# Teaching Word Problem Solving to Students With Autism and Intellectual Disability

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*Ms. Dobson teaches 10 students with autism and intellectual disability in Grades 3 through 5 during her math block. She knows problem solving is a foundational skill that can be applied across domains of math, but more importantly, she knows it is a critical skill for students to be able to generalize to solving problems in the real world. Like most special education teachers, her students have a wide variety of strengths and skills in math. Some have memorized math facts and know procedures for computation of multidigit numbers while others are working on expanding number recognition and counting skills. Last year, she noticed that one of her students relied on keywords to solve problems, insisting on adding anytime she heard the word "more" in the problem. However, when that strategy did not work, such as when there were no keywords in the word problems or the problem was making a comparison like "how many more," she quickly became frustrated. This year, Ms. Dobson wants to try a strategy called modified schema-based instruction (MSBI) she learned about over the summer at a professional development workshop. She hopes it will help her students gain the conceptual and procedural knowledge they need.*

**MSBI teaches students to recognize the pattern or structure of the mathematics problem (i.e., schema) and use that knowledge to develop a plan for finding the solution.**

There is growing recognition of the importance of mathematical problem-solving skills for all students, including those with autism spectrum disorder (ASD) and intellectual disability (ID). Word problem solving is one way that mathematical problem solving is often taught and assessed in school settings. MSBI is an evidence-based practice for teaching students with ASD/ID to acquire and generalize problem-solving skills (Clausen et al., 2021; Root, Ingelin, & Cox, 2021). Like all schema-based instructional techniques, MSBI teaches students to recognize the pattern or structure of the mathematics problem (i.e., schema) and use that knowledge to develop a plan for finding the solution. There are three general schemas for additive problem types (i.e., group or combine, change, and compare) and four general schemas for multiplicative problem types (i.e., equal groups, multiplicative comparison, ratio, and proportion). With MSBI, students learn to identify and model "what is happening" in the word problem using a schematic diagram (i.e., graphic organizer) that represents the problem structure.

This approach differs from the common keyword strategy that teaches students to base their plan for solving a problem on key or "signal" words, such as choosing addition because a word problem had the words "total" or "more." A recent analysis of word problems from high-stakes assessments across third to eighth grade found the keyword strategy only works for a little more than half of one-step problems if they have one keyword, a third of one-step problems with multiple keywords, and less than 10% of multiple-step word problems (Powell et al., 2022). Furthermore, real-world scenarios involving problem solving will not have keywords.

Teaching skills with limited transfer is problematic at best. In contrast, if students are taught each additive and multiplicative schema, understand how to discriminate between them, and can select and apply an attack strategy, they should be able to solve most word problems they encounter. If students conceptually understand why addition is used to solve a problem involving combining two parts to find a whole (e.g., a group or total problem) and subtraction is used to compare two quantities to find the difference (a difference or compare problem) rather than relying on the presence of signal words, they are more likely to be able to recognize when and why to use those operations to solve real-world situations.

The three components of schema instruction are: (a) teaching the key features of each schema, (b) teaching a solution strategy for each schema, and (c) teaching important vocabulary and language related to the schema (Powell & Fuchs, 2018). Although these are important for all students, learners with ASD/ID may need additional support for working memory, language, reading level, and numeracy skills. It is important to proactively consider the strengths, needs, and preferences of students with ASD/ID as well as what they will need to be engaged, motivated, and able to "show what they know" while problem solving (Root et al., 2020).

The Universal Design for Learning (UDL) framework is helpful for minimizing potential barriers in the environment and curriculum (CAST, 2018). MSBI uses the principles of UDL—multiple means of engagement, representation, and action and expression—to layer evidence-based practices for teaching students with ASD/ID on top of the key components of schema instruction to address barriers students may face (Root et al., 2020). For example, in MSBI, students are provided with a bank of high preference themes for word problems (engagement), schematic diagrams with visual supports that show the relationships between quantities in the word problem (representation), and a student-friendly task analysis with a checklist to help self-guide them through the problem-solving process (action and expression). Although these are three specific examples, MSBI strategies cover a multitude of the UDL framework's nine guidelines and 31 checkpoints that fall under the three broad principles (see CAST, 2018).

The intention of MSBI is to support students in accessing the problem, conceptually comprehending the problem and mathematical content, procedurally solving the problem, and generalizing problem-solving skills in multiple ways (Spooner et al., 2017). In this article, we provide guidance for using MSBI to teach word problem solving to students with ASD/ID based on a body of experimental research with students from elementary through transition age with ASD/ID (for systematic reviews, see Clausen et al., 2021; Root, Ingelin, & Cox, 2021). Practitioners can use these seven steps to plan instruction that proactively supports learners with ASD/ID during word-problem-solving instruction.

## Step 1: Engage Students in Meaningful Problem-Solving Tasks

*Ms. Dobson has never felt particularly confident using the math tasks and word problems she finds online or in textbooks. The quantities are often unrealistic or beyond her students' current numeracy level, and scenarios are not personally relevant or meaningful for her students. Ms. Dobson is going to create contextualized word problems that are relevant for her students and their interests. She will begin by asking students about their interests and future goals.*

Students need to understand the "why" behind learning activities to be engaged and motivated to learn (UDL guideline for "recruiting interest"; CAST, 2018). One way to proactively plan for variability in student interests and make sure instruction is personally relevant is by having them solve meaningful real-world problems. This contextualized approach to designing mathematics tasks makes explicit connections between real-life activities and routines in students' current and future environments and the targeted mathematics skills (Root, Clausen, & Spooner, 2021). The first step to making tasks meaningful is to identify real-world contexts in which students could use the targeted math skills that are high interest, such as familiar locations in their community, preferred activities, or family routines (e.g., cooking, cultural celebrations). These can be generated by students or solicited from families. Incorporating student experiences and understandings from their homes and communities into problem-solving tasks leverages their funds of knowledge and identities as mathematics doers and thinkers (Driver & Powell, 2017). Creating opportunities for student choice in the development or selection of themes can further increase engagement (CAST, 2018). For example, students can take turns selecting the theme for word problems when given a menu or array of options.

The real-world context of word problems lays a foundation for making connections between the word problems and use of mathematics in everyday situations. Anchoring instruction using an anticipatory activity is an additional strategy for maintaining engagement and promoting generalization. This can be done by having students watch a short video, view pictures, or engage with materials that are related to their selected theme.

> **This contextualized approach to designing mathematics tasks makes explicit connections between real-life activities and routines in students' current and future environments and the targeted mathematics skills.**

## Step 2: Make Word Problems and Materials Accessible

*Ms. Dobson wants to make sure that all her students learn how to solve word problems but recognizes some face barriers that will impact their progress. For example, one of her students can rote count, identify numbers, and make sets up to 5 but is still working on developing those skills up to 10. Another student can read almost anything you put in front of them, but they struggle with comprehension. She will ensure these two students have access to manipulatives and text to speech to reduce barriers in their learning environment.*

Students with ASD/ID may experience barriers to accessing word-problem-solving tasks because of their literacy and numeracy skills. Specifically, these could include the reading level, problem structure, quantities/content, and vocabulary demands of the task (Root, Clausen, & Spooner, 2021). Many of these aspects are related because word problem solving can be thought of as a form of text comprehension (Fuchs et al., 2018). Spooner et al. (2017) published guidelines for writing word problems using considerate text that has familiar and easily decodable words. The reading level and quantities used in problems can be adjusted to meet differing abilities of students while still addressing the same learning goals (UDL guideline "language & symbols"; CAST, 2018). Related to the reading level and language that is used, the structure of a problem (e.g., how information appears, order of information, presence of extraneous information) also influences accessibility and how challenging it will be for a student to solve it independently. As shown in ***Table 1***, the supports students need to access problem-solving tasks and materials can be viewed as being on a continuum and are not static (UDL guideline "provide options for perception"; CAST, 2018). Students should not be pigeon-holed into one column. For example, a student may require the most support needs in early numeracy skills, but that does not automatically place them in the most support needs for reading skills. The student may be a proficient reader and need less support in that area. As instruction progresses and students move past acquisition and become more fluent and generalize skills, the level of support they need will likely decrease.

## Step 3: Intentionally Sequence Instruction to Focus on Problem Types

*Ms. Dobson is committed to helping all her students improve their word-problem-solving skills. She once believed all students needed to master addition before learning subtraction and only after mastery of those operations could she introduce multiplication or division to her students. After her professional development experience, however, she now understands that these skills should not be prerequisites and that she should sequence her instruction based on problem types (i.e., schemas) instead. She needs to determine the best sequence for each of her students.*

When problem-solving instruction focuses on schemas, the instructional sequence and learning goals focus on the problem types. This may be a change for teachers who typically sequence instruction by operations, assuming students must master addition and subtraction before being exposed to multiplication and division. This developmental mindset can prevent students with ASD/ID from accessing grade-level mathematics by perpetuating the "not ready for" mindset. In contrast, ***Table 2*** shows the recommended

*Table 1* **Continuum of Supports**

| | *Most support* ⟵ | ⟶ | *Least support* |
|---|---|---|---|
| Reading | • Teacher reads problem and steps aloud (may be requested by student verbally or through picture exchange) | • Student may read first, teacher rereads; or technology reads as activated by student | • Student reads problems and steps independently |
| Numeracy | • Student uses manipulatives to represent quantities; may use jig or match-to-sample for quantities | • Student progresses through C-R-A/V-R-A sequence possible use of a calculator | • Student uses fact fluency and/or larger numbers, decimals, fractions, or percentages |
| Word-problem structure | • Word problems written in four-line format with picture support above referent nouns | • Word problems written in four-line format; picture supports may be faded as student shows proficiency | • Word problems written in paragraph format; quantities may not appear in separate lines; extraneous information may be included |
| Task-analysis modifications | • Task analysis broken into maximum number of steps with picture supports for steps; checkoff for steps completed | • Task analysis modified to number of steps student needs to solve; picture support faded; checkoff for steps completed; self-management of “on my own” or “with help” checklist optional (Gilley et al., 2021) | • Task analysis modified to number of steps needed or student transitions to attack strategy (Powell & Fuchs, 2018) |
| Schematic-diagrams modifications | • Schematic diagrams are color coded with different colors representing different quantities and consistent color used for final amount across problem types (e.g., blue); rules appear at top; picture support for key characteristics of problem types (e.g., symbol for same; see ***Figure 2***) | • Schematic diagrams are premade and are line drawings (Powell & Fuchs, 2018); rule may or may not appear at the top | • Students are taught to draw diagrams on their own to represent problem types; drawing of diagrams may be faded as student shows proficiency |
| Fine motor supports | • Pull offs are provided for numbers and labeling the word problems; manipulatives are three-dimensional for grasp; problems are presented on tablet device with draggable or touch functions | • Use of virtual manipulatives; problems are presented on tablet device with draggable or touch functions; adapted keyboard or pens | • Pencil and paper or tech |
| Executive functioning/ cognitive load demands | • Primary focus is on solving for missing ending quantity only; task analysis, manipulatives, student-friendly checklists, and any other supports as needed | • Missing ending quantity taught first and then progresses to missing medial and missing initial quantities; supports are faded as student shows proficiency | • Flexible strategies for solving are taught (Cox & Root, 2021) |

*Note.* C-R-A/V-R-A = concrete-representation-abstract / virtual-representation-abstract.

*Table 2* **Suggested Modified Schema-Based Instruction Instructional Sequence for Additive Problem Types.**

| *Unit* | *Instructional goals* | *Example problems* |
|---|---|---|
| Introduction to additive reasoning | • Vocabulary and concept introduction: same, equal, different, more, less, symbols (=, +, − )<br>• Compose and decompose sets<br>• Read[a] and write [a] equations | |
| Group (total) | • Represent group problems with all values known in equation and on schematic diagram (e.g., 4 + 2 = 6 animals)<br>• Follow problem-solving routine to solve group problems with sum known (e.g., 4 + 2 = *X* animals) | Ramona fed animals at the zoo.<br>She fed 4 hippos.<br>She fed 2 giraffes.<br>How many animals did she feed? |
| Compare (difference) | • Represent compare problems with all values known in equation and on schematic diagram (e.g., 5 − 3 = 2 more churros)<br>• Follow problem-solving routine to solve compare problems with difference unknown (e.g., 5 − 3 = *X* more churros) | Bethenny bought snacks for her friends.<br>She bought 5 churros.<br>She bought 3 pretzels.<br>How many more churros did she buy than pretzels? |
| Group and compare | • Discriminate between group and compare problems<br>• Follow problem-solving routine to solve intermixed group and compare problems with sum or difference unknown | |
| Change: addition | • Represent change-addition problems with all values known in equation and on schematic diagram (2 + 6 = 8 birds)<br>• Follow problem-solving routine to solve change-addition problems with sum unknown (e.g., 2 + 6 = *X* birds) | Luann saw different birds at the zoo.<br>First she saw 2 flamingos.<br>Then she saw 6 penguins.<br>How many birds did she see? |
| Change: subtraction | • Represent change-subtraction problems with all values known in equation and on schematic diagram (9 − 4 = 5 dollars)<br>• Follow problem-solving routine to solve change-subtraction problems with difference unknown (e.g., 9 − 4 = *X* dollars) | Tom bought a key chain at the zoo.<br>He had $9.<br>He spent $4 on the key chain.<br>How much money does he have left? |
| Change: addition and subtraction | • Discriminate between change-addition and change-subtraction problems<br>• Follow problem-solving routine to solve change-addition and change-subtraction problems with sum or difference unknown | |
| All | • Discriminate between all additive problem types<br>• Follow problem-solving routine to solve all additive problem types | |

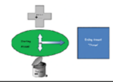
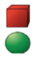
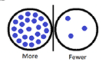
[a]Student symbolic level and communication preferences should be accommodated (e.g., alternative and augmentative communication, indicating correct reading or answer, dictate to scribe, using response options).

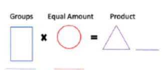
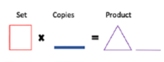
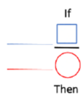
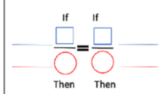
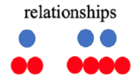
*Table 3* **Suggested Modified Schema-Based Instruction Instructional Sequence for Multiplicative Problem Types**

| *Unit* | *Instructional goals* | *Example problems* |
|---|---|---|
| Introduction to multiplicative reasoning | • Vocabulary and concept introduction: same, equal groups, different, variable, equation, expression, factor, product, multiply, symbols (=, ×)<br>• Create equal groups of manipulatives<br>• Read,[a] write,[a] and discriminate between expressions and equations | |
| Equal group | • Represent equal group problems with all values known in equation and on schematic diagram (e.g., $4 \times 2 = 8$ pickles)<br>• Follow problem-solving routine to solve equal group problems with product unknown (e.g., $4 \times 2 = X$ pickles) | Kelly helped make sandwiches for a trip.<br>She made 4 sandwiches for her family.<br>She put 2 pickles on each sandwich.<br>How many pickles did she use? |
| Multiplicative comparison | • Represent multiplicative comparison problems with all values known in equation and on schematic diagram (e.g., $3 \times 4 = 15$ )<br>• Follow problem-solving routine to solve multiplicative comparison problems with product unknown (e.g., $3 \times 4 = X$) | Leah packed things to read for her trip.<br>She packed 3 books.<br>She has 4 times as many magazines as books.<br>How many magazines did she pack? |
| Equal group and multiplicative comparison | • Discriminate between equal group and multiplicative comparison problems<br>• Follow problem-solving routine to solve intermixed equal group and multiplicative comparison with product unknown | |
| Ratio | • Represent ratio problems on schematic diagram and with manipulatives<br>• Follow problem-solving routine to solve ratio problems<br>• (e.g., 1 snack : 2 hours) | Bobby needs snacks for his trip.<br>His mom lets him have 1 snack every 2 hours they are driving in the car.<br>What is the ratio of snacks to hours? |
| Proportion | • Represent proportion problems with all values known in equation and on schematic diagram (1 snack : 2 hours = 2 snacks : 4 hours )<br>• Follow problem-solving routine to solve change-subtraction problems with product unknown (e.g., 1 snack : 2 hours = 2 snacks : 4 hours) | Bobby needs snacks for his trip.<br>His mom lets him have 1 snack every 2 hours they are driving in the car.<br>If he will be driving for 4 hours, how many snacks should he pack? |
| Ratio and proportion | • Discriminate between ratio and proportion problems<br>• Follow problem-solving routine to solve ratio and proportion problems with product unknown | |
| All | • Discriminate between all multiplicative problem types<br>• Follow problem-solving routine to solve all multiplicative problems | |

[a]Student symbolic level and communication preferences should be accommodated (e.g., alternative and augmentative communication, indicating correct reading or answer, dictate to scribe, using response options).

*Figure 1* **Distinctions between additive and multiplicative schemas**

| | Schema | Group | Compare | Change |
|---|---|---|---|---|
| Additive | Example Schematic Diagram |  |  |  |
| | Key Characteristic | Combining groups of different things  | Comparing quantities of things   | Dynamic action with 1 group of same thing  |
| | "What is happening?" | "Do we have groups of different things we can combine into a larger group?" | "Are we comparing things using more or fewer?" | "Did something happen to make the amount of things increase or decrease?" |
| | Rule | Small group plus small group combines into BIG group | Bigger number, smaller number, difference between the two | One thing, add to it OR take away, change |

| | Schema | Whole Number Equal Group | Whole Number Comparison | Ratio | Proportion |
|---|---|---|---|---|---|
| Multiplicative | Example Schematic Diagram |   |   |   |   |
| | Key Features | Same amount in each group  | Copies of sets  | If- Then relationship  | Equal If-then relationships  |
| | What is happening? | "Are there groups with an equal number in each group?" | "Is a set copied a number of times?" | "Are there relationships among quantities - if – then ?" | "Are there relationships among quantities – if then, if-then?" |
| | Rule | Groups with equal amount in each group multiply to find the product | Start with a set, make copies, multiply to find the product | If-then | If-Then is the same as If-Then |

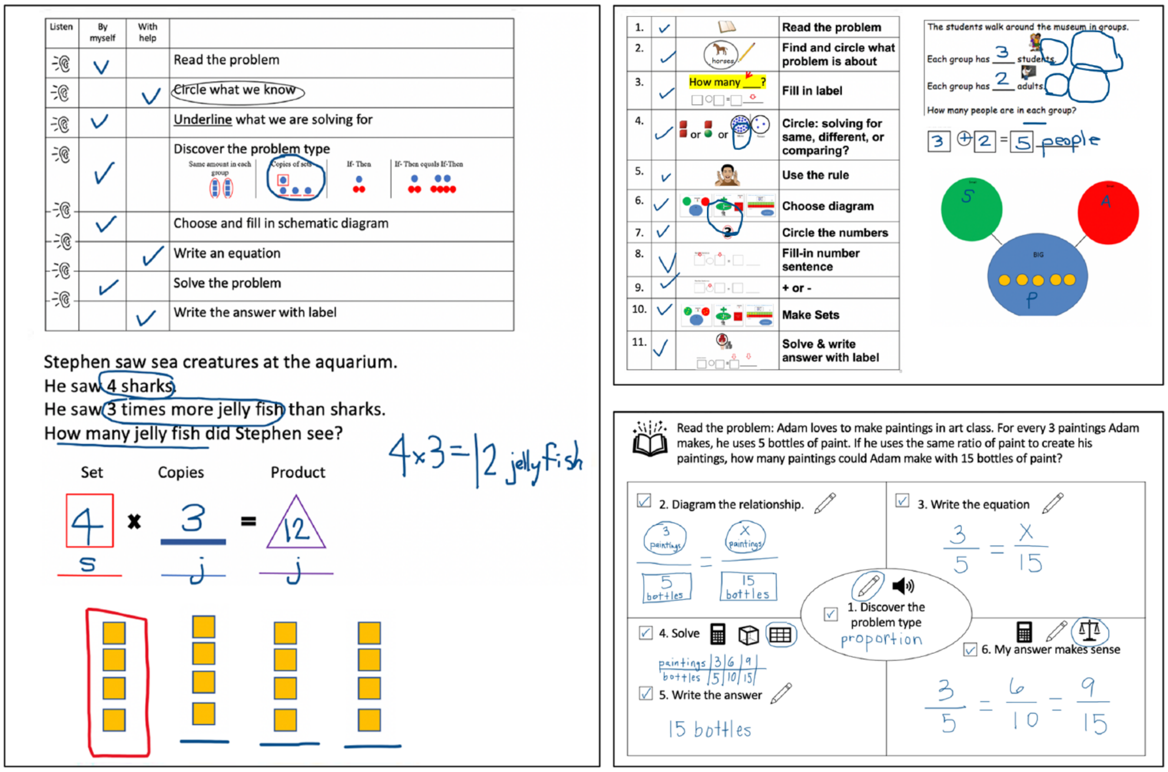
*Note.* These rules may change for negative numbers, fractions/decimals, or when missing initial or medial quantity.

instructional sequence for additive problem types based on Browder et al. (2018), and ***Table 3*** shows the sequence for multiplicative problem types. The intention of this sequence is twofold: (a) to reduce cognitive load by teaching one problem type at a time and (b) to provide opportunities to explicitly teach how to discriminate between problem types (Browder et al., 2018; Spooner et al., 2017; UDL guideline "comprehension"; CAST, 2018). Percent should be taught as a special type of rate problem (Jitendra & Star, 2016). Although conceptual understanding of and fluency with additive schemas would likely positively impact student progress through multiplicative schemas, it should not be viewed as a prerequisite for multiplicative schema instruction.

Although the sequence may be similar for students with learning disabilities or mathematics difficulties (e.g., Powell & Fuchs, 2018), the pace will be different, and students with ASD/ID will benefit from repeated opportunities for practice to move from acquisition to fluently solving problems. Students with ASD/ID may also need some preteaching of new vocabulary, how to represent quantities in number sentences with sets of manipulatives, and how to perform operations (i.e., +, −, ×) prior to adding in the cognitive demands of word problem solving (UDL Checkpoint 3.1 "activate or supply background knowledge"; CAST, 2018). The vocabulary should be clear, concise, and mathematically correct (see Hughes et al., 2016; Powell et al., 2019). Once introduced, vocabulary and concepts should be consistently referenced and highlighted throughout problem-solving instruction.

Students with ASD/ID also will likely need explicit discrimination training to recognize key features of schemas (i.e., problem types) and prevent overgeneralization of strategy use (e.g., solving all problems using the most recently taught strategy; UDL Checkpoint 3.1 "activate or supply background knowledge"; CAST, 2018). ***Figure 1*** illustrates the key features of both additive and multiplicative problem types, including example schematic diagrams, key characteristics, and rules. These can be sorted on a T-chart or three-column chart when discriminating between problem types, allowing for multiple means of representation and accessibility for learners with a variety of communication modalities. After students have sorted key features, they can practice sorting word problems by schema. The focus of this activity is not on solving the problems but instead gives multiple opportunities to practice identifying the schema based on the key features.

*Figure 2* **Example problem-solving routines**




*Note.* Left is a multiplicative comparison problem, top right is additive comparison problem, and bottom right is proportion problem.

## Step 4: Provide Options for Metacognition and Engaging in Mathematical Discourse

*Ms. Dobson wants to encourage her students' mathematical discourse, so she teaches them rules with hand gestures that mimic the schematic diagrams to use with the different schemas. One student who communicates primarily through scripted phrases excels at this! Ms. Dobson worked with the speech language pathologist to add the rules to her students' speech-generating devices. Ms. Dobson also points out schemas using the hand gestures as they naturally occur throughout the day. For example, during snack she says, "We have 6 students in our class, and everyone has 5 grapes. This is an equal group problem because we have 6 groups with an equal amount in each group. Six students times 5 grapes each equals 30 grapes in all."*

"Metacognition" is a term that describes the way a person plans, monitors, and assesses their own thinking (Özsoy & Ataman, 2009). Teaching strategies to solve word problems with explicit ways to remember the strategies aids metacognition. "Think-alouds" are a metacognitive strategy that teachers use to model how to engage in mathematical discourse. Teachers explicitly model their thinking processes and plans for solving out loud using clear, concise, and consistent language that serves as a model for students who may have difficulty with expressive language (Hughes et al., 2016; Powell et al., 2019). The variability in expression of learners with ASD/ID is accounted for in MSBI by giving multiple options for demonstrating metacognition and engaging in mathematical discourse (Cox & Root, 2021; Root et al., 2020). For example, "rules" representing the underlying structure of each problem type are taught using verbal statements (i.e., chants) paired with physical movements (i.e., hand gestures) and icons/visual supports (see ***Figure 1***; UDL guideline "provide options for expression & communication"; CAST, 2018). For example, the group problem rule uses the chant "small group plus small group combined equals BIG group" with hand motions that mimic the problem structure on the schematic diagram. Rules and chants help students remember the key characteristics of the problem type (in this case, a part-part-whole relationship) and corresponding mathematical action (combining parts to make a whole using addition).

Mathematical discourse is verbal and nonverbal communication about mathematical procedures and/or concepts (Xin et al., 2020). Language is a critical component of student learning because students need to understand language receptively to learn new concepts and use language expressively to convey their thoughts and demonstrate understanding (Xin et al., 2020) Students need instruction that provides consistent, clear, and concise language for mathematical concepts (Hughes et al., 2016; Powell et al., 2019). MSBI facilitates both receptive and expressive mathematical language for students in several ways, promoting mathematical discourse for a deeper understanding and expression of mathematical concepts. First, MSBI uses consistent mathematical language to model how to communicate mathematical reasoning. Second, MSBI provides visual supports through task analyses or heuristics to engage students in the problem-solving process (as seen in in ***Figure 2***). Third,

*Figure 3* **Teacher data sheet and corresponding student goal setting sheet and graph**

Student: Tiffany

Interventionist: Mrs. Keporos

Goal: Given multiplicative word problems and MSBI instruction, the student will be able to independently complete 7 out of 8 steps correctly for three sessions.

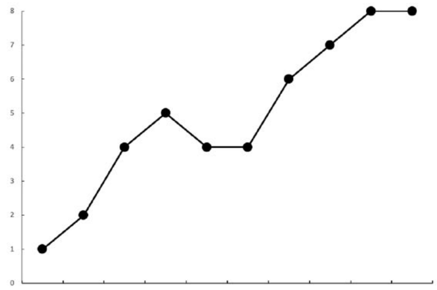
Date Mastered: 12/18 ⇒ Mastered MC, next step EG/MC Discrimination

| Steps to solve a percent of change problem | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8. Counted total steps for self-monitoring | IC | IC | GV | IC | IC | GV | IC | IC | IC | IC | | | | |
| 7. Correctly wrote the product in GO | GV | SV | IC | IC | IC | IC | IC | IC | IC | IC | | | | |
| 6. Correctly calculated the product | EC | M | EC | SV | SV | GV | IC | IC | IC | IC | | | | |
| 5. Labeled the product in GO | M | EC | SV | M | EC | M | SV | GV | IC | IC | | | | |
| 4. Completed the multiplier in GO | SV | SV | IC | IC | IC | IC | IC | IC | IC | IC | | | | |
| 3. Labeled the referent unit in GO | SV | GV | SV | GV | GV | EC | GV | IC | IC | IC | | | | |
| 2. Filled in the referent unit value in GO | SV | IC | IC | IC | GV | IC | IC | IC | IC | IC | | | | |
| 1. Read or listen to the problem out loud | GV | GV | IC | IC | IC | IC | IC | IC | IC | IC | | | | |
| Session number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| Session Date | 11/27 | 11/29 | 12/02 | 12/04 | 12/06 | 12/09 | 12/11 | 12/13 | 12/16 | 12/18 | | | | |

Prompting Key: IC = Independently Correct GV = General Verbal SV = Specific Verbal M = Model EC = Error Correction

**Directions**: For each step of the task analysis, write the final level of prompt required before the student completed the step correctly. After all the steps are complete, count the number of independently correct (IC) steps for that day, and circle the corresponding number on the chart. Note, the number of steps IC in one day may not land on an IC for the specific step, circle based on the overall number of IC steps for that session. Draw a connecting line from the circle from the previous session to the completed session, this will form a graph of the student's independently correct steps over sessions.

| Date | Steps Independent Correct | Next Goal |
|---|---|---|
| 11/27 | 1 | 2 |
| 11/29 | 2 | 3 |
| 12/2 | 4 | 5 |
| 12/4 | 5 | 6 |
| 12/6 | 4 | 6 |
| 12/9 | 4 | 6 |
| 12/11 | 6 | 7 |
| 12/13 | 7 | 8 |
| 12/16 | 8 | 8 |
| 12/18 | 8 | |




MSBI allows for multiple means of engaging in discourse by pairing gestures with the verbal chants for each schema to support the development and expression of both procedural and conceptual knowledge.

## Step 5: Implementing a Problem-Solving Routine

*Ms. Dobson developed a problem-solving routine for her students to follow when solving additive word problems by first task analyzing the process of solving word problems and then creating a "student-friendly" visual support for the routine in the form of a task analysis. The basic steps are to (1) read the problem, (2) identify the important details, (3) find the problem statement, (4) identify the problem schema or type, (5) diagram the problem, (6) write an equation, (7) solve the problem, and (8) answer the problem statement. Ms. Dobson adds or deletes steps as needed for specific students. To support emerging readers, she represented each written step with a picture cue to help the student remember what to do.*

An integral component of all schema-based instructional strategies is providing explicit instruction in a problem-solving routine to build student independence in the problem solving process (Browder et al., 2018; Powell & Fuchs, 2018). Students with mild disabilities or who are at risk for math difficulties may benefit from learning to use an attack strategy, such as the mnemonic "DISC" (Discover the problem type, Identify information in the problem to represent in the diagram, Solve the problem, Check the answer; Jitendra & Star, 2016; for additional attack strategies, see Powell & Fuchs, 2018). Students with ASD/ID, who may have limited literacy skills and memory skills, will likely need more support to be independent and fluent. In the following, we describe how problem-solving routines can proactively account for these learning characteristics in the way they are developed and taught.

### Develop Student-Friendly Problem-Solving Routine

The problem-solving routines in MSBI are developed by task analyzing the problem-solving process (i.e., identifying the sequence of discrete behaviors/decisions that make up the chained task of solving a word problem) and presenting them to students in a way that supports their independent use (i.e., is "student friendly"). It is critical to note that these problem-solving routines must support both the conceptual and procedural knowledge needed to solve problems. ***Figure 2*** shows three different variations of problem-solving routines adapted from MSBI research studies that show varying levels of support. Common features across these three examples include (a) sequence of steps for the problem-solving routine, (b) visual supports, and (c) checkoff areas to facilitate self-monitoring.

### Teach Problem-Solving Routine

Like traditional schema-based instruction, explicit instruction using a model-guided, practice-independent practice approach remains the primary teaching strategy. In addition, principles of systematic instruction, such as prompting with feedback, should be strategically incorporated. For example, constant time delay is an effective strategy for teaching students the mathematics vocabulary and symbols they will need to know for independence in the problem-solving routine (Root & Browder, 2019). As previously mentioned, students with ASD/ID may need explicit preteaching of math skills, concepts, and vocabulary (see ***Tables 2*** and ***3***) to reduce their cognitive load during problem-solving instruction. Next, teachers use explicit and systematic instruction to teach students how to follow the problem-solving routine. During the modeling portion of the lesson, the teacher is leading the decision-making using think-alouds with clear and concise language, but students should be actively engaged and provided with multiple opportunities to respond. Students could have their own materials to follow along with or take turns using one shared set of materials (e.g., taking turns checking off steps).

As the lesson moves to guided practice, the teacher should fade their modeling by providing students with an opportunity to respond independently before prompting. The emphasis during guided practice is to give students the opportunity to lead problem solving while teachers provide immediate feedback and error correction. This mastery-oriented feedback ensures the student has an accurate understanding before moving on to independent practice. Some students may need the support of a system of least prompts during guided and/or independent practice (Spooner et al., 2017). While specific prompting hierarchies should be tailored to the student and task, MSBI studies have generally used three levels of prompts if the student does not respond independently: (a) generic verbal prompt (read/point to step of task analysis), (b) direct verbal prompt (explanation of how to complete step), and (c) model-retest (model how to complete step and then give student chance to repeat). Consistent behavior-specific praise is an important component of guided practice while students are in the acquisition phase of learning. Rather than just saying "good job," it is critical to state specifically what the student did correctly. For example, for Step 6 in the additive problem example shown in ***Figure 2***, the teacher could say, "Fantastic! This is a group problem. It's about combining a small group of adults and a small group of children into one big group of people!" After a few days of instruction, the teacher can use data to determine steps the student is consistently getting independently correct and fade behavior-specific praise on those individual steps. An example data sheet for a multiplicative problem is shown in ***Figure 3***.

Finally, independent practice is another opportunity for students to lead the problem-solving process and for teachers to take data. Independent practice can include prompting and feedback (e.g., using the system of least prompts) after each step, or students can go through the entire problem-solving process and then check their work with the teacher or use a video model. The teacher may need to use pacing prompts or words of encouragement if needed to keep the student on track (e.g., "Keep going!" or "You are working so hard!"). Students may ask the teacher to read things again if needed or use technology to access a read aloud.

## Step 6: Embed Self-Monitoring and Goal Setting

*One of Ms. Dobson's students has demonstrated they are able to independently solve problems and complete all steps independently, but they aren't consistent and often require intensive adult support to stay on task and complete the independent practice problem. Ms. Dobson wants to start fading some of that adult support and increasing the student's independence. She decided to add a self-graphing and goal-setting component for everyone to build self-regulation, an important component of self-determination for all students. The students monitor their progress checking off steps completed independently, and she provides meaningful reinforcement (e.g., behavior-specific praise, access to preferred activity, edible treat).*

Self-regulation and executive functioning play an important role in academic learning. The UDL framework outlines considerations for scaffolding in these areas through self-assessment and self-reflection, increasing capacity for monitoring progress, and guiding appropriate goal setting (CAST, 2018). MSBI organically incorporates UDL guidelines in these areas (Gilley et al., 2021; Root et al., 2020). For example, lessons can begin with students reviewing progress from the prior day to identify their goal (i.e., how many steps or problems they want to get independently correct). They can then self-monitor progress through the problem-solving routine during independent practice (e.g., steps solved "by myself" in ***Figure 2***). Finally, lessons can end with students self-graphing their progress to identify the next lesson's goal with teacher support (see ***Figure 3***).

## Step 7: Use Data to Adjust Instruction

*Mrs. Dobson has learned several ways to individualize MSBI for her students' diverse abilities, such as using manipulatives to support conceptual understanding of the problem type. She plans to take the data and graph it using an upside-down task analysis that will help her make immediate decisions about her students' progress across units, including when to fade prompts and provide positive feedback.*

Solving a mathematics word problem is a chained skill that is made up of multiple discrete behaviors. Therefore, we recommend taking data at the step level to capture detailed student progress toward learning goals. Formatting the data sheet to be an upside-down task analysis, as seen in ***Figure 3***, is an efficient method of simultaneously collecting and graphing data. See McConomy et al. (2021) for more details on how to develop and use a task analysis for assessment and instruction. Teachers can choose to take data on independent steps completed or prompt level required to show incremental growth and to make data-based decisions. ***Figure 3*** shows prompt-level data taken for a student until they mastered solving equal group problems (seven out of eight steps correct for three sessions) and the instructional decision to move to the next schema

> **This contextualized approach to designing mathematics tasks makes explicit connections between real-life activities and routines in students' current and future environments and the targeted mathematics skills.**

(multiplicative comparison). Conversely, data may show a lack of progress, indicating instruction needs to be adjusted through making changes to the materials, methods, or task difficulty (see ***Table 3***).

Problem solving requires a tremendous amount of executive functioning skills, such as storing numbers and key information in working memory, distinguishing relevant from irrelevant information, organizing information, and selecting and flexibly using strategies for solving. One goal of MSBI is to reduce barriers caused by working memory demands and cognitive load during skill acquisition (Browder et al., 2018). The continuum of supports provided in ***Table 3*** are examples of how mathematical tasks can be scaffolded to meet students where they are and make problem solving accessible (UDL guidelines "provide options for perception" and "provide options for executive functions"; CAST, 2018). Instructional supports and strategies should be matched to the student's phase of learning (i.e., acquisition, fluency, maintenance, generalization; Jimenez et al., 2021). For example, schematic diagrams with color coding and picture supports and word problems with single-digit quantities that can be easily represented with concrete manipulatives are appropriate for initial acquisition instruction. This way students can use manipulatives to "see" the mathematical action (e.g., combining, taking apart, comparing differences, making equal groups, scaling up quantities) when using these planned supports. We recommend that even students who have the calculation skills and/or math fact fluency use concrete or virtual manipulatives during the acquisition phase to gain conceptual understanding. As data indicates, students who are fluent (e.g., consistently and accurately solving problems) can focus on generalizing to other variations of the schema. For example, once students are fluent in solving multiplicative comparison problems with missing product (e.g., $2 \times 5 = ?$), the strategy can be generalized to a factor unknown, either the number of sets or multiplier (e.g., $? \times 5 = 10$ or $2 \times ? = 10$).

## Conclusion

Word problem solving is the premise for students with ASD/ID to generalize their problem-solving skills to real-world problems. The keyword strategy is an ineffective strategy for teaching students to problem solve, especially because problems in everyday life do not contain keywords. MSBI, which was derived from traditional schema-based instruction, is an effective strategy for teaching mathematical problem solving to students with ASD/ID and utilizes teaching strategies tailored to this population. This article provided a practitioner-friendly seven-step strategy for implementing MSBI that can give students with ASD/ID access to meaningful problem-solving instruction.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