three component skills: (a) attending to children's strategy details, (b) interpreting children's understandings reflected in their strategies, and (c) deciding how to respond on the basis of children's understandings (Jacobs et al., 2010). This final skill—deciding how to respond—refers to teachers' intended responses because teacher noticing is invisible, happening prior to teachers' observable responses. The three component skills are conceptually and temporally linked, and in the midst of instruction, they often occur almost simultaneously. They are not ends in themselves, but collectively are foundational for making productive instructional responses that build on children's thinking.

In this study, we extended our earlier work by identifying profiles of noticing expertise that include consideration of teachers' expertise with each of the component skills. By better understanding how teachers in multiyear professional development (PD) take up and engage in the complex practice of teacher noticing, we should be better able to support them in developing this expertise. Thus, we investigated the following research question: *What meaningful profiles of teachers' expertise in professional noticing of children's mathematical thinking exist among teachers engaged in multiyear PD?*

Methods

The data were drawn from a larger PD design study in which the goals included building a model of teaching that is responsive to children's mathematical thinking (Empson & Jacobs, 2021, these proceedings). In this paper, we focus on one instructional practice in the model—professional noticing of children's mathematical thinking—and use teachers' responses to a noticing assessment to identify profiles of expertise across the component skills.

Participants

We assessed the noticing expertise of 72 upper elementary school teachers—68 classroom teachers (grades 3–5) and 4 teaching specialists (instructional facilitators, resource teachers, etc.)—who had voluntarily enrolled in our PD. The teachers (64 females and 8 males) were generally experienced, with their teaching experience ranging from 2 to 36 years ($M = 11.8$).

To develop our noticing profiles, we purposefully studied teachers who were at different points in our 3-year PD and worked in a variety of contexts. Specifically, data were collected during one school year when teachers were at the end of their first ($N = 22$), second ($N = 26$), or third ($N = 24$) year of PD. Teachers worked in 3 districts in a state in the southern United States. The districts had varied instructional histories in that all administrations had endorsed teaching that was responsive to children's thinking, but for different amounts of time. Further, teachers were drawn from 36 schools that reflected a range of student demographics. Across the schools, students who qualified for free or reduced-cost lunch ranged from 10%–98% ($M = 59.7\%$) and students classified as Limited English Proficiency ranged from 2%–85% ($M = 33.3\%$). Student race and ethnicity classifications also varied. White students ranged from 6%–85% ($M = 49.6\%$), Hispanic students ranged from 4%–81% ($M = 34.8\%$), Black students ranged from 0%–20% ($M = 4.3\%$), Hawaiian and Pacific Islanders students ranged from 0–31% ($M = 5.4\%$), and students with race and ethnicity classifications of "other" ranged from 0%–14% ($M = 6.0\%$).

Professional Development

Our PD consisted of more than 150 hours of face-to-face workshops offered over 3 years, and the overall goal was to help teachers develop expertise in teaching that is responsive to children's mathematical thinking, with special emphasis on the teaching and learning of fractions (Jacobs, Empson, Pynes, et al., 2019). Key resources included research-based frameworks of children's mathematical thinking (Carpenter et al., 2015; Empson & Levi, 2011) and research-based frameworks of instructional practices, such as noticing children's mathematical thinking (Jacobs et al., 2010) and questioning to support and extend children's mathematical thinking (Jacobs & Ambrose, 2008; Jacobs & Empson, 2016).

Noticing Assessment

We captured teachers' noticing expertise using a written assessment that was structured around three instructional scenarios in which teachers had opportunities to notice children's thinking linked to fraction story problems. The scenarios were conveyed via authentic, strategically selected artifacts of practice—a classroom video, a set of children's written work, and a video of a teacher's conversation with one child. We chose the three scenarios because we wanted to capture teachers' noticing expertise throughout the multiple facets of their work.

For each instructional scenario, teachers responded, in writing, to prompts linked to the component skills of noticing children's mathematical thinking (see Table 1). Note that we included two prompts for the final skill of deciding how to respond. We chose to keep separate the prompts (and scores) for deciding on follow-up questions and deciding on next problems because the two categories of deciding how to respond are conceptually distinct, and we wanted to better understand their relationship to teachers' overall noticing expertise.

Table 1: Writing Prompts for the Noticing Assessment

Noticing Component Skills		Sample Writing Prompts
Attending to children's strategy details		Please describe in detail what you think each child did in response to this problem.
Interpreting children's understandings		Please explain what you learned about these children's understandings.
Deciding how to respond on the basis of children's understandings	Deciding on follow-up questions	Imagine that you are the teacher of these children and you want to have a one-on-one conversation with one of them. Which child would you choose? Describe some ways you might respond to their work on this problem, and explain why you chose those responses.
	Deciding on next problems	Imagine that you are the teacher of these children. What problem or problems might you pose next? What is your rationale?

Scoring

Each teacher received 12 noticing scores—4 scores for the noticing component skills within each of the 3 instructional scenarios. Drawing on our past research (Jacobs et al., 2010), scoring was done holistically on a 0–2 scale indicating the extent to which we had evidence for teachers' engagement with children's mathematical thinking: lack of evidence (0), limited evidence (1), or robust evidence (2). We double-coded all data (in a blinded format) and interrater reliability for all 12 noticing scores was 80% or higher. Discrepancies were resolved through discussion.

For the attending-to-children's-strategy-details score, we looked for inclusion of mathematically significant details such as how children used drawings to represent and partition quantities, how they combined fraction amounts, or how they described amounts using fraction names or notation. For the interpreting-children's-understandings score, we did not seek a single best interpretation but instead looked for an emphasis on what children understood (versus did not understand) and reasoning that was consistent with and grounded in the children's strategy details. For the deciding-on-follow-up-questions score, we did not seek a single best set of follow-up questions but instead looked to see if the questions and rationales were reasonable, meaning that they were consistent with the children's strategies and understandings. We also looked to see if the questions centered children's thinking not only by asking about details *inside* their existing strategies (Jacobs, Empson, Jessup, & Baker, 2019) but also by leaving room for children's ways of thinking (versus funneling children toward a particular strategy or answer [Wood, 1998]). For the deciding-on-next-problems score, we did not seek best next problems but instead looked for problems that were consistent with teachers' rationales. We further looked to see if the rationales linked to children's understandings and left room for children's thinking.

Findings

Our goal was to identify profiles of noticing expertise across the noticing component skills. We began by determining that the internal consistency for the noticing assessment was adequate, as indicated by Cronbach's alpha of .77. We then conducted a latent class analysis to empirically identify subgroups of the 72 teachers displaying similar patterns of responses across their 12 scores—4 scores for the noticing component skills within each of the 3 instructional scenarios. We considered the response patterns for these subgroups as profiles of teachers' expertise in professional noticing of children's mathematical thinking. Our goal was not to "label" teachers but instead to better understand variation in teachers' expertise in this practice.

We considered a 3, 4, and 5-profile solution, and we chose the 3-profile solution based on (a) the lowest Bayesian Information Criteria goodness-of-fit statistic (Schwarz, 1978), (b) conceptually interpretable profile patterns, and (c) sufficient sample sizes for comparison among profiles. We then assigned each teacher to the profile for which they had the highest probability based on their response pattern across the noticing assessment. The 3-profile solution generated ordered profiles that we labelled *Accomplished Noticing* ($N = 14$), *Mixed Noticing* ($N = 33$), and *Emerging Noticing* ($N = 25$). The profile means of teachers' overall noticing scores (computed as a mean of their 12 scores) reflected this ordering: 1.42, 0.98, and 0.61, respectively. Our assessment design allowed us to further characterize the expertise associated with each profile in terms of the noticing component skills, and we were especially interested in whether mean scores were above or below a score of 1—the midpoint in our 0–2 scale that indicated limited evidence of engagement with children's mathematical thinking (see Figure 1).

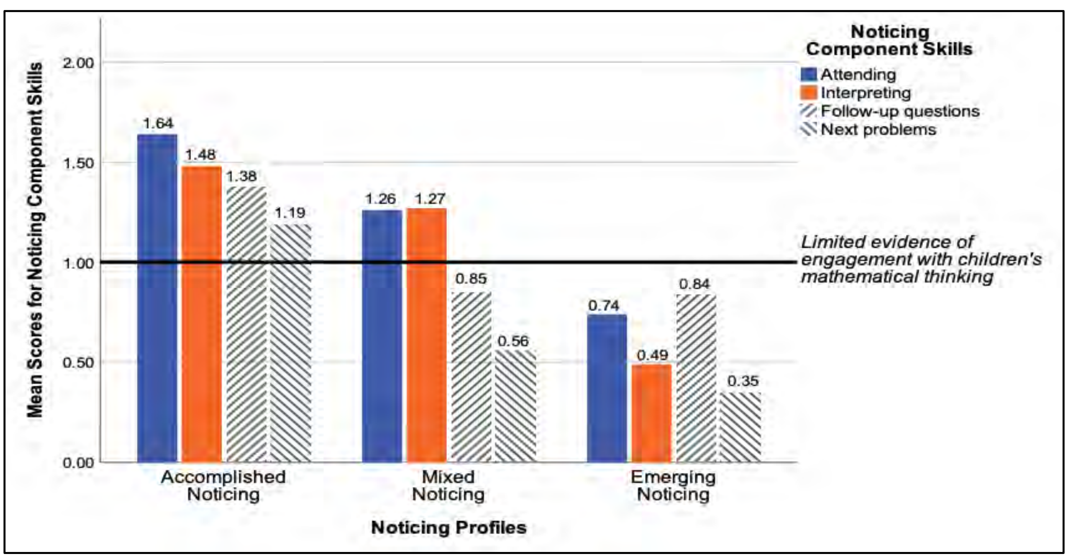

Figure 1: Mean Scores for Noticing Component Skills by Noticing Profile

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The *Accomplished Noticing* profile was characterized by consistently strong expertise, with all mean scores above 1. For these teachers, expertise in attending to children's strategy details was their strongest skill and deciding on next problems was the skill for which they showed the most room for growth. The *Mixed Noticing* profile was characterized by a split performance, with mean scores above 1 for attending to children's strategy details and interpreting children's understandings, and mean scores below 1 for the two deciding how-to-respond skills. Thus, these teachers had developed substantial expertise in making sense of children's strategies, but they were still learning what to do with that information in terms of an instructional response. Finally, the *Emerging Noticing* profile was characterized by consistently weak expertise, with all mean scores below 1. However, their pattern of means scores showed that they were beginning to notice the details of children's thinking and pose follow-up questions about those details, but that they particularly needed support in making sense of what those details meant in terms of children's understandings and how to craft problems that built on those understandings.

We also noted two major patterns across the profiles. First, the mean score for deciding on follow-up questions was higher than the mean score for deciding on next problems for all three profiles, reinforcing the importance of our separate consideration of these two categories. A related finding was that the mean score for deciding on next problems was the lowest score for all three profiles, which is consistent with earlier findings documenting this skill's challenging nature (Jacobs et al., 2010, 2011). Second, the mean score for attending to children's strategy details was one of the top scores for all profiles, suggesting the foundational role that details play in teachers' ability to make sense of and build on children's thinking (Jacobs & Spangler, 2017).

Figure 2 illustrates this second pattern for a teacher with an *Accomplished Noticing* profile. We share samples of her responses linked to Joy's written work for the pancake problem. Joy had a correct solution with a non-traditional final answer—she used words and pictures of fraction pieces (rather than fraction symbols) and did not combine her amounts into a single total. All sample responses were scored as robust evidence of engagement with children's mathematical thinking. For attending to children's strategy details, the teacher richly described Joy's strategy, highlighting details such as multiple partitions (4ths, 8ths, and 24ths) and why Joy might have made those partitions. For interpreting children's understandings, she focused on what Joy *did* understand, drawing on Joy's strategy details of (a) repeated halving, which is a common strategy for young children (Empson & Levi, 2011), and (b) correctly naming a fractional amount (1/3 of 1/8) which is challenging for many children. For deciding on follow-up questions, she made extensive use of strategy details, asking Joy how she partitioned, how she named the 1/24th-size pieces, and whether she had a sense of the amount each child would receive. She consistently centered Joy's thinking, and even her last question that moved beyond Joy's strategy to explore other possible partitions left room for Joy's thinking (versus funneling it toward the teacher's thinking). For deciding on next problems, she posed the same problem with new numbers that built on Joy's initial strategy that involved fourths (and repeated halving). Her rationale drew on this strategy detail to anticipate Joy's new strategy, and problem numbers were chosen strategically—9 pieces (for 9 sharers) cannot be reached by partitioning into fourths thereby making visible whether Joy could partition differently, such as by the number of sharers.

Across this teacher's responses for the component skills, strategy details were visible and integral to her reasoning. In contrast, for responses with scores of limited evidence or lack of evidence, we saw progressively less attention to strategy details in the initial strategy descriptions and throughout the other component skills. At times, teachers even focused on changing the child's strategy to one that they preferred.

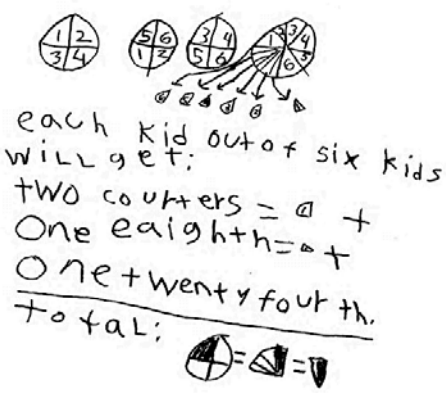
Joy's Strategy for the Pancake Problem *The teacher has 4 pancakes to share equally among 6 children. How much pancake does each child get?*	
Attending to Joy's strategy details	Joy drew her 4 pancakes and cut them into 1/¼ I believe she did that because she is comfortable with 1/¼ When she reached her last pancake, she realized 1/¼wouldn't get each person a pancake piece. I think she then divided it into 1/8s. Again, I think 1/¼and 1/8s are comfortable for her. After she numbered 6, she realized she had 2 pieces left so she divided the last two 1/8s into 6 pieces. I believe she counted the pieces as if thirds were in each 1/8 to come up with twenty-fourths.
Interpreting Joy's understandings	**…** Joy has a good grasp of repeated halving (or fourth-ing). She continued to use 1/¼and 1/8s until she was able to solve the problem. She was however able to identify what [1/3] of 1/8 was. That impressed me and would be a question I'd pose to her.
Deciding on follow-up questions for Joy	• Can you tell me what you did? (To understand the thinking behind the work) • Why did you split the first 3 pancakes into 4 pieces? (To understand the rationale, to see if she saw the relationship with the people) • Tell me about the last pancake. (I want to see what she was thinking when she split this pancake) • You wrote here 1/24. Can you show me 1/24 in the picture? How do you know that is 1/24? (What thinking was behind this decision to split the pieces? What understanding does she have about it?) • Do you know how much the kids will get altogether? (Can she add her pieces?) Is it more than 1/½r less? More than 1 or less? • Is there another way to split the pancakes? (Does she see the connection now?)
Deciding on next problems for Joy	*The teacher has 5 pancakes to share equally among 9 children. How much pancake does each child get?* … I was curious to see if Joy would start with 1/¼and divide the pancakes into smaller pieces to solve the problem.

Figure 2: Sample Responses Linked to Joy's Strategy (*Accomplished Noticing* Profile)

Discussion

We began this study with the assumption that all participating teachers had strengths as teachers. They chose to engage in our PD to enhance their teaching by learning about children's mathematical thinking and its pivotal role in instruction—learning about noticing children's mathematical thinking was a piece of that learning. By assessing teachers' noticing expertise and

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empirically identifying three profiles of expertise, we hoped to better understand how teachers were taking up and engaging in this practice so that we could better support their development. We purposefully chose to assess teachers with varying amounts of PD because we know that teachers learn, and implement what they learn, at different rates, and we wanted to capture as much variety as possible. Our findings replicated our earlier work (Jacobs et al., 2010) in new grade levels (upper elementary grades versus primary grades) and with new mathematical content (problem solving with fractions versus problem solving with whole numbers). We also extended this work in two main ways: (a) elaboration of the deciding-how-to-respond component skill, and (b) identification of profiles of noticing expertise.

Elaboration of the Deciding-How-to-Respond Component Skill

In our earlier work, we introduced the inclusion of the deciding-how-to respond component skill in teachers' noticing of children's mathematical thinking, and we explored either decisions about follow-up questions or decisions about next problems, but not both together (Jacobs et al., 2010, 2011). In this study, we asked teachers to make both decisions for each instructional scenario so that we could compare teachers' engagement with children's thinking in the two categories of deciding how to respond with the same set of children's strategies. We found that teachers consistently showed more expertise when deciding on follow-up questions than when deciding on next problems, and this relationship held for each of the three profiles. This finding may reflect how teachers often have little experience deciding on next problems that build on children's understandings, and they may even wonder if they have the freedom to craft their own problems (or adjust existing problems) given the widespread, systemic use of resources such as pacing guides and mandated textbook materials.

In short, we would encourage the theoretical bifurcation of deciding how to respond because teachers engaged differently with each category, and both are important to teachers' work. We would also suggest including opportunities to practice both categories in PD, with an awareness that deciding on follow-up questions may initially be more accessible. Gaining expertise in posing these follow-up questions has other benefits as well because these questions can serve as leverage points for teachers' learning. Follow-up questions not only provide teachers with information about a specific child's thinking, but over time they also help teachers increase their understanding of children's mathematical thinking in general (Franke et al., 1998, 2001).

Identification of Noticing Profiles

Each profile had strengths and room to grow, and thus they provide snapshots of developing expertise. Theoretically, the profiles extend our earlier work in which we characterized expertise in each component skill but did not provide a conceptualization for how the skills might work together differently for individual teachers (Jacobs et al., 2010). Our profiles provide this conceptualization and suggest that teachers in different profiles may need different types of support (see also, Munson, 2020). We provide some initial suggestions for customization.

Teachers with an *Accomplished Noticing* profile demonstrated strong expertise across component skills, but still with room to grow. Focusing on challenging examples—complex or ambiguous strategies—could provide these teachers with opportunities to refine their expertise (Jacobs, Empson, Pynes, et al., 2019). Teachers with a *Mixed Noticing* profile demonstrated some expertise, with more expertise in attending to children's strategy details and interpreting children's understandings than with the two deciding-how-to-respond skills. Focusing on typical and straightforward strategies could provide these teachers with opportunities to easily make sense of children's strategy details and related understandings so that they could concentrate on how to build on these understandings with follow-up questions and next problems. Teachers with an *Emerging Noticing* profile had substantial room to grow in all component skills, but their mean scores for attending to children's strategy details and deciding on follow-up questions were relatively higher. Focusing on typical and straightforward strategies could provide these teachers with opportunities to solidify their ability to recognize strategy details and generate follow-up questions. Further, providing access to research on children's mathematical thinking could help them begin to interpret children's understandings reflected in strategy details, learn how those understandings are likely to develop, and consider next problems to support this development.

In addition to providing insights for PD, our profiles provide a starting point for conversations about teachers' developmental trajectories with respect to noticing expertise. However, caution is warranted given that our data are not longitudinal. Assuming that teachers are moving toward an *Accomplished Noticing* profile, the question is whether the other two profiles represent two separate paths or a single, connected path. Specifically, one possibility is that, as teachers learn about children's mathematical thinking, some may develop skills consistent with an *Emerging Noticing* profile and others with a *Mixed Noticing* profile, and then each group follows a different path toward an *Accomplished Noticing* profile. Another possibility is that, as teachers learn, they move from an *Emerging Noticing* profile to a *Mixed Noticing* profile and finally to an *Accomplished Noticing* profile in a single, connected path. We have some evidence to suggest that this second possibility may be more apt.

We looked at the relationship between the number of years of PD that teachers completed and their noticing profile. Teachers who had completed 1, 2, and 3 years of PD were found in all three profiles, but the distribution varied as one might expect with a single, connected path for development—there were more teachers who had 3 years of PD with an *Accomplished Noticing* profile and more teachers with only 1 year of PD with an *Emerging Noticing* profile. In fact, the membership of the two profiles were essentially mirror images of each other. The *Accomplished Noticing* profile had 7%, 36%, and 57% of teachers who had completed 1, 2, or 3 years of PD respectively, whereas the *Emerging Noticing* profile had 56%, 36%, and 8% of teachers who had completed 1, 2, or 3 years of PD respectively. The *Mixed Noticing* profile was in-between, with a more even distribution. These findings support earlier findings that teachers usually do not gain expertise in noticing children's mathematical thinking from teaching experience alone, but it can be developed with sustained time and support (Jacobs & Spangler, 2017).

Final Thoughts

We provided an initial exploration into profiles of teachers' expertise in professional noticing of children's mathematical thinking. The profiles we identified differed in terms of the overall expertise demonstrated and in the constellations of strengths and needed areas of growth related to the noticing component skills. Not only do these profiles help us better understand the construct of professional noticing of children's mathematical thinking, but they also form a basis for customizing PD to support growth in teachers' noticing expertise. Overall, the profiles increased our appreciation for the complexity of noticing expertise and raised our awareness that teachers may display inconsistent expertise across the component skills as they are learning.

Acknowledgments

This research was supported in part by the National Science Foundation (DRL–1712560). The opinions expressed do not necessarily reflect the position, policy, or endorsement of the supporting agency. We thank the participating teachers and our research team—Amy Hewitt, Naomi Jessup, Gladys Krause, and D'Anna Pynes—for help with data collection and scoring.

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