A Systematic Review and Meta-Analysis of Rational Number Interventions for Students Experiencing Difficulty with Mathematics

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Author Note

The data set compiled for this project (Krowka & Haymond, 2021), along with all quantitative data collected or generated, will be deposited in the Harvard Dataverse Network upon publication. One of the study's authors (R.S.) is a co-author on some of the primary reports included in this meta-analysis. Thus, this author was not involved in the screening or data extraction procedures for those reports.

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Abstract

This systematic review and meta-analysis appraised and synthesized mathematics interventions research addressing rational number concepts for students with mathematics difficulties (MD) in Grades 3–9. Included studies (n = 28) meet the What Works Clearinghouse Group Design standards (Version 4.0) for study quality. Across all studies, we found significant mean effects (g = 0.68) favoring intervention. Understanding which instructional components within interventions are associated with effects was a fundamental purpose of the study. Random-effects meta-regression models with robust variance estimation (RVE) were used to examine which instructional components moderated effects. The teaching and use of accurate mathematical language emerged as significant (b = 0.50, p = .042) when controlling for other instructional components (visual representations, review, student explanations, and fluency building activities) and three control variables (group size, duration, nature of the control). Intervention characteristics and study design features were also explored through moderator analysis. Group size, duration, nature of the control, type of measure, interventionist, provision of ongoing training, and grade level emerged as significant moderators ($p < .05$).

Keywords: intervention, mathematics difficulties, meta-analysis, rational numbers

A Systematic Review and Meta-Analysis of Rational Number Interventions for Students Experiencing Mathematics Difficulties

Proficiency in mathematics is critical for academic and occupational success (e.g., Koedel & Tyhurst, 2012; National Mathematics Advisory Panel [NMAP], 2008), and understanding rational number concepts (e.g., fractions, decimals, percentages, proportions) is a vital component (Tian & Siegler, 2018). The importance of rational numbers was established in Siegler et al. (2012), where knowledge of fractions in children as young as 10 (i.e., Grade 5) predicted performance in algebra in Grade 11 above and beyond other aspects of mathematical knowledge (e.g., proficiency with whole number arithmetic, geometry), intellectual ability, or socioeconomic status. Since then, several other studies have demonstrated the predictive relationship between early rational number knowledge and mathematics achievement in middle and high school (e.g., DeWolf et al., 2015; Jordan et al., 2017; Resnick et al., 2016; Siegler et al., 2012; Siegler & Pyke, 2013).

Unfortunately, many students are not adequately learning rational number concepts (Kloosterman, 2010, 2012; National Assessment of Educational Progress [NAEP], 2019). One explanation for this may be that understanding rational numbers requires a level of abstract reasoning that understanding whole numbers does not (Wu, 2011). Fractions, for example, are the first type of rational number students learn in school and require understanding the bipartite relationship between the numerator and denominator to determine numerical magnitude. This differs from understanding whole numbers where magnitude corresponds to counting discrete objects. Proportions and ratios then build on initial fraction understandings and require students to work with two linked quantities that vary and scale concurrently (Lamon, 2011), introducing even more complexity.

The topic of rational numbers continues to increase in complexity throughout middle school as students build their knowledge of fractions, decimals, percentages, and proportions and are asked to apply this knowledge when learning measurement and geometry or when solving word problems (e.g., California Department of Education [CDE], 2013; Texas Education Agency [TEA], 2012). It is therefore not surprising that the achievement gap in mathematics between students with and without mathematics difficulties (MD) tends to widen in middle school as content becomes more demanding (Wagner et al., 2003).

The achievement gap between students with and without MD has worsened in the last ten years. The results from the 2019 National Assessment of Educational Progress (NAEP) shed light on which student groups are most impacted; students below the 10^{th} percentile in mathematics performed even worse than a decade ago, whereas those at the 25^{th} percentile showed stagnant growth and those at the 50^{th} percentile improved (NAEP, 2019). Rethinking current intervention practices is an important step in repairing this widening achievement gap.

School-based interventions have historically focused on whole number calculations and on remedial content from previous grades (Schumacher et al., 2020). However, this approach may only lead to short-term benefits and leave students unprepared to pass Algebra 1 (Hoffer et al., 2007), a requirement for high school graduation in many states. An alternative approach is to increase the complexity of content provided during intervention so that students are able to access grade-level material, which aligns with the goals of Individuals with Disabilities Education Act [IDEA] (IDEA, 2004). Providing this type of targeted early intervention on complex topics like rational numbers, may help close the achievement gap in middle school, which could better prepare students for high-school mathematics and provide access to college and desired occupations (e.g., nursing, automotive technicians; Hoyles et al., 2001; Sformo, 2008).

Over the last decade, and especially in the last five years, there has been a growing body of high-quality, experimental research focused on rational number interventions that target students with MD in Grades 3–9 (e.g., Barbieri et al., 2019; Dyson et al., 2020; Fuchs, Wang, et al., 2021; Jayanthi et al., 2021). The primary goal of this article is to delve into this research to explore and communicate which aspects of rational number interventions are associated with better outcomes for students with MD (Therrien et al., 2020). To achieve this goal, we used meta-analytic techniques to investigate which *instructional components* (e.g., visual representations, fluency activities, review), *intervention characteristics* (e.g., group size, grade level, duration), and *study design features* (e.g., comparison condition, type of measure) are associated with effective interventions to tease out the "active ingredients" for improving the performance of struggling students.

Prior Syntheses and Meta-analyses Related to the Topic

To date, no one has conducted a meta-analysis focused on rational number interventions for students with MD. However, there have been several meta-analyses focused on students with MD across a range of mathematics topics (e.g., Dennis et al., 2016; Jitendra et al., 2018; Jitendra et al., 2021; Stevens et al., 2018). Fractions interventions, an important aspect of rational numbers, have been the focus of three qualitative reviews (Misquitta, 2011; Roesslein & Codding, 2019; Shin & Bryant, 2015) and two meta-analyses (Ennis & Losinski, 2019; Hwang et al., 2019). Here, we discuss these two meta-analyses and how they are similar to and differ from the current study.

Ennis and Losinski (2019) included 21 studies that focused on fractions interventions for students with or at risk for disabilities in Grades 3–11 published between 1986 to 2017. They synthesized both single-case and group design studies and found a large overall effect (g = 1.17) for fractions interventions, with individual study effects ranging from g = 0.42 to g = 11.51. Using study design as a moderator, authors reported that single-case designs significantly predicted larger effect sizes compared to group design studies. However, the inclusion of both single-case and group design studies in a meta-analysis introduces methodological issues that may inflate effect sizes (Busse et al., 1995), which may have happened in this case.

To understand more about instructional approaches, Ennis and Losinski (2019) coded each study as belonging to one of five overarching instructional categories (e.g., anchored instruction, explicit instruction, strategy instruction). The Council for Exceptional Children Standards for Evidence-Based Practices (CEC EBP; CEC, 2014) were then applied to each set of studies and were used to identify which instructional approaches are grounded in evidence. Explicit instruction was identified as an evidence-based practice after the application of the CEC EBP standards; anchored instruction had mixed evidence.

Hwang et al. (2019) also conducted a meta-analysis on fractions interventions in Grades 3–12 and included 22 group design studies (randomized control trials [RCTs] and quasi-experimental designs [QEDs]) from peer-reviewed journals published between 1990 and 2015. Studies were included if they were implemented in mathematics classes (i.e., whole-class or large-group) and were intended to enhance fractions achievement for all learners. The authors stratified students in their sample into five achievement levels: high-achieving, typical-achieving, low-achieving, students with disabilities, and a mixture of achievement levels in an inclusive setting. Thus, findings are not directly relevant to the set of studies on targeted interventions for students with MD. Authors did, however, conduct a sub-analysis of the impact of Tier 1 rational number instruction on students identified as low-achieving. Like Ennis and Losinski (2019), they categorized studies by their overall instructional approach (e.g., use of multiple representations, computer-based instruction). Authors examined whether effects varied by instructional approach and achievement group. Moderator analysis revealed that instructional approach was significant—particularly, contextualized video instruction and multiple representations—with effect sizes ranging widely (0.60 to 2.27). Additionally, the authors concluded that specific aspects of whole class fractions instruction were more effective for low-achieving students (effect sizes ranged from 0.56 to 1.85) compared to high- and typical-achieving students. Among the student groups, these interventions were least effective for students with disabilities.

Even though the current meta-analysis involves a population similar to the two prior meta-analyses, there are several key differences. First, unlike both meta-analyses, this study includes a larger, more varied set of interventions on all rational number topics (e.g., fractions, decimals, percentages, proportions), and explored practices that may be effective across areas of rational number, rather than focusing only on fractions. Given the importance of learning more advanced rational number topics in anticipation of taking Algebra 1, we wanted to capture interventions that focused on any aspect of rational number learning prior to or during Grade 9, when Algebra is most often taken.

Second, unlike (Hwang et al., 2019) only interventions intended to improve performance for students with MD were included. Students with MD were defined as those scoring below the 35th percentile on a valid screening measure or students with a special education designation from their school. Cut-off at the 35th percentile is commonly used in the literature (e.g., Fuchs, Schumacher, et al., 2016; Malone et al., 2019) to identify students who experience difficulties in mathematics. Even though both prior meta-analyses were interested in students with MD, Ennis and Losinski (2019) did not report use of specific inclusion criterion indicating student performance level (i.e., cut-off score) and Hwang et al. (2019) included study samples from all achievement levels and used achievement level as a potential moderator.

A third difference is that we wanted to include studies that were free of issues related to study quality (e.g., group equivalence at pretest, measure reliability). Even though meta-analyses often include all identified studies regardless of design quality to synthesize as much data on a topic as possible, the inclusion of studies with potential design flaws may compromise the interpretation of meta-analytic findings (Graham et al., 2020). Therefore, the present study only included group design studies that meet the What Works Clearinghouse (WWC) standards (Version 4.0; U.S. Department of Education [U.S. ED] & What Works Clearinghouse [WWC], 2017). This approach was used so that the conclusions drawn from this meta-analysis are taken from the most empirically rigorous evidence available.

Finally, this meta-analysis will include studies available through March 2021, whereas Ennis and Losinski (2019) included studies through 2017 and Hwang et al. (2019) through 2015. This distinction from the prior two meta-analysis is important, given the large number of studies that were conducted on rational number interventions during the past five years. Among the 28 studies that were included, half of the studies (14) were published in 2016 or later. Hence, this meta-analysis will include a more current set of studies than the prior two meta-analyses.

Purpose of the Present Meta-Analysis

The purpose of the present meta-analysis was to evaluate the most recent research on rational number interventions to inform the field about which instructional components are associated with better outcomes for students with MD. To accomplish this, we synthesized the rigorous, empirical research on rational number interventions in Grades 3–9 to identify the "active ingredients" for improving outcomes for students with MD. We investigated which *instructional components* are associated with impacts on mathematical knowledge for students with MD. Additionally, we explored the conditions under which interventions were effective by exploring *intervention characteristics* and *study design features* associated with successful intervention.

Instructional Components

Based on prior research and meta-analyses that have focused on instructional components in mathematics interventions (e.g., Fuchs, Wang, et al., 2021; Gersten, Beckman, et al., 2009; Gersten, Chard, et al., 2009), we identified 12 instructional components of interest. Explicit instruction, visual representations, strategic prompting tools (sometimes referred to as heuristics), review, fluency building activities, ongoing feedback, teaching and use of mathematical language, student explanations, use of number lines, comparison of worked examples, technology delivered intervention, and guided inquiry were explored. We were especially interested in whether any of these components were directly related to rational number learning.

Six of these components—explicit instruction, visual representations, strategic prompting tools, review, fluency activities, ongoing feedback—we expected to recur across the set of studies because they occur in many contemporary mathematics intervention programs (e.g., Fuchs, Newman-Gonchar, et al., 2021; Gersten, Chard, et al., 2009; National Center on Intensive Intervention [NCII] Tools Chart, 2019). Explicit instruction, for example, is often used as an overarching framework of mathematics interventions and has been found to be especially effective for students with MD (Ennis & Losinski, 2017; Gersten, Chard, et al., 2009). Different from other components, it can involve a combination of instructional practices that involve the use of clear and concise explanatory language (i.e., strategic prompting tools, ongoing feedback;

Fuchs et al., 2013).

Teaching mathematical language and supporting student explanations have only been serious components of mathematics interventions more recently, likely coinciding with the focus in contemporary state standards (e.g., CDE, 2013; TEA, 2012) and trends in mathematics education (e.g., Karp et al., 2016) that require students to explain their mathematical understanding using accurate mathematical language. Teaching and using accurate mathematics language and supporting students in their use of this language consistently was included as a recommendation in the recently updated practice guide focused on mathematics interventions (Fuchs, Newman-Gonchar, et al., 2021). Among the set of studies included in this meta-analysis, mathematical language and student explanations were identified only in studies conducted in the past seven years, apart from Xin et al. (2005). Given these components primarily appear in recent studies, examining their association to effects on mathematical outcomes is timely.

The use of number lines to represent and teach rational number magnitude understanding (e.g., determining which is greater, $\frac{4}{12}$ or $\frac{1}{2}$; estimating fractions on a number line with two endpoints) was also examined. Like the teaching and use of mathematical language, we identified the use of number lines only among studies that were conducted more recently. Number lines are increasingly used in U.S. mathematics curricula and intervention programs and have been used for instruction in many Asian curricula for years (e.g., Singapore Mathematics Curriculum; Ministry of Education [MOE], Singapore, 2007). The use of number lines was the only component we explored that is directly related to learning rational numbers, therefore, we were especially interested in the relationship of number lines to student outcomes.

The comparison of worked examples has been included in research from cognitive psychology to promote mathematical analysis and understanding and has been shown to increase learning (Durkin et al., 2017; Durkin & Rittle-Johnson, 2012; Renkl, 2017). This practice was also stressed in the IES practice guides focused on mathematical problem solving (Woodward et al., 2012) and algebra (Star et al., 2015). Hence, exploring the extent of its use among the intervention studies on rational numbers, and whether it was associated with outcomes, was of interest.

Interventions delivered with technology, or a hybrid technology approach were investigated. With greater emphasis on virtual and asynchronous learning due to the COVID-19 pandemic, understanding the role of technology across such a complex mathematics topic, like rational numbers, was of interest.

Guided inquiry—a student-driven approach that is typically aligned to the mathematics educator perspective—was also explored to determine whether it is included in mathematics interventions and whether this component is associated with improved outcomes. In guided inquiry, students are provided a framework for discovering or gaining insight into various mathematical ideas and principles, rather than having the teacher explicitly model and teach (e.g., Carpenter et al., 2015; Smith et al., 2013).

Intervention Characteristics and Study Design Features

We also explored intervention characteristics and study design features that may provide information about the conditions under which interventions were most effective. We explored the level of support interventionists received for implementation, whether intervention was provided by the research team or school personnel, the duration and setting of the intervention, and differences in content, grade-level, and group size. In terms of study design features, the nature of the comparison condition and type of measure were explored. We were interested in whether RCTs and QEDs were associated with different effects, but only one study in the sample used a QED.

Research Questions

Through this meta-analysis, answers to the following research questions were sought to better understand which *instructional components*, *intervention characteristics*, and *study design features* might lead to stronger, weaker, or even non-existent impacts.

1. What are the impacts of rational number interventions on mathematical knowledge for students with MD in Grades 3–9?
2. Do *instructional components* moderate impacts of rational number interventions for students with MD on mathematical knowledge?
3. Do *intervention characteristics* and *study design features* moderate impacts of rational number interventions for students with MD on mathematical knowledge? (Exploratory)

Method

Identification of Studies

We conducted a search of all studies published from January 1987 to March 2021 that focused on rational number interventions for struggling students in Grades 3–9. Eligible studies were published as a dissertation, ERIC document, or an article in a peer-reviewed journal. First, a targeted keyword search of the following databases was conducted using ProQuest: ERIC, PsycINFO, Dissertation Abstracts, and the Social Sciences Citation Index.

To identify all studies that potentially met the eligibility criteria, we searched titles and abstracts of studies using the following set of keywords and their variants, and the Boolean operators "*," "AND," and "OR": *fraction, rational number, proportion, ratio, decimal, intervention, response to intervention, tutor, multi-tiered system of support, third grade, fourth grade, fifth grade, sixth grade, seventh grade, eighth grade, ninth grade, elementary, upper* *elementary, middle school, junior high, studies, experimental, random, experiment*. We then conducted supplementary searches of the bibliographies of relevant screened studies, and prior narrative reviews and meta-analyses to identify studies that may not have been retrieved from the electronic searches. Finally, we solicited recommendations from content experts in the fields of mathematics and special education. These search procedures identified 1,654 candidate reports to be screened for inclusion.

Criteria for Inclusion

Under the supervision and training of the principal investigators, studies were screened for eligibility using the PICOS framework (i.e., population, intervention, comparison, outcomes, and study design). This framework guided the formulation of eligibility criteria and development of literature search strategies.

Intent of the Study

To be eligible, the study needed to be an evaluation of an intervention designed to improve rational number knowledge. For this meta-analysis, the term *intervention* was defined as an instructional program aimed at helping students experiencing difficulties in learning rational number topics. Interventions were eligible when implemented in three instructional settings: (a) the general education classroom, (b) supplemental intervention settings (e.g., pull-out for additional instruction), or (c) special education settings, including resource rooms. Data for those studies with interventions conducted in the general education classroom were reported as subgroup analyses (i.e., data for students with MD were disaggregated from the full sample). Eligible interventions addressed key concepts from previous grades or supported the learning of challenging grade-level material and they were designed with a specific scope and sequence.

Rational number topics included fractions, decimals, percentages, or proportions. If topics unrelated to rational numbers were also part of an intervention, rational number topics were at least 50% of the content. In addition, to meet inclusion criteria, studies needed to provide details about the knowledge and skills that were targeted, the instructional approaches that were used, and how intervention was delivered.

Study Design

Only RCTs and QEDs were eligible for inclusion. Eligible studies were required to include at least one student outcome that assessed general knowledge of rational number topics, procedural computation, arithmetic word problems, or more complex problem solving, and demonstrated sufficient content validity and reliability (i.e., internal consistency of at least 0.50; temporal stability/test-retest reliability of at least 0.40; or inter-rater reliability of at least 0.50; U.S. ED & WWC, 2017).

Study Quality

Only studies demonstrating adequate methodological rigor (i.e., those meeting WWC group design standards) were eligible for inclusion. The WWC group design standards provide a structured review process to appraise the causal validity of findings reported in intervention research. Two WWC-certified reviewers independently reviewed each study following the procedures outlined in the *WWC Procedures and Standards Handbook* (Version 4.0; U.S. ED & WWC, 2017). The WWC standards were chosen because they are an established, rigorous set of standards put forth by the Institute of Education Sciences and interventions passing WWC standards are recommended for school-based use by the Every Student Succeeds Act (2015).

Population

Eligible samples included students in Grades 3–9 who experience mathematics difficulties. Studies that included students identified with a learning disability in mathematics by their school or district or students who scored below the 35th percentile on a valid screening measure of general mathematics knowledge or rational number topic(s) were eligible. Even though some argue that the 25th percentile is a more accurate representation of students with MD (e.g., Swanson et al., 2018), many recent studies use the 35th percentile as a cut-off to increase the likelihood that students who may develop serious mathematics problems are not missed (Hanich et al., 2001; Jordan et al., 2003). Studies were included if authors implemented an intervention with a mix of achievement levels and/or grade levels and presented results that were disaggregated by grade level or achievement status that fit the inclusion criteria.

Screening

We conducted screening using an Excel-based codebook to apply the PICOS framework. During the first screening phase, two study authors independently screened the 1,654 candidate reports for eligibility based on the title, keywords, abstracts, and publication dates; all disagreements were reconciled. A majority of reports (n = 1,553) were excluded, leaving 101 reports to be screened in a second phase using the full text of the article. Two study authors independently reviewed each article for eligibility and all disagreements were reconciled via final determination by the senior study author. Interrater reliability for study inclusion exceeded 95% and was calculated by dividing the number of agreements by the number of agreements plus disagreements multiplied by 100.

After full-text screening, 73 ineligible reports were excluded, primarily due to: (a) a sample that did not include a sufficient number of students meeting eligibility; (b) failure to meet WWC group design standards; (c) lack of relation of intervention content to rational number concepts or operations; (d) ineligibility of research design; or (e) absence of eligible outcomes in the study. A total of 28 studies (with 3,853 unique participants and 90 effect sizes) were deemed eligible for inclusion in the final meta-analysis (see Figure 1).

The 28 studies included in the analysis represent interventions that differ on several intervention characteristics and study design features (see Table 1). Sample sizes ranged from 22 to 755 students, with publication dates ranging from 1993 to 2021. Of the included studies, 25 were journal articles, and three were dissertations.

Study Coding

All instructional components, intervention characteristics, and study design features were independently coded by two researchers. Reliability exceeded 95%; all discrepancies were reconciled with 100% agreement.

Coding for Instructional Components

Studies were coded using an Excel-based codebook. The presence or absence of the instructional components were coded by relying on authors' descriptions of the interventions. Operational definitions for each instructional component are included in Table 2. Only eight of the instructional components were examined in the analysis. We were unable to reliably code for some instructional components we were initially interested in exploring (e.g., ongoing feedback, guided inquiry). Two components (e.g., technology delivered intervention, comparing worked examples) did not occur in a sufficient number of studies (i.e., 10 or more studies) to be included in the analyses (Borenstein et al., 2009). The instructional components included in our analysis varied across the set of studies (see Table 3). If authors reported an instructional component to have occurred in both treatment and comparison conditions, it was not included in the analysis.

Coding for Intervention Characteristics and Study Design Features

Data were extracted for intervention characteristics and study design features to be explored as potential moderators or used as control variables in the meta-analysis. For grade level, the sample of studies was divided evenly between students in Grades 3–6 (*n* = 14) and Grades 7–9 (*n* = 14). We grouped grade levels in this manner because Grades 7–9 require higher level rational number concepts involving proportional reasoning, measurement, or solving word problems, and the building of early rational number concepts continues through Grades 3–6, especially for students with MD.

Five interventions were implemented in general education classes and authors presented findings disaggregated for students with MD, eight were implemented in special education settings, and 15 were supplemental to core instruction and implemented outside of a typical classroom structure (e.g., during an intervention block, during optimal times identified by the teacher). Small groups of 2–6 students were used in 17 studies and large groups of six or more students were used in nine studies—two studies delivered instruction individually via computer. Interventions were provided by the research team in 15 studies, by school personnel in eight studies, by computer in two studies, and three studies included a mix of research team staff, school-based personnel, or computer-delivered instruction.

The duration of the initial interventionist training (prior to the beginning of intervention implementation) was reported as more than one day (8 hours) in 17 studies, less than or equal to one day in six studies and did not occur or was not reported in five studies. Ongoing training (i.e., implementation monitoring/feedback that continued throughout intervention) was reported in 15 studies. Intervention length varied across studies. Fifteen interventions lasted more than 20 hours, seven were less than nine hours, and six occurred between 10–19 hours. Content focused on fractions-only instruction in 19 studies, while the remaining nine studies also included decimals, ratios, or proportions. Sixteen studies targeted foundational knowledge combined with skills related to grade-level content (i.e., integrated content), six focused only on foundational content, and six focused only on grade-level content.

Twenty-two studies meet WWC standards without reservations (i.e., RCTs with low attrition and no design flaws). The remaining six studies meet WWC standards with reservations (i.e., QEDs or RCTs with high attrition and/or a lack of equivalence at baseline). Except for Turner (2012), a QED, all studies were RCTs. For 20 studies, treatment was compared to a business-as-usual (BAU) condition. In eight studies, the comparison condition used an alternative treatment condition implemented by the research team and in five of those studies the contrast between treatments was minimal (e.g., in Morano et al., 2020, the content sequence differed across conditions, yet the instructional components were identical). Those five studies were coded as having a closely aligned alternative treatment for the analysis. The other three were grouped with BAU for the analysis because the comparison condition was different from the treatment.

Data Analysis

Meta-Analytic Procedures

The outcomes of interest in the meta-analysis were calculated using means and pooled standard deviations, which includes an adjustment for small samples (Hedges, 1981). All positive effect size values indicate stronger performance on outcome measures.

Most studies in our sample (n = 25) provided multiple effect sizes relevant for the meta-analysis. To include all effect sizes from each study and account for dependencies within the dataset, random-effects robust variance estimation (RVE; Hedges et al., 2010) was used. This approach handles clustered data (i.e., effect sizes nested within studies or samples) by applying a correction to the standard errors to account for the correlations between effect sizes from the same sample. When studies included multiple treatment arms and shared the same control group, generally the two treatments were aggregated for the analyses when they both included the same instructional components. For example, in Malone et al. (2019), 25 of the 30 minutes was identical; one treatment devoted an extra five min to word problem instruction and the other treatment to decimals. The common portion of intervention included the instructional components explored in the meta-analysis. However, in two studies, aggregating treatments did not make sense because the differing portion of the two treatments included one of the instructional components. In those cases, the most relevant treatment condition was chosen for the meta-analysis. For example, Fuchs, Malone, et al. (2016) delivered the same core intervention across two conditions and one condition had an additional focus on mathematical language and student explanations. Because of our interest in those two components, we included effects for the condition with mathematical language and student explanations in the analysis and did not aggregate.

Robust variance estimation does not result in the loss of any information, does not require knowledge of the underlying correlation structure, and can accommodate multiple sources of dependencies. Thus, RVE was used to estimate the overall effect size and to conduct moderator analyses. All RVE analyses were run in Stata version 16.0 (StataCorp, 2019) using the *robumeta* package (Hedberg, 2014). *Robumeta* provides a small-sample correction, allowing RVE to estimate meta-regression models even when the sample includes fewer than 40 studies and provides trustworthy p values if the degrees of freedom are larger than four (Tanner-Smith & Tipton, 2013). Because RVE requires an estimate of the mean correlation (ρ) between all pairs of effect sizes within a cluster (i.e., study) to be specified, we first conducted sensitivity analyses using ρ values of 0 to .90. Results showed that findings were robust across differing reasonable estimates of ρ. Thus, we estimated τ^2 using a value of .80 (Tanner-Smith & Tipton, 2013).

Results

Overall Intervention Effect

To answer the first research question on overall impact, we used the intercept from the RVE meta-regression models to estimate the mean effect sizes and examined heterogeneity by using τ^2 as the between-study variance component. All analyses used random-effects modeling to account for the expected heterogeneity in effect sizes.

The estimate of the mean effect size across all 28 studies (90 effect sizes) included in the analysis was 0.68 and differed statistically significantly from zero (SE = 0.04, $p < .001$, 95% CI [0.51, 0.85]). The τ^2 estimate of the true variance in the population of effects was 0.28. Students receiving rational number interventions had larger impacts than those in control conditions.

Efficacy of Instructional Components

To better understand the associations between instructional components that relate to the efficacy of rational number interventions and effect sizes, as posed in research question two, we examined eight instructional components. We estimated a series of both multivariate and univariate random-effects meta-regression models using *robumeta* (Hedberg, 2014). Only instructional components for which we could identify 10 or more studies were included (Deeks et al., 2021). First, we examined through univariate meta-regression, the significance of each instructional variable alone. Explicit instruction, student explanations, fluency activities, mathematical language, strategic prompting tools, visual representations, and the number line were all significant moderators of outcomes ($p < .05$). Review of previously taught content was not significant ($p = .14$).

Second, many of the studies examined multi-component interventions (e.g., a study may have included the teaching and use of mathematical language, fluency activities, and review [see

Table 3]); therefore, to account for collinearity among the instructional components, the correlations among the eight instructional variables were examined (see Table 4). It is common practice to consider two variables to be collinear if they have a pairwise correlation of .80 or greater. Collinearity at this level can lead to unstable and unreliable estimates of the regression coefficients and variables with high correlations should not be included in the same model.

We also tested for indicators of multicollinearity, which may be present when at least two highly correlated predictors are included simultaneously in a regression model. We examined multicollinearity among all eight instructional variables using a variance inflation factor (VIF) threshold of 10 (Vittinghoff et al., 2012). A VIF above 10 suggests high correlations and is cause for concern. Five instructional variables—student explanations, review, teaching and use of mathematical language, fluency activities, and visual representations—had VIF values below 10 and three variables were excluded due to VIF values higher than 10: number line, strategic prompting tools, and explicit instruction. When examined with these five variables alone, the individual VIF values were each below 5 and the mean VIF was 2.38. A mean VIF threshold of 2.5 is recommended when detecting multicollinearity in multivariate meta-regression models (Johnston et al., 2018).

Third, a multivariate meta-regression model with RVE was used to assess the possible moderating effects of the five instructional components within a single model while controlling for important study design features and intervention characteristics (Pigott & Polanin, 2019). The control variables included in the model were group size, duration of the intervention, and whether the study used a closely aligned alternative treatment condition for the comparison. These variables were chosen as controls because prior evidence has demonstrated that group size (e.g., Jitendra et al., 2021) and duration of intervention (e.g., Jitendra et al., 2018; Stevens et al.,

2018) are associated with increased outcomes. When the comparison condition is closely aligned to the treatment, effect sizes tend to be smaller (e.g., Gersten, Chard, et al., 2009).

As indicated by the mean effect sizes (i.e., the intercepts from the RVE meta-regression model; see Table 5), most of the instructional components were non-significant ($p > .05$) when included together, with the exception of mathematical language ($p = .042$). Twelve studies with 51 effect sizes included mathematical language. Studies that included this instructional component were associated with statistically significant positive effects on the outcomes.

Fourth, sensitivity analyses were conducted by analyzing each instructional component separately, with the three control variables from the meta-regression (group size, duration of the intervention, and comparison condition). In these sensitivity analyses, only mathematical language remained significant ($p = .026$), demonstrating the same finding as the meta-regression that included all instructional variables.

Exploratory Moderator Analysis of Intervention Characteristics and Study Design Features

A series of univariate RVE meta-regression models were run as exploratory analyses to address research question three. When variables included more than two categories, one was chosen as the *reference category*. Univariate analyses were conducted rather than multivariate analyses due to the large number of variables of interest.

Eleven categorial moderator variables related to intervention characteristics and study design features were explored. Significant moderator effects related to intervention characteristics and study design features were found (see Table 6). Here we report only those moderators statistically significant at a Bonferroni corrected critical p value of .0045.

Studies with students in elementary school (Grades 3–6) had larger effects than those for students in middle school (Grades 7–9; $p < .001$). Interventions delivered in small groups had statistically significantly larger effects than those delivered in large groups ($p < .0045$).

Interventions delivered by research project personnel were more effective ($p < .0045$) than those delivered by school personnel. Interventions longer than nine hours (i.e., interventions 10–19 hours and interventions 20 hours or longer) were more effective than shorter interventions (0–9 hours); however, only interventions lasting 20 hours or longer was statistically significant ($p < .001$). Interventions for which the interventionists participated in ongoing training were more effective than those without ongoing training ($p < .001$). Finally, interventions that were contrasted against a closely aligned alternative treatment yielded smaller effects ($p < .0045$).

Risk of Bias

Publication Bias. To assess the potential presence of publication bias, we constructed a funnel plot to note any asymmetry in the distribution of effects, conducted trim-and-fill analyses (Duval & Tweedie, 2000), and conducted Egger's regression tests (Egger et al., 1997). Results from Egger's regression tests were non-significant and provided no evidence of small-study bias ($p = .06$). Trim-and fill analyses also provided no strong evidence of publication bias, such that the average effect sizes for intervention were substantively unchanged (effect-size estimate = 0.71, $p < .001$). All analyses were run in Stata version 16.0 (StataCorp, 2019) using the *metafunnel*, *meta trimfill,* and *metabias* commands.

Other Sources of Potential Bias. One anomaly among the 28 studies available for this meta-analysis is the large representation from certain research groups (see Table 1); for example, those of Fuchs and colleagues (i.e., Fuchs, Wang, Malone; $n = 8$) and Bottge and colleagues ($n = 5$). Since research groups tended to use similar instructional components across their sets of studies, we ran a univariate meta-regression on whether *research group*—defined as two or more studies conducted by the same research team—moderated effect sizes. Research group showed no significant differences (*p* = .58, 95% CI [-0.08, 0.13]). As such, sources of potential bias among the studies are likely only a minor concern.

This meta-analysis adheres to the Preferred Reporting Items for Systematic Review and Meta-Analysis (PRISMA) statement (Moher et al., 2009). The methods were based on a protocol developed prior to data collection and analysis.

Discussion

Enhancing rational number knowledge for students with MD has become a vibrant topic for educational research over the past decade. This meta-analysis synthesized the rigorous, empirical evidence that evaluated rational number interventions for students with MD in Grades 3–9. Given that a working knowledge of rational number concepts appears vital to achieve success in algebra and other advanced mathematics classes (NMAP, 2008; Siegler et al., 2012), a current meta-analysis on this topic is valuable to communicate to educators what is effective for students with MD. Studies were published from 1987 to March of 2021.

With respect to ***research question* 1**, meta-analytic findings indicated that the set of interventions demonstrates a rather large and statistically significant and positive impact on students' performance with a mean effect size of 0.68. Recent literature on interpreting effect sizes of education interventions provides evidence that an effect size greater than 0.20 can be considered large and educationally meaningful (e.g., Kraft, 2020). The 0.68 effect size from the current study is somewhat larger than other syntheses of mathematics intervention that address all mathematics topics (*g* = 0.53, Dennis et al., 2016; *g* = 0.37, Jitendra et al., 2018; *g* = 0.41, Jitendra et al., 2021; *g* = 0.49, Stevens et al., 2018). This large effect size for rational number intervention aligns to the finding from Stevens et al. (2018) where fractions interventions were associated with higher impacts among their set of upper elementary and secondary mathematics interventions.

Instructional Components Associated with Stronger Intervention Effects

A fundamental purpose of this meta-analysis, as posed in ***research question 2***, was to examine instructional components that are frequently associated with effective mathematics interventions on rational number knowledge. When each component was analyzed by itself in univariate meta-regression analyses, each was statistically significant ($p < .05$), except for review ($p = .14$). Due to issues with multicollinearity, three instructional components (explicit instruction, strategic prompting tools, and number lines) were excluded from the final model, leaving a set of five instructional components (i.e., visual representations, mathematical language, review, student explanations, and fluency activities) for the analysis.

In the multivariate meta-regression focused on the set of five instructional components, only one component—mathematical language—significantly moderated effects ($b = 0.50$, $p = .042$). This significant finding for mathematical language signals that mathematical language remains a strong predictor of student outcomes even when accounting for the inclusion of four other effective instructional components.

The approach to examining instructional approaches using moderator analysis, was similar to Gersten, Chard, et al. (2009) in their meta-analysis of mathematics interventions for students with MD. In that study, explicit instruction and use of heuristics significantly moderated effects; meaning, they appeared to boost student outcomes. Visual representations, student verbalizations, and sequence of examples, were not significant on their own in the full model; however, visual representations was found significant when combined with explicit instruction.

The current study included a related, yet somewhat different set of instructional components from Gersten et al. (2009) that reflect current trends in intervention research. We speculate that explicit instruction and strategic prompting tools (similar to heuristics)—both of which were statistically significant in Gersten, Chard, et al.—may not have been statistically significant in our model because they recurred in a large portion of the studies (see Table 3), which decreases the variability of those components and the association to impacts is likely dampened. In Gersten, Chard, et al., out of 42 studies, only four studies included heuristics and 11 studies included explicit instruction. Since 2009 more mathematics intervention research has been conducted that is grounded in explicit instruction and utilizes strategic prompting tools. It is important to note that these two components were significant moderators when analyzed individually and should not be discounted as important components of effective interventions.

Use of number lines to teach magnitude was of key interest because of its direct relationship to understanding rational number magnitude and its increased use across recently conducted research (e.g., Barbieri et al., 2019; Fuchs, Wang, et al., 2021; Jayanthi et al., 2021). As mentioned, number lines were not available for the meta-regression examining the set of instructional components because of collinearity with other instructional components (see Table 4). When tested alone, number lines were statistically significant ($b = 0.61$, $p < .001$) and were not statistically significant ($b = 0.43$, $p = .12$) with control variables (i.e., group size, duration, nature of comparison condition). This was somewhat surprising given the established relationship between estimating fractions magnitude on number lines and overall understanding of fractions concepts (Resnick et al., 2016; Siegler et al., 2012).

To further explore the relationship of number lines to magnitude understanding specifically, we tested the association of number lines to outcomes focused only on magnitude understanding. Among the set of studies that included number lines, the effect size for assessments of magnitude understanding was large and significant ($b = 1.02$, $p = .01$). This strong relationship further demonstrates the important role number lines play for teaching rational number magnitude. When more studies become available in the future, we anticipate gaining a better understanding of the role number lines play within multi-component rational number interventions.

Intervention Characteristics and Study Design Features as Potential Moderators

To address ***research question 3***, exploratory analyses were conducted to better understand the conditions under which rational number interventions were effective. Each variable was modeled separately without control variables. This approach was chosen to explore the associations of each variable to student outcomes.

Grade Level, Group Size, and Setting

Interventions in the elementary grades (Grades 3–6) were more effective than those in middle school (Grades 7–9), and those conducted in smaller groups (2–6 students) were more effective than those conducted in larger groups (more than six students). The finding for grade level aligns with prior work, which shows that elementary students make greater gains than secondary students in many content areas (Bloom et al., 2008). In this set of studies, most large-group interventions were conducted at the middle school level, and most small-group interventions were conducted in the elementary grades, reflecting current common practice in schools (Hollo & Hirn, 2015). Reduction in instructional group size has often been shown to improve outcomes for struggling learners at the elementary level (e.g., Lou et al., 1996) and may also be important for students in middle school. However, secondary mathematics intervention is often provided in large groups as an elective due to scheduling and logistical needs (e.g., Nomi & Allensworth, 2009). Therefore, the reality of providing intervention in small groups is likely a barrier for many middle schools.

There was no evidence of moderator effects related to instructional setting (i.e., general education classrooms, special education classrooms, or pull-out supplemental interventions provided during intervention blocks). The lack of significance for setting could indicate that setting does not matter for achieving robust outcomes if the intervention is of high quality and includes effective instructional components or other aspects of effective intervention (e.g., small group size, sufficient duration of 19+ hours).

Intervention Delivery, Training, and Ongoing Support

Interventions provided by researchers were found to be more effective than those provided by school-based personnel, which has been found in other meta-analyses for interventions focused on both reading (e.g., Wanzek et al., 2010) and mathematics (e.g., Dennis et al., 2016). One explanation may be that researchers are focused on providing intervention with fidelity and may devote more time to prepare and practice each lesson (Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2016; Jayanthi et al., 2021). School-based personnel likely have less time to devote to planning and practice because they teach a variety of classes and topics during the day and have other responsibilities at the school.

Although the amount of time spent on training prior to the intervention was not a statistically significant moderator, the provision of ongoing training throughout intervention (e.g., frequent meetings, observations, and feedback) did statistically significantly moderate effects. A similar finding in early reading intervention was found Gersten et al. (2020).

Of the 13 studies that included ongoing training of interventionists, 10 were provided by research project personnel, two by school-based personnel, and one included a combination of researcher and school-based personnel. Perhaps school-based interventionists might have been more effective had they received ongoing support from the research team during intervention.

Duration

Interventions that lasted more than 19 hours were more effective than those lasting fewer than 19 hours. This resonates with prior work (e.g., Kidron & Lindsay, 2014) demonstrating the importance of increased or additional learning time for struggling students. We hypothesize that adequate time may be especially important for the material on rational numbers, because it is among the most challenging for American students (NMAP, 2008).

The Nature of the Control Condition

Only five studies included a closely related alternative treatment condition. These comparison groups varied from the experimental treatment in subtle ways, often to answer a specific question of practice. Studies that utilized a closely related comparison condition significantly predicted smaller effect sizes ($b = -0.50$, $p = .004$). This finding was not surprising, given the relatively minor differences between the treatment and comparison groups.

Type of Measure

Researcher-developed measures tend to focus on content covered in depth by the intervention. By contrast, independently developed measures often cover a wider range of mathematics topics and include content that was not a focus of the intervention. We expected researcher-developed measures to demonstrate greater impacts than measures that were independent (Cheung & Slavin, 2016), as some recent reviews of research have found (e.g., Jitendra et al., 2021; Pellegrini et al., 2018; Wolf et al., 2020). However, this variable did not significantly moderate effects in this study. One potential reason for this was the application of RVE which controls for multiple effect sizes for the same study. Gersten et al. (2020), a meta-analysis on reading interventions, used RVE and had similar results. When using meta-regression without RVE for a sensitivity analysis, researcher-developed measures did significantly predict larger effect sizes (*b* = 0.27, *p* = .03). The application of RVE is one theory for these non-significant findings; yet, Jitendra et al. (2021) used RVE and the type of measure moderated effects in their study. Whether or not the application of RVE is related to the moderator analysis on measures is still unclear given the mixed pattern of results.

Limitations

Due to the complex nature of these interventions and the considerable variation in details provided on instructional components within the available research, accurately identifying intervention components presented some challenges. Therefore, it is possible that authors may have included an instructional component we did not identify from the description in the study.

Operational definitions of instructional components were developed for coding purposes so that the presence or absence of a component was identified systematically across studies. Even still, coding some variables was difficult due to the variations in author descriptions. For example, whether intervention emphasized mathematical language was difficult to identify through some author descriptions, so we refined the operational definition. For mathematical language, the definition specified two approaches: (a) teaching mathematical vocabulary (b) teaching specific terminology used in word problems to explain mathematical relationships. Refining definitions helped increase agreement and coding reliability. This approach may also have missed studies that taught mathematical language.

The studies that used an alternative intervention for the comparison group sometimes included instructional components that could not be analyzed due to their presence in the both conditions. For example, in Xin et al. (2005) both treatment conditions taught word problems and used some overlapping features (explicit instruction, strategic prompting tools). Also, in Morano et al. (2020), both conditions were focused on concrete and semi-concrete representations and the order of their presentation was the key difference; therefore, coding for visual representation in the analysis did not seem appropriate since both treatments included this component. Additionally, author descriptions of BAU conditions in many studies lacked sufficient detail to reliably code for the presence or absence of the instructional components. These limitations in coding emerge across educational research and likely occur because access to the instruction provided in the comparison condition is limited, page limits in journals don't allow for full intervention descriptions, and authors describe similar intervention practices in different ways.

Practical Implications

Determining which interventions to implement and which instructional practices and mathematical content to prioritize during intervention can be difficult for schools and teachers. The results from this meta-analysis indicate that there are several high-quality, effective interventions focused on rational number that could be useful for enhancing instruction for students with MD. Specifically, findings suggest that intervention programs devoting time to teaching mathematical language can substantially enhance student outcomes.

Mathematical language is a type of abstract, academic language—terms such as *equivalent*, *unit-fraction*, *reciprocal, circumference*—that represents mathematical ideas accurately. The studies we included represent an assortment of different approaches for learning and developing the language of mathematics. Some studies focused on understanding difficult mathematical language used in word problems (e.g., Jitendra et al., 2017), while others focused on learning accurate mathematical terminology and vocabulary (e.g., Fuchs, Malone, et al., 2016; Jayanthi et al., 2021) to help students understand and explain the mathematics they are doing when solving problems. For example, in Fuchs, Malone, et al. (2016), instruction on specific terms (i.e., equivalent, magnitude) was used when teaching students to compare the relative magnitude of fractions and students were supported in using these words when explaining their thinking.

When students understand and use mathematical language accurately, it is believed that students will more deeply understand the mathematics they are learning (Purpura et al., 2017). Students who have a poor grasp of mathematical language tend to overly use pronouns or use incorrect or vague words when explaining their mathematical reasoning (Schumacher et al., 2018). Therefore, the findings suggest that teachers should encourage their students to use mathematical language when responding to questions and when providing explanations of their work. One support that teachers might use is a *word wall* to serve as a reminder of the precise mathematical language they have been learning (e.g., Jayanthi et al., 2021). Another support might be a verbal prompt or question that helps steer students toward a more precise and well-reasoned explanation for their decisions made in problem solving (e.g., Fuchs, Malone, et al., 2016).

Connecting the mathematical language students are learning across intervention and general mathematics settings may bolster students' overall learning of the language of mathematics (Karp et al., 2016). Building students' mathematical language requires teachers across settings to integrate mathematical language consistently and to offer their students frequent opportunities to use mathematical language in their verbal and written explanations of mathematical concepts.

Contemporary state standards require that students explain their reasoning when solving a mathematics problem, using mathematical language (e.g., CDE, 2013; TEA, 2012). Thus, these two instructional components are related in that student explanations should include a focus on mathematical language. It was surprising that student explanations were not significantly associated with larger effects in the full model ($b = -0.02$, $p = .97$). Student explanations was, however, significant when analyzed alone ($b = 0.47$, $p < .001$). Thus, we urge future researchers to continue to include instruction on mathematical language and student explanations during intervention, because it is likely that both of these components play an important role when used together to teach rational number concepts.

Exploring the conditions under which intervention was effective may help schools improve upon their current intervention implementation. To no surprise, we found that intervention provided in small groups (between 2–6 students) as opposed to large groups (more than 6 students) was most effective. Additionally, students who received intervention for 19 hours or more had greater learning outcomes. A recommendation here might be for schools to frequently schedule intervention, over an extended period, in small groups. Yet, decreasing group size and elongating intervention time may not be available to some schools due to lack of resources. Even though cost and resource demands may increase, in the end this could be more cost efficient if more students pass algebra coursework and fewer remedial courses in high school are needed.

Even still, changing group size and duration is unlikely to increase learning in the absence of effective intervention. So, what should schools do to change the learning trajectories of their students? One suggestion might be to focus on identifying and adopting interventions with strong evidence and to embrace and acknowledge the fact that most effective intervention takes time, training, and investment to implement well.

Conclusion

The present study offers insights into which instructional components are associated with improved outcomes for students with MD when working with complex rational number material. Through the systematic investigation of “active ingredients” within multi-component, rational number interventions, mathematical language emerged as critical. We advocate for mathematical language to be considered a critical aspect of rational number interventions and be used in combination with other important, effective instructional components that are linked to improved student learning. Investigating which combination of instructional components are associated with improved outcomes for students with MD needs further research and attention.

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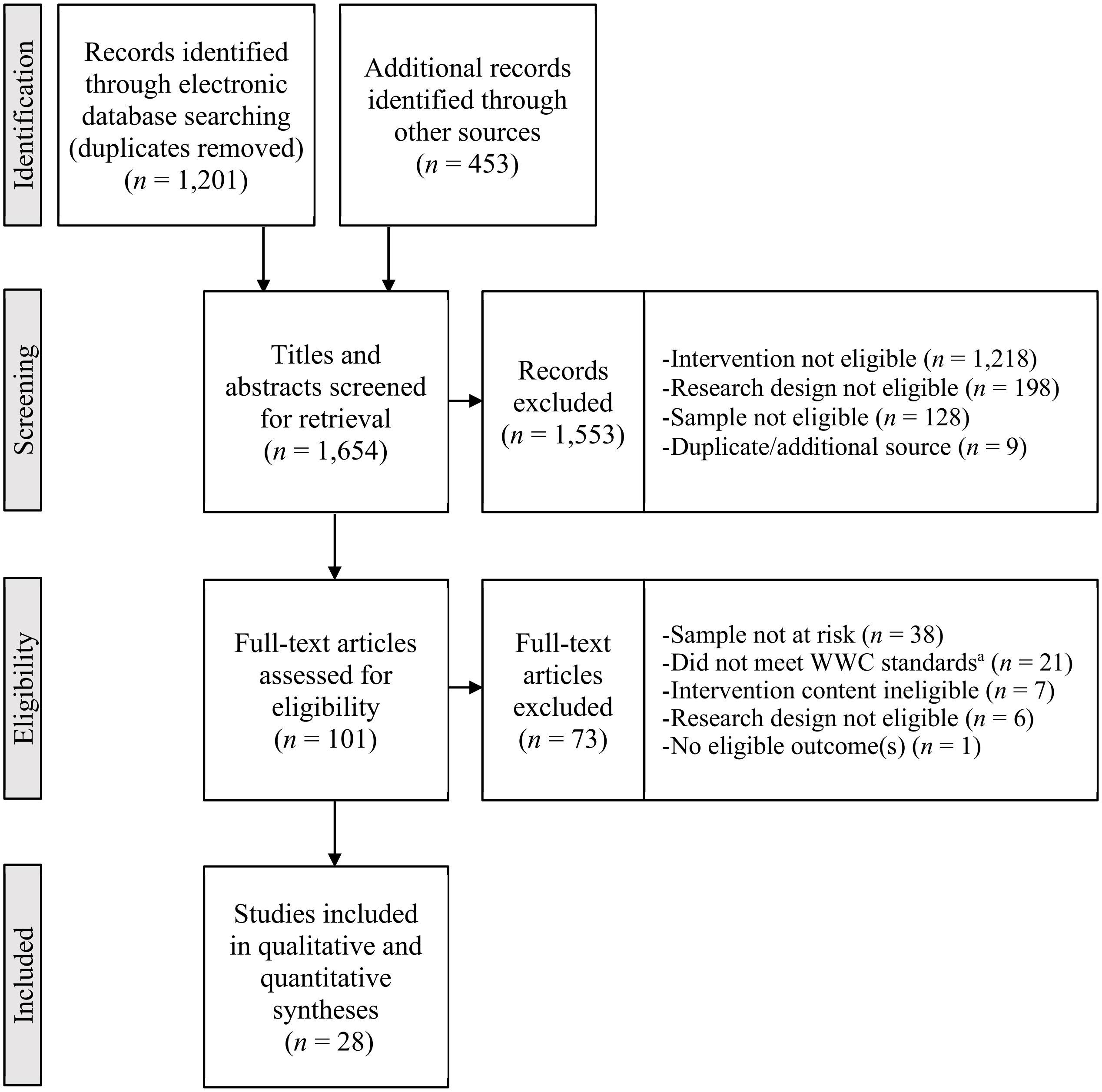
Figure 1

Study Identification Flow Diagram Following PRISMA Guidelines



[a] These studies were randomized controlled trials with high attrition or a quasi-experimental design studies with analysis groups that were not shown to be equivalent.

Table 1

Studies Included in the Meta-Analysis and Their Intervention Characteristics and Study Design Features

Author	*N*	Grade level	Group size	Setting	Interve-ntionist	Content	Content level	Initial training	Ongoing training	Interv-ention Duration	Control
Adams et al. (2014)	107	7th–9th	Ind	Supp	Virtual	D	Int	None	None	≤ 9	ALT
Barbieri et al. (2019)	51	3rd–6th	SG	Supp	RT	F	Fnd	>1 day	None	≥ 20	BAU
Bottge et al. (1993)	29	7th–9th	SG	SpEd	RT, SB	F	GL	None	None	≤ 9	ALT[a]
Bottge et al. (2007)	90	7th–9th	LG	SpEd	SB	D, RR	GL	>1 day	None	≥ 20	BAU
Bottge et al. (2010)	54	7th–9th	LG	SpEd	SB, Virtual	F	GL	>1 day	None	≥ 20	ALT[a]
Bottge et al. (2014)	317	7th–9th	LG	SpEd	SB	F, D, Per, Pro, RR	Int	>1 day	None	≥ 20	BAU
Bottge et al. (2015)	123	7th–9th	LG	GenEd	SB	F, D, Per, Pro, RR	Int	>1 day	None	≥ 20	BAU
Butler et al. (2003)	50	7th–9th	LG	SpEd	SB	F	Fnd	<1 day	Yes	≤ 9	ALT
Dyson et al. (2018)	52	3rd–6th	SG	Supp	RT	F	Fnd	<1 day	Yes	10−19	BAU
Fuchs et al. (2013)	259	3rd–6th	SG	Supp	RT	F	Int	>1 day	Yes	10−19	BAU
Fuchs et al. (2014)	164	3rd–6th	SG	Supp	RT	F	Int	>1 day	Yes	10−19	BAU
Fuchs, Malone et al. (2016)	143	3rd–6th	SG	Supp	RT	F	Int	>1 day	Yes	≥ 20	BAU
Fuchs, Schumacher et al. (2016)	213	3rd–6th	SG	Supp	RT	F	Int	>1 day	Yes	≥ 20	BAU
Fuchs et al. (2019)	143	3rd–6th	SG	Supp	RT	F	Int	>1 day	Yes	≥ 20	BAU
Fuchs et al. (2021)	84	3rd–6th	SG	Supp	RT	F	Int	>1 day	Yes	≥ 20	BAU

Author	*N*	Grade level	Group size	Setting	Interve-ntionist	Content	Content level	Initial training	Ongoing training	Interv-ention Duration	Control
Hughes (2011)	35	7th–9th	LG	GenEd	SB	F	Fnd	<1 day	Yes	10−19	BAU
Jayanthi et al. (2021)	185	3rd–6th	SG	Supp	RT	F	Int	>1 day	Yes	≥ 20	BAU
Jitendra et al. (2016)	260	7th–9th	LG	GenEd	SB	Per, Pro, RR	GL	>1 day	None	≥ 20	BAU
Jitendra et al. (2017)	755	7th–9th	LG	GenEd	SB	Per, Pro, RR	GL	>1 day	None	10−19	BAU
Kiuhara et al. (2020)	90	3rd–6th	SG	SpEd	RT	F	Int	>1 day	Yes	≥ 20	BAU
Malone et al. (2019)	152	3rd–6th	SG	Supp	RT	F, D	Int	>1 day	Yes	≥ 20	BAU
McLaren et al. (2015)	200	7th–9th	Ind	Supp	Virtual	D	Int	None	None	≤ 9	ALT
Morano et al. (2020)	28	3rd–6th	SG	SpEd	RT	F	Int	<1 day	None	≤ 9	ALT
Turner (2012)	88	7th–9th	LG	GenEd	SB	F	Fnd	NR	None	≥ 20	BAU
Wang et al. (2019)	84	3rd–6th	SG	Supp	RT	F	Int	>1 day	Yes	≥ 20	BAU
Watt et al. (2016)	32	7th–9th	SG	Supp	RT	F	Int	<1 day	None	≤ 9	BAU
Westenskow (2012)	43	3rd–6th	SG	Supp	RT	F	Fnd	NR	Yes	≤ 9	ALT
Xin et al. (2005)	22	7th–9th	SG	SpEd	RT, SB	F, Pro, RR	GL	<1 day	Yes	10−19	ALT[a]

Note. 1 day = 8 hours; ALT = alternative treatment; BAU = business-as-usual; D = decimals; F = fractions; Fnd = foundational; GenEd = general education; GL = grade level; Ind = individual; Int = integrated; LG = large group; NR = not reported; Per = percentages; Pro = proportions; RR = rates and ratios; RT = research team; SB = school-based; SG = small group; SpEd = special education; Supp = supplemental.

[a] These three studies were coded as BAU for the analysis because the comparison condition differed in several ways from treatment.

Table 2

Operational Definitions for Instructional Components

Variable	Definition
Explicit Instruction	Yes = Intervention was delivered by an interventionist. Instruction included at least one of the following features: clear explanatory language, the teaching and modeling of concepts and/or procedures, or scripted protocol. No = Intervention was constructivist in nature, computer-mediated intervention, discovery/explore description, student-driven rather than teacher-driven.
Fluency Activities	Yes = Intervention included timed activities for building students' fluency. No = Authors did not report including timed fluency activities.
Mathematical Language	Yes = Intervention included instruction on mathematical language and was described as teaching and using specific words to describe rational number concepts or learning the meaning of mathematical language commonly found in word problems that represents numerical relationships. No = Authors did not report teaching mathematical language.
Number Lines	Yes = Intervention incorporated the use of the number lines to teach rational number magnitude understanding. No = Authors did not report the use of a number lines to teach rational number magnitude during intervention.
Review	Yes = Intervention was described as including review of previously taught content. No = Authors did not report including review of previously taught content.
Strategic Prompting Tools	Yes = Intervention included cue card or notes, a step-by-step strategy, a monitoring strategy, strategic prompt cards, strategic cards, or a description of a repeated process for approaching mathematical tasks. No = Authors did not report the use of prompting tools or repeating a process for solving mathematical tasks during intervention.
Student Explanations	Yes = Intervention had students provide verbal or written explanations of their solutions to solving mathematical problems. No = Authors did not report students providing verbal or written explanations of their solutions to solving mathematical problems.
Visual Representations	Yes = Intervention included use of *both* concrete (3-dimensional; e.g., fraction tiles) and semi-concrete (2-dimensional; e.g., number lines, shaded models) representations to teach mathematical concepts. No = Authors did not report using representations or reported use of either concrete or semi-concrete, rather than both, to teach concepts.

Table 3

Instructional Components Included in Each Study

	Explicit instruction	Fluency activities	Mathematical language	Number lines	Review	Strategic prompting tools	Student explanations	Visual representations
Adams et al. (2014)								
Barbieri et al. (2019)	X	X		X	X	X	X	X
Bottge et al. (1993)					X			
Bottge et al. (2007)								X
Bottge et al. (2010)					X			X
Bottge et al. (2014)								X
Bottge et al. (2015)								X
Butler et al. (2003)								X
Dyson et al. (2018)	X	X		X	X	X	X	X
Fuchs et al. (2013)	X	X	X	X	X	X		X
Fuchs et al. (2014)	X	X[a]	X	X	X	X	X	X
Fuchs, Malone et al. (2016)	X	X	X[a]	X	X	X	X[a]	X
Fuchs, Schumacher et al. (2016)	X	X	X	X	X	X	X	X
Fuchs et al. (2019)	X	X	X	X	X	X	X	X
Fuchs et al. (2021)	X	X	X	X	X	X	X	X
Hughes (2011)	X				X			X
Jayanthi et al. (2021)	X		X	X	X	X	X	X
Jitendra et al. (2016)	X				X	X		
Jitendra et al. (2017)	X				X	X		
Kiuhara et al. (2020)	X		X	X	X	X	X	X
Malone et al. (2019)	X	X	X	X	X	X	X	X
McLaren et al. (2015)								
Morano et al. (2020)								
Turner (2012)								X
Wang et al. (2019)	X	X	X	X	X	X	X	X
Watt et al. (2016)	X		X		X			X
Westenskow (2012)								
Xin et al. (2005)			X					

Note. If an intervention used an instructional component in both the treatment and comparison conditions, the study was not considered to explore the effect of the component.

[a] This is a multi-contrast study and only one contrast included the instructional component.

Table 4

Correlation Matrix for Instructional Components and Controls

	Effect Size	Comparison condition	Duration	Explicit instruction	Group size	Number line	Review	Strategic prompting tools	Student explanations	Mathematical language	Fluency activities	Visual representations
Effect Size	1.00											
Comparison condition	−0.33	1.00										
Duration	0.24	−0.75	1.00									
Explicit instruction	0.42	−0.53	0.47	1.00								
Group size	0.30	0.16	−0.22	0.38	1.00							
Number line	0.51	−0.47	0.50	0.89	0.50	1.00						
Review	0.30	−0.63	0.42	0.83	0.28	0.74	1.00					
Strategic prompting tools	0.46	−0.50	0.52	0.95	0.40	0.93	0.80	1.00				
Student explanations	0.38	−0.43	0.55	0.81	0.46	0.91	0.68	0.85	1.00			
Mathematical language	0.56	−0.45	0.43	0.76	0.48	0.82	0.61	0.75	0.73	1.00		
Fluency activities	0.45	−0.40	0.39	0.76	0.43	0.85	0.63	0.80	0.76	0.67	1.00	
Visual representations	0.23	−0.50	0.63	0.44	−0.25	0.56	0.36	0.42	0.51	0.41	0.48	1.00

Table 5

Relationships Between Instructional Components and Effect Sizes Using Multivariate Meta-Regression Models

Variable	*k*(*n*)	*b* [95% CI]	SE	*df*	*p*	τ^2
Instructional components						0.25
Fluency activities	10(46)	0.34 [−0.67, 1.35]	0.32	3.02	0.36[a]	
Mathematical language	12(51)	0.50 [0.03, 0.98]	0.18	4.77	0.04*	
Review	18(65)	−0.48 [−1.04, 0.07]	0.23	5.90	0.08	
Student explanations	11(49)	−0.02 [−1.48, 1.44]	0.51	3.72	0.97[a]	
Visual representations	20(74)	−0.11 [−0.67, 0.46]	0.23	6.06	0.66	

Note. Three variables were included in this model as controls: group size (small group, large group), duration (≤ 9 hrs, 10–19 hrs, ≥ 20 hrs), and comparison condition (BAU, active alt. treatment). *k* = number of studies; *n* = number of effect sizes.

[a] Degrees of freedom are less than four, which may yield an inaccurate p-value.

* $p < .05$.

Table 6

Relationships Between Intervention Characteristics or Study Design Features and Effect Sizes

Intervention characteristics	$k(n)$	b [95% CI]	SE	df	p	τ^2
Grade level						0.19
Elementary (3–6)	14(57)	0.49 [0.23, 0.75]	0.13	25.14	0.00^{***}	
Middle school (7–9)	14(33)	0 (Ref)				
Group size						0.21
Large group (> 6)	9(23)	0 (Ref)				
Small group (2–6 students)	17(63)	0.49 [0.19, 0.79]	0.14	16.79	0.00^{***}	
Interventionist						0.21
Research project personnel	15(58)	0.48 [0.17, 0.78]	0.14	17.06	0.00^{***}	
School-based personnel	9(23)	0 (Ref)				
Duration of intervention						0.30
≤ 9 hours	7(16)	0 (Ref)				
10−19 hours	6(14)	0.58 [0.01, 1.16]	0.26	10.47	0.05^{*}	
≥ 20 hours	15(60)	0.49 [0.24, 0.73]	0.11	10.88	0.00^{***}	
Initial training						0.29
None or not reported	5(11)	0 (Ref)				
< 1 day	6(13)	0.05 [−0.49, 0.58]	0.24	8.68	0.85	
> 1 day	17(65)	0.35 [−0.05, 0.77]	0.17	6.17	0.07	
Ongoing training						0.19
No	13(30)	0 (Ref)				
Yes	15(60)	0.52 [0.25, 0.78]	0.13	24.74	0.00^{***}	
Instructional setting						0.26
Special education	8(22)	0 (Ref)				
General education classroom	5(10)	−0.27 [−0.75, 0.21]	0.21	9.00	0.24	
Supplemental intervention	15(58)	0.18 [−0.27, 0.62]	0.21	13.21	0.41	
Intervention content						0.25
Decimals only	2(4)	−0.41 [−1.55, 0.73]	0.14	1.25	0.17[b]	
Fractions only	19(65)	0 (Ref)				
Multiple content topics	7(21)	−0.12 [−0.56, 0.32]	0.20	10.81	0.55	
Content level						0.29
Foundational	6(14)	−0.31 [−0.69, 0.08]	0.17	8.46	0.10	
Grade level	6(13)	−0.23 [−0.76, 0.31]	0.23	8.34	0.36	
Study design features	**$k(n)$**	**b [95% CI]**	**SE**	**df**	**p**	**τ^2**
Comparison condition						0.28
Business as usual	23(78)	0 (Ref)				
Aligned alternative treatment	5(12)	−0.50 [−0.76, −0.23]	0.11	5.63	0.00^{***}	
Type of measure[a]						0.29
Researcher developed	23(47)	0.17 [−0.11, 0.44]	0.13	26.21	0.22	
Independent	22(43)	0 (Ref)				

Note. Study counts for group size and interventionist excluded the two studies for which the intervention was entirely conducted on a computer. Ref = reference category to which others are compared; k = number of studies; n = number of effect sizes; b = unstandardized coefficient.

[a] Meta-regression model was estimated at the outcome level, not the study level.

[b] Degrees of freedom are less than four, which may yield an inaccurate p-value.

$^{*} p < .05$. $^{**} p < .01$. $^{***} p < .0045$ (the Bonferroni corrected critical p value).