TEACHER QUESTIONING STRATEGIES IN SUPPORTING VALIDITY OF COLLECTIVE ARGUMENTATION: EXPLANATION ADAPTED FROM HABERMAS' COMMUNICATIVE THEORY

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Facilitating productive mathematical argumentation is challenging; it is critical to develop a specific guiding vision of practices to help teachers learn to teach argumentation. However, what counts as acceptable classroom-based mathematical argumentation remains an open question. In this study, building on Habermas' theory of communicative action, we developed two analytic frameworks to examine questioning strategies used to support the validity of collective mathematical argumentation. Habermas' three components of rationality allowed us to focus on fine-grained rationality components of teacher questioning as well as teachers' intentions of asking these questions. The theory of validity claims was used to capture different forms of validating argumentation. The frameworks may help teachers to be aware of the types of questions that they are asking when aiming at supporting valid argumentation.

Keywords: Research Methods, Classroom Discourse, Reasoning and Proof

Rationale and Purpose

Current research discusses many benefits of incorporating mathematical argumentation in classroom discourse (e.g., Nussbaum, 2008) and emphasizes the essential role of teacher questioning in facilitating collective mathematical argumentation — teacher and students (or a small group of students working independently) working together to determine the validity of a claim (Conner et al., 2014). Studies (e.g., Kazemi & Stipek, 2001; Kosko et al., 2014; Wood, 1999) have highlighted teachers' questioning as a pivotal factor shaping argumentative discourse and as strongly influencing students' engagement in productive mathematical argumentation. However, most of these studies placed more emphasis on documenting current situations or difficulties that teachers had in using questioning to regulate argumentative discourse than on developing effective ways to address some of these difficulties. For example, Sahin and Kulm (2008) analyzed types of questions two teachers used in two sixth-grade classes over two months. They found that the majority of questions teachers posed were factual, even when using a reform-based textbook, which included probing and guiding questions in the teaching guides. Scaffolding argumentation is not an easy task, and it is not clear precisely what actions of the teacher provide the desired results of argumentation. Further, no consensus exists in the field of mathematics education concerning the characteristics of successful argumentative discourse. Some researchers (e.g., Stylianides et al., 2016) have called for more research in the field to design practical tools for use in the classroom to address teachers' difficulties or particular learning goals in orchestrating argumentative discourse.

The goal of this study is to investigate how a beginning secondary mathematics teacher uses *rational questioning* as a didactical tool to support the validity of collective mathematical argumentation according to Habermas's (1984) theory of *validity claims*.

Theoretical Framework

Two concepts from Habermas' theory of communicative action are used in this study to investigate how teachers could support valid argumentative practices with a particular focus on teachers' questioning strategies. The first is Habermas' (1998) perspective on three interrelated components of

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rationality: epistemic (inherent in the control of validation of statements), teleological (inherent in the strategic choice of tools to achieve the goal of the activity), and communicative (inherent in the conscious choice of suitable means to communicate understandably within a given community). Boero (2006) advocated that Habermas' three components of rationality account for students' rational behavior in proving and argumentative activities. Corresponding to these components, students are expected to strategically choose tools to achieve a goal (teleological rationality) on the basis of specific knowledge (epistemic rationality) and communicate in a precise way with the aim of being understood by the classroom community (communicative rationality). Douek suggested that it was beneficial to develop argumentative discourse along the three components of rationality (i.e., epistemic, teleological, or communicative) and that the teacher should support students to meet the requirements of rationality, thus dialectically forming argumentation (Boero & Planas, 2014). In order to reach such aims, Douek further proposed the idea of using "rational questioning" as a method to "organize the mathematical discussion according to the three components of rationality" (Boero & Planas, 2014, p. 210). Following Douek's idea, we developed a Teacher Rational Questioning Framework (see Table 1) to classify types of rational questioning from teachers' perspectives to engage student participation in argumentation with different kinds of rationality (For more details, see Zhuang & Conner, 2018). We defined *rational questioning* as a question that contains at least one component of rationality. At times, for clarity, we call a question epistemic rational questioning if it contains an epistemic rationality component.

Table 1: Teacher Rational Questioning Framework

Components of Habermas' Rationality	Features	Examples
Epistemic Rationality (*ER*)	The questions intended to allow students to reason and justify their arguments; to clarify/challenge students when they gave unclear or incorrect responses.	Can you tell me why?
Teleological Rationality (*TR*)	The questions intended to allow students to show or reflect on the strategic choices that they used to achieve their arguments or ideas; to point students towards the specific means or tools.	How did you figure that out?
Communicative Rationality (*CR*)	The questions intended to allow students to communicate or reflect on the steps involved in their reasoning and arguments to ensure that their ideas can be understandable in the given community; to point students towards the correct use of mathematical terminology.	How would we write this correctly mathematically?

In terms of validation of argumentation, we adapted Habermas (1984) theory of *validity claims*, which proposed that three forms of validity claims exist: to truth, to rightness, and to sincerity (see Table 2). By adapting Habermas' theory of validity claims to collective argumentation in mathematics classrooms, we identified three parallel dimensions of validating argumentative practice (For more details, see Zhuang & Conner, unpublished).

Table 2: Validity Claims and Corresponding Validating Argumentative Practice

Validity Claims	Features	Corresponding Validating Argumentative Practice
Truth	Concern the way things are in the external world of objects and spatiotemporal entities, thus constituting a constative (fact-stating) speech act.	Argumentation results in correct mathematical conclusions (T). The truth of an argumentation was judged by the researcher's perspective according to shared mathematical theorems, axioms, and principles in the given mathematical classroom community.
Rightness	Concern the way things are in the social world of shared duties, norms, values, thus constituting a regulative speech act.	Argumentative practices conforming to the social norms (N-S) and sociomathematical norms (N-M) (Yackel & Cobb, 1996) in a given classroom social context.
Sincerity	Concern the way things are in the subjective world consisting of personal self-understandings, thoughts, intentions, feelings, thus constituting an expressive speech act.	We assume when the students engaged in the argumentation, they satisfied the sincerity of argumentation unless there is clear evidence demonstrating that the argumentative discourse deteriorates into oppositional or confrontational talk and interpersonal conflicts spill over into the intellectual content.

Further, Habermas (1984) argued that the acceptance of valid argumentation not only links to the referred mathematical objective world, to norms, but also to the use of language. If a speaker cannot present comprehensible and accepted language, then there is no way to establish a shared understanding through communication. This concern about the fundamental use of language gives rise to a new dimension for validating argumentation, which focuses on communicative validity of argumentation and the participants' intentions on reaching a shared understanding within an argumentative practice, that is: Argumentation is communicated by using appropriate mathematical language and representations with participants' intentions to reach a consensus or a shared understanding (*C*).

In this study, we adapted two developed frameworks on the basis of Habermas' theory of communicative action to investigate how rational questioning supports the validity of collective argumentation in a 9th-grade algebraic mathematics classroom.

Data and Methods

The participant, Jill (a pseudonym), was in her third year of teaching and was purposefully selected based on her good understanding of argumentation and willingness to support student engagement in argumentation. We video recorded two consecutive days of Jill's instruction per month, which translated into eight classes a semester. The primary data sources in this study included video recordings and transcriptions of two consecutive days of Jill's instruction, focused on factoring and expanding binomials with integer coefficients.

Each lesson was first divided into multiple argumentation episodes. An argumentation episode was located by identifying the final claim of an argument and the accompanying data, warrants, and data/claims supporting the final claim the collective attempted to establish. Therefore, if there were arguments or claims that supported or refuted the initial argument, these arguments were viewed as connected with each other and included in an episode of argumentation. The next step was analyzing all teacher questions within each chosen argumentation episode in order to identify and categorize rational questioning based on our Teacher Rational Questioning Framework (see Table 1). Each rational question was also categorized according to the valid argumentation analytic framework (see Table 2) to explore how teachers used rational questioning to support the validity of argumentative practices. The classification of teacher questioning started with developed frameworks, but we kept an open mind by using a grounded theory approach (Glaser & Strauss, 1967) to ensure the inclusion of additional themes that were not included initially in the framework. A simple enumerative approach was finally used to quantify rational questioning in order to explicate the patterns that emerged from the open-coding process.

An Example of Using Habermas' Frameworks

As an illustration, let us consider an argumentation episode on the second day where students had reviewed the greatest common factor and expansion of binomials with form $(x \pm a)(x \pm b)$ on the first day of the lesson. During this episode, the students were learning about factoring trinomials with integer coefficients in a small group:

Given $x^2 + \_\_\_ x + 12$ *, what are the possible values for the blank?*

1 T: **All right, what do we think?** (Questioning without a rational component: *N*).
2 S1: It's six or nine.
3 T: Six or nine.
4 S2: Yup.
5 T: **Tell me why.** (*ER:* contains epistemic rational component).
6 S1: Tell her why S2.
7 S2: Why do I have to tell her. Oh. Um, okay, so couldn't, couldn't like...
8 T: **Hang on. I want to hear 6 or 9 explanations first.** (*N*)
9 S2: Oh gosh. Could you say the 9 explanation and I say 6 explanation?
10 T: **Tell me the 6 explanation.** (*ER*)

Interpretation. At the beginning of this episode, both students provided incorrect answers. Instead of giving direct corrective feedback, the teacher challenged students' arguments by asking them to provide an explanation of incorrect answers (Lines 5, 8 and 10). Thus, we coded these three questions as rational questioning that contains epistemic rational components (*ER*). These questions also illustrated that the teacher has a special role to play in trying to develop classroom social norms (*N-S*) to address expectations for student participation in argumentative practices through ongoing negotiations. In this context, students were expected to provide warrants, reasons, or backings to justify their claims. Thus, we coded these rational questions as facilitating the validity of argumentative practices in regard to classroom social norms.

11 S2: Okay. So, 6 times 2 is 12.
12 T: Yes, 6 times 2 is 12. That's true.
13 S2: Yeah, and then 6 might not work, 6 wouldn't work.
14 T: **Why not? Talk to me about why 6 might not work.** (*ER*)
15 S2: Because 6 plus 2 is 8 and you have to have 12 and so because [mumbling]
16 T: **Hang on, hang on. You are saying things that are on the very right track.**
17 **Think through it.** (*ETCR:* contains all three rational components).

Interpretation. Through the explanation of her arguments, S2 noticed that 6 was incorrect and worked towards the correct answer 8. However, she lacked the confidence to further articulate the justification in her thinking. At this point, the teacher encouraged her to explain her reasoning (Lines

16 to 17) which revealed again that students are expected to provide reasons to justify their claims (*N-S*).

S2: Okay. 6 plus 2 is 8 but yeah do not even know where 12 like, how are you supposed to like, do you know what I am saying it's like
S1: You can put 8 in here. That's the point.
S2: Yeah.
S1: So we are trying to find the line, what goes on the line up here.
S2: Yeah.
S1: You can put 8 but 6 times 2 is 12 and then 6 plus 2 is 8.
S2: So it's 8.
S3: Yeah.
T: **Yes. You are thinking about it in the right way. You said 8. That's okay. That's why I want you to think about it. Now does that make sense?** (*ETCR*)
S2: Yeah.

Interpretation. The student-student interactions (Lines 18 to 24) illustrated that pushing students to justify why their arguments hold served to support students to understand that the acceptable claims are based on mutual understanding and agreement on epistemic reasons. The question "Now does that make sense?" showed Jill's intention to provide students with opportunities to make sense of other students' epistemic, teleological and communicative requirements of argumentative practices (*ETCR*). It also pushed students to be able to learn from each other which promotes their productive disposition towards mathematics to reach a consensus or a shared understanding in argumentation (*C*).

T: **So does anyone come up with another number besides 8 that could go there? Anybody come up...S4, why could you do 7?** (*ER*)
S4:Oh gosh.
T: **You said you could do it. Why?** (*ER*)
S4:4 plus 3 is 7 and 4 times 3 is 12.
T: Very good. **Is there anything else?** (*TR*)

Interpretation. When S4 came up with answer 7, which was different than others, the teacher intentionally called on her to explain why this could work (Lines 30 to 31, *ER*), which established the expectations for students in the class to share their thinking, ideas, and solutions, even if they have answers that differ from other students' answers (*N-S*). At this point, the teacher's directive question served to help students understand what counts as mathematically different solutions (*N-M*). The final claims of this argumentation are 8 and 7 could work while 6 cannot. Notice that the answers provided here do not include all possible values. At the end of this episode, the teacher asked students to keep thinking of any other possible values that might exist (Line 35, *TR*). From the researcher's point of view, all the rational questions used in this episode also support the requirement of truth argumentation (*T*). Therefore, rational questioning can support multiple forms of validating argumentation.

Results

Based on our definition of argumentation episodes, Jill's two consecutive days of lessons contained 23 argumentation episodes. Within argumentation episodes, Jill asked 136 questions, and 81% (110/136) of questions involved rational questioning. According to our analysis, Jill used a variety of combined forms of rational questioning: some questions included two or three components of rationality and others only involved one. For the purpose of this study, we looked across epistemic, teleological, and communicative rational questioning and examined how different types of rational questioning were related to these components of validity of argumentative discourse. Table 3 gives the numbers in each type of rational questioning and how it related to engagement in valid argumentation.

Table 3: Rational Questioning Supports Validity of Collective Argumentation

Type of Rational Questioning	Number of Questions	Truth	Norms	Communication
Epistemic	46	27	45	9
Teleological	70	54	31	15
Communicative	32	22	17	15

Note. The number of questions in each category is not discrete; a question might be categorized in several categories.

The content of these lessons included multiple problem-solving mathematical activities to teach students how to factor and expand binomials with integer coefficients (see Figure 1), and the teleological (i.e., producing strategies to achieve the aim of the activity) was the most common rationality component among all rational questioning, in which over 60% (70/110) of rational questioning contained a teleological component. Most of Jill's teleological rational questions (54/70) were strategically goal-oriented to support students achieving the *truth* of arguments in regard to filling in an area model (e.g., "Alright now I have the inside of my area model filled out. How do I get the outside?") and finding the greatest common factor in each row and column of the area model so as to solve the problem (e.g., "What is the greatest common factor of the bottom row?"). Jill also intentionally used some teleological rational questioning to encourage other students to join the discussion: "Okay at this point we have two empty boxes. Somebody else, I want you to tell me how we find what goes into those two empty boxes that we have." By continuing to ask other students to respond to particular students' answers, Jill developed *norms* that every student in the class was expected to pay attention to what other students say and be ready to share solutions. On a few occasions, Jill wanted students to be able to use precise mathematics language to *communicate* their ideas and communicated this by asking them to be more specific about their solutions.

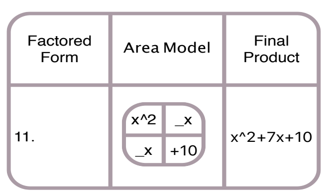

Figure 1: An Example of Task in Lesson One

Epistemic rationality followed as the second most common component of rational questioning (46/110). Jill used most of her epistemic rational questioning (45/46) to encourage her students to justify why their arguments hold (e.g., "You are correct; it's not three, but why?") or challenge her students to provide reasons for their arguments, especially when they gave incorrect answers. Epistemic rational questioning presses students to provide evidence to support the claims that contribute to the development of another *norm*: when engaging in argumentation, constructing a claim is not enough, you are expected to provide your reasoning for the claim. Through analysis we also noticed that not all epistemic rational questioning resulted in correct responses (27/46 prompted correct answers). Jill used sequences of epistemic rational questioning to make students' implicit ideas more explicit and help students to revise their incorrect answers (as shown in the example above). In this way, the teacher also supports the development of students' ability to form comprehensive and acceptable speech acts (i.e., *communication*) based on mutual understanding and agreement. This result provides empirical evidence to support Frank and colleagues' (2009) contention that a single specific question is not enough to elicit a complete explanation or justification; sequences of questions that concentrate on students' explanations are required.

Based on our analysis, 29% (32/110) of the rational questioning contained communicative components. In this unit, Jill's goal for students was apparently less focused on communicative rationality than on teleological and epistemic rationality. Most of the communicative rational questions (22/32) served to introduce graphic representation (the area model) to help students reason and to pull out correct answers (*truth*). For example, she asked, "If this is an area model, what could I call this [point at length] and what could I call this [point at width]?" Sometimes Jill intentionally asked students to rewrite a mathematical expression so that they could easily find the greatest common factor (e.g., "x^4, how do I rewrite this one?"). Occasionally, Jill wanted to highlight her expectations for students to use correct mathematical representations and ensure their representations can be understood in the given classroom community (i.e., *norms and communication*). An example of this type of question would be as follows: "Have I actually finished...I need to write it in the factor form. So tell me what to write."

Conclusions and Implications

Drawing on two different concepts from Habermas' theory of communicative action, in this study we developed two frameworks focused on teacher questioning strategies to facilitate valid argumentative practices. Our definition and classification of *rational questioning* came from Habermas' three components of rational behavior. Many researchers (e.g., Boero, 2006; Cramer, 2015) applied this construct as a tool to analyze students' participation in argumentation; our study shows it could also be used to analyze teachers' ways of dealing with argumentation in the classroom. Habermas' theory of validity claims provides a tool to develop an analytic framework to capture "validation" of an argumentative discourse according to the three forms of validity claims. The two analytic lenses from Habermas' theory provide us with a more comprehensive perspective to shed light on teacher questioning that supports collective mathematical argumentation. Habermas' threefold perspective on epistemic, teleological and communicative rationality helps us to identify fine-grained rationality components of teachers' questions and how teachers' questioning is constrained in relation to the three components of rational behavior; the teacher's use of rational questioning to control the validation of argumentation is seen through Habermas' theory of validity claims.

Classroom-based argumentative discourse is a form of collaborative discussion, and classroom discussions are complex, messy (Frank et al., 2007), and sometimes the argumentation may not happen in the intended way. In order to facilitate productive collective mathematical argumentation, it is critical to understand what constitutes successful argumentative practice. We view rational questioning as a teaching intervention to enrich different levels of argumentation and help students to meet the requirements of rationality, thus dialectically forming productive collective argumentation. Our analytic framework for valid argumentation supports the analysis of classroom instruction related to argumentation and identifies different forms of valid argumentation. It considers students as mathematics learners to participate in argumentation throughout the grades and emphasizes the validity of argumentation as context-dependent. For future research, we should continue to find effective ways to support students' participation in appropriate local acceptance criteria for argumentation and study the role of teachers in regulating valid collective argumentation.

More importantly, in this study, we investigated the effectiveness of classroom-based rational questioning as a didactical tool to support validating argumentation, which responds to the call from the field to use theoretical ideas to design practical tools for teachers to use in the classroom. Our results indicated that although epistemic rational questioning may not always elicit correct or complete reasoning, it served as a way for teachers to set social norms that students were expected to provide reasons to justify their claims when engaging in argumentative activities. Through leading students to work towards a specific method or foreground a particular piece of mathematics for consideration, teleological rational questioning worked well in constructing correct claims. Further, by calling on a particular student to share a different solution, the class worked on what counted as a mathematically different solution, which facilitated the establishment of sociomathematical norms. Communicative rational questioning contributed to the development of students' communicative competencies by asking students to make sure their representations were correct and to use appropriate mathematical terminology to communicate ideas. Questioning focused on communicative rationality also cultivated norms that students were expected to ensure their use of mathematical language and representation can be understood in the given mathematical classroom community. Teacher questioning is one of the most frequently used ways of orchestrating students' reasoning and a key factor in promoting argumentation (Kosko et al., 2014). The fine-grained analysis of teacher questioning in regard to Habermas' three components of rationality served as a method to help us understand how collective argumentation could be initiated and sustained, which thereby contributes to the construction of the culture of rationality in argumentative discourse. This study only focused on general types of rational questioning; it will be interesting to examine what combinations of components of rational questioning appeared to be more supportive and which are less supportive of the validity of argumentation.

In summary, the findings of this study have implications for both theory and professional development in mathematics education. This study illustrates how an important theoretical construct from outside mathematics education can be interpreted and flexibly adapted to offer a new and promising perspective into the study of discursive practices that are related to mathematical argumentation. The frameworks provide a new perspective to understand the roles of teacher questioning in supporting mathematical argumentation. As for professional development, the types of questions provide information about how a beginning mathematics teacher used questions to support mathematical argumentation. In addition, the work of this study contributes to illustrate the link between theoretical and classroom-based research and can be applied in teacher professional programs as a means to develop teachers' awareness about using rational questioning to support the rationality and validity of argumentative discourse.

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