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The Effects of Dynamic Strategic Math on English Language Learners' Word Problem Solving

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Abstract

English language learners (ELLs) struggle with solving word problems for a number of reasons beyond math procedures or calculation challenges. As a result, ELLs may not only need math support but also reading and linguistic support. The purpose of this study was to assess the effectiveness of a math comprehension strategy called Dynamic Strategic Math (DSM) on word problem solving for Latino ELLs. The strategy provides performance-contingent feedback based on the student's reading and language comprehension level. A multiple baseline design was used to assess 6 second-grade Latino ELLs at risk for math failure/math disability. As compared with the baseline phase, DSM increased word problem solving for all the participants. All students' level of performance was maintained during follow-up sessions. The results suggest the intervention facilitated math problem–solving performance.

Keywords

Latino English language learners, mathematics, dynamic assessment, word problem solving

English language learners (ELLs) represent more than 5 million students in K-12 public schools in the United States, of which 75% are Spanish speaking (Planty et al., 2009). The challenges for many ELLs are not only overcoming a language barrier but also achieving academically (Garcia & Cuéllar, 2006). According to The Nations Report Card (National Center for Education Statistics, 2009), the average math score for White fourth graders was 248 with only 9% below basic math skills, whereas the average math score for ELL fourth graders was 218 (43% below basic skill level). By eighth grade, this gap had widened with White students averaging 291 (17% below basic) and ELLs averaging 243 (72% below basic). These gaps challenge school authorities to achieve equitable math outcomes for ELLs.

Although ELLs come to school with varying degrees of experiences that foster their math abilities, they face many comprehension obstacles. Math content is unforgiving in terms of the constant need to build specific working math and English knowledge that is dependent on reading comprehension skills (Solano-Flores & Trumbull, 2003). Abedi, Lord, and Plummer (1997) suggested that these comprehension challenges are related to understanding the conventions of English (e.g., voice of verb phrase, conditional clauses, and question phrases), irrelevant numerical information, vocabulary, and the mathematics language of word problems. As an example, the following word problem requires multiple comprehension skills for problem solving: "15 toy soldiers are for sale, 7 soldiers have hats. The soldiers are large. How many soldiers do not have hats?" The problem involves various cognitive processes simultaneously that require the learner to access prestored information (e.g., 15 dolls), access the appropriate algorithm (15 minus 7), and finally, apply problem-solving processes to control its execution (Barrouillet & Lépine, 2005). In the above example, when a student answers this problem incorrectly, the teacher may lack information on which specific processes or steps in the problem-solving process led to the error. Given the multistep nature of word problems, strategy instruction is emerging as an important intervention approach to improve solution accuracy.

A number of studies suggest that strategy instruction in word problem solving can be an effective approach in helping learners improve their word problem–solving accuracy (e.g., Baker, Gersten, & Lee, 2002). Instructional strategies that researchers have found effective for improving word problem–solving accuracy include (a) direct and explicit strategy instruction that teaches conceptual understanding of a word problem (e.g., math concepts and principles; Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004; Griffin & Jitendra, 2008; Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra,

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Table 1. Linguistic Modification.

Linguistic modification (Level)	Description	Example
Beginning (Level 1)	Math terms used in everyday conversation	After, altogether, and, before, combine, face, together, more, more than, together, in all, less, fee, fewer than, fewer, take away, common, and stock
Intermediate (Level 2)	Math terms not directly associated with a specific math content area	Addition, subtraction, multiplication, division, addend, and digits
Advance intermediate (Level 3)	Math terms directly associated with a specific math content area	Divisor, divisible by, least common denominator, and least common multiple
Technical vocabulary (Level 4)	Math terms associated with a specific math content area topic	Decimal point, perimeter, area, cylinder, inch, meter, centimeter, mile, rectangle, square, triangle, cube, and right triangle

Note. Adapted from G. Ernst-Slavit and D. Slavit (2007).

Griffin, Deatline-Buchman, & Sczesniak, 2007; Swanson, Hoskyn, & Lee, 1999; Xin & Jitendra, 1999; Xin, Jitendra, & Deatline-Buchman, 2005), (b) strategy training that helps to develop visual spatial skills (e.g., Van Garderen & Montague, 2003), (c) instructional feedback (e.g., Fuchs, Seethaler, et al., 2008), and (d) using peer-assisted collaborative learning strategies during instruction (e.g., Fuchs, Fuchs, Yazdin, & Powell, 2002).

One challenge that arises from the word problem–solving literature is the high dependency on static assessment procedures that measures a student's current problem-solving competencies by presenting scripted tasks that require the student to access previous acquired knowledge (e.g., Haywood & Lidz, 2007). As a consequence of this, these testing procedures have not been able to incorporate instructional feedback as part of the assessment process and how such procedures can help students overcome problem-solving inefficiencies. Thus, there is a gap in the word problem–solving literature in developing an assessment model that can be used to identify learning deficits in word problem–solving accuracy, make diagnostic decisions, and propose interventions (e.g., strategies) that address these learning challenges. The purpose of this study was to assess the effectiveness of an intervention procedure based on a dynamic assessment (DA) framework.

DA is a procedure that determines whether substantive changes occur in child behavior if instructional feedback is provided across an array of increasing complex or challenging tasks (Vygotsky, 1978). In DA, a teacher facilitates (via verbal prompts and cues) a student's ability to build on prior knowledge and use this knowledge to internalize new information by providing activities and instruction that are just beyond the level of what the learner can do alone. When a student is having difficulty, the teacher attempts to move the student from failure to success by modifying the format for more clarity, providing more trials, providing information on successful strategies, or offering increasingly more direct cues, hints, or prompts. Intervention effectiveness is determined by measuring the student's potential to attain higher levels of comprehension and comparing this potential with the range of performance along their zone of proximal development (ZPD; i.e., what the student can accomplish with and without instructional support; Grigorenko, 2009). Thus, "potential" for learning new information (or accessing previously presented information) is measured in terms of the distance, the difference between, and/or change from unassisted performance to a performance level with assistance.

Although there is a vast DA literature, the empirical validity of dynamic math assessment is sparse (e.g., Fuchs, Compton, et al., 2008), particularly regarding assessments with ELLs with word problem–solving challenges. The purpose of this study was to investigate a comprehension strategy called Dynamic Strategic Math (DSM). DSM was operationally defined in this study as the researcher systematically modifying the vocabulary (via a four-level linguistic modification procedure, Table 1) to the individual student's understanding level of the word problems and then providing strategy instruction with probes that assessed students' ability to solve problems. This study addressed two research questions with Latino ELLs.

Research Question 1: To what extent does DSM facilitate children's comprehension of math text as measured by word problem–solving performance?

Research Question 2: To what extent does DSM maintain performance level in follow-up sessions?

Method

Setting and Participants

Six second-grade Latino ELLs (English/Spanish) at risk for math failure/math disability from a Southern California English-as-a-second-language elementary classroom participated in this study. This school's population consisted of 453 students (55% Hispanic [all Latino ELLs], 22% African American,

Table 2. Demographic, School-Related Data, and WJ NU III-Ach Test 10 Pre- and Posttest Scores.

Student	Gender	Age	District reading assessment level	District math assessment level	WJ pretest percentile (%)	WJ pretest standard score	WJ posttest percentile (%)	WJ posttest standard score
Ely	Male	7.6	1.4	Below basic	34	94	38	96
Liz	Female	8.4	1.4	Below basic	33	93	38	96
Rey	Male	8.1	1.7	Basic	34	94	39	96
George	Male	7.9	1.5	Below basic	35	94	40	97
Doris	Female	8.3	1.8	Basic	35	94	40	97
Beth	Female	7.7	1.8	Basic	35	94	41	97
M					34.3		39.4	
SD					0.82		1.21	

Note. WJ NU III-Ach Test 10 = Woodcock–Johnson NU Tests of Achievement 3rd Edition, Achievement Test 10: Applied Problems.

14% White, 5% Asian, and 4% other). The school was considered a high-poverty school, as it had approximately 75% of its population in the free or reduced-price lunch program.

Students were selected based on language and math criteria. ELL standardized eligibility criteria included (a) the *California English Language Development Test* (CELDT; Marr, Rodden, & Woods, 2009; we used a composite cutoff score of 397–446, which indicates early intermediate English proficiency); (b) Spanish spoken as their native language, as determined by the school's home language survey; (c) teacher student recommendation for intervention based on student's previously experiencing difficulties with mathematical word problem–solving assessments and homework; and (d) parent consent. It is important to note that although there has been a debate on how well the CELDT measures English proficiency in listening, speaking, reading, and writing, reliability scores from year to year for the CELDT are between 0.73 and 0.94 across all grades (Marr, Rodden, & Woods, 2009).

Students' mathematical skills were measured with The *Woodcock–Johnson NU Tests of Achievement 3rd Edition, Achievement Test 10: Applied Problems* (WJ NU III-ACH Test 10; Woodcock, McGrew, & Mather, 2007). This subtest is an aggregate measure of problem solving, analysis, reasoning, and vocabulary. The WJ NU III has a mean standard score of 100 and a standard deviation of 15. The WJ NU III was nationally standardized on a stratified normative sample of 8,782 participants (356 second-grade students) with 12% Hispanic in the norming sample and has a reported internal reliability coefficient of 0.85 for ACH Test 10 ages 8 to 10 (Woodcock et al., 2007). The same test was also administered at postassessment. Pre- and posttest data were compared with multiple baseline data, in determining whether DSM positively mediated learners' word problem–solving skills (i.e., math comprehension) over time. Table 2 provides descriptive, school-related information and WJ NU III-ACH Test 10 data.

Instrument

DSM procedure is built on a collaborative teacher-directed and student-based instructional and assessment foundation (Palincsar & Brown, 1984). DSM provides a set of systematic scaffolding procedures (three levels) that seek to improve ELLs' math performance. In the first level, students are pretaught math ideas, concepts, and relationships by modeling prior to introducing the math vocabulary. As students become familiar with specific math concepts and terminology for that lesson, a teacher then begins to integrate the second level, which is comprehension strategy instruction. At this level, DSM integrates five common strategies (i.e., find the question(s), find the key vocabulary/numbers, set it up, solve it, and check it) that are modeled through explicit instruction that show an ELL how to solve the word problem (for an example see Figure 1). Finally in Level 3, DSM provides a collaborative approach that allows the teacher to gradually release the leader role, which allows the student to practice this method. In this stage, the student is assigned the leadership role and imitates the teacher's role. Along this process, the student generates and asks questions to check for understanding. The student then solves the problem and evaluates it to see if it was answered correctly. If not, the teacher and student go over this problem-solving process again, to see where mistakes may have been made. As they review, the teacher monitors the student's effectiveness. If learning challenges persist, the teacher may need to reciprocate and teach specific concepts or terminology over again until the student can transfer these understandings in progressing to the next learning level (Palincsar & Brown, 1984).

> Examiner, "A word problem asks a question. Can you find the question in the following word problem?"
>
> Examiner, "In each question there are always important words. Can you underline words in this question that you think are important to solving this problem?"
>
> Examiner, "In each math problem there are always numbers that you need to solve the problem. Can you circle the numbers that you need to solve this problem?"
>
> Examiner, "Numbers are used to set-up and solve a math problem. Can you use these numbers to set-up the problem so that you can solve the word problem?"
>
> Examiner, "After solving the math problem, you need to check your answer. Can you check your answer?"

Figure 1. Dynamic strategic math probe (DSMAP).

Experimental Design

A multiple baseline across participants was used to evaluate the effects of DSM on mathematical word problem–solving performance (Kennedy, 2005). Students were selected based on low math and reading scores and the amount of support needed by teacher recommendation, and from this, a list was generated that ranked students in order based on the most need (e.g., Ely was first). After response rates were stable in the baseline phase, the intervention phase was staggered across individuals and the number of sessions necessary to establish response stability (a minimum of three sessions above the baseline mean). All students were individually administered four word problems for each session similar to those used during the preassessment phase. This study was conducted as a pullout program for 17 sessions (average 20–25 min per session) over a 5-week period and was a supplementary intervention to the 50 min of general education math students received per day. Their general education instruction consisted of teacher-centered instructional approaches using the Houghton Mifflin California Math Grade 2 (Hill, 2009) that focused on basic math skills development.

Word Problems

All word problems applied were similar to the problems used in classroom instruction. Although the research is limited on the effect of linguistically modified complexity on ELL word problem–solving skills, word problems were linguistically modified using four levels (see Table 1 for a description). These levels followed the premise that the language of mathematics is embedded within various discourse processes (Ernst-Slavit & Slavit, 2007; Moschkovich, 2002) that first begins with basic interpersonal skills and then proceeds to cognitive academic language proficiency (Cummins, 1979). Level 1 word problems incorporated math terms used in everyday conversation (high frequency), Level 2 word problems incorporated math terms not directly associated with a specific math content area (general math words), Level 3 word problems incorporated math terms directly associated with a specific math content area (specialized math vocabulary), and Level 4 (technical math vocabulary) incorporated math terms associated with a specific math content area topic (Ernst-Slavit & Slavit, 2007). Linguistic modification was achieved by minimizing sentence length, rephrasing math problems, and removing irrelevant language or sentences (Abedi, 2008; Abedi, Hofstetter, & Lord, 2004; Sato, 2008; Sato, Rabinowitz, Gallagher, & Huang, 2010). As an example, a Level 1 word problem such as "Lisa saw 5 butterflies. Juana saw 7 butterflies. Erin saw some flies. Erin also saw 9 butterflies. How many butterflies did the three girls see in all?" was linguistically modified by guiding the students to cross out the unnecessary phrase, "Erin saw some flies." In this case, this word problem was made less linguistically complex by removing an irrelevant sentence without altering the math construct. Interrater agreement of linguistic modification of word problems at all levels was established between the participants' homeroom teacher and the first author comparing and cross-checking modified word problems with the original word problems in the math curriculum. Agreement was 100% at baseline, intervention, and maintenance time points.

Probing

A probing procedure called DSM Assessment Probe (DSMAP; Figure 1) was designed by the researchers. The dependent variable in this study was word problem level achieved with the strategy intervention. DSMAP was designed to reveal differing levels of word problem–solving skill through the use of a series of five prompts (scaffolds) to determine the student's word problem performance with and without instructional assistance. Scoring involved the assignment of points: a 0 was given for every incorrect response and a 1 was given for every correct response to a prompt. As part of DSMAP procedures, each student attempted to solve linguistically modified grade-level math word problems at their ZPD level. If the student was able to solve problems without prompts (questions), he or she was given full credit (5 points) and assigned word problems that were more difficult. After 3 min, if the student could not solve the problem, he or she was given prompts. After each prompt, the student was given 1 min to answer the prompt. If the student was unable to answer the prompt, he or she was administered the remainder of the prompts so that he or she could solve the problem. An administration of a prompt lasted for an average duration of 4 to 5 min. The number of prompts needed to solve the problem established the child's intervention level.

Procedures

Baseline procedure. During the baseline phase, each student was individually given four second-grade math word problems that contained four progressive levels of word problem difficulty (e.g., addition, subtraction; see Figure 1 for a Level 1 example). Participants were given as much time as needed to solve the problem and were told to do their best with no assistance. None of the participants required more than 15 min in attempting to solve all four problems. Individual scores were recorded for each participant. Baseline (i.e., the word problem level the student could not accurately solve) established the starting point in treatment. Five of the six participants (Ely, Liz, Rey, George, and Doris) started at a baseline Level 1. Only Beth was able to establish a Level 2 word problem level.

Intervention procedure. Intervention consisted of three steps as follows: (a) preteaching math concepts, (b) comprehension strategies instruction that integrate math concepts, and (c) hints or scaffolds to improve word problem–solving accuracy. DSM intervention was delivered one to one. Each instructional session lasted 20 to 25 min. After each session, each student was given time to solve word problem(s) based on their current level of function without help. The purpose of this was to measure generalization and maintenance of word problem–solving skills. Each student was required to obtain 100% mastery (i.e., solving four consecutive problems at the current level) prior to progressing to the next problem level.

Step 1: Preteaching concepts and terminology. In this step, the student was introduced to key concepts and terminology (e.g., add, more, and take away) from the math word problems that they were asked to solve for that session. Each student was provided 3- × 5-inch index cards to practice vocabulary. The researcher modeled the activity by holding up an individual flash card, looking at the word, pronouncing the word, providing various definitions through contextualization, writing these on a vocabulary chart, and then applying them in a math problem. The researcher stated,

> This is the word *stock.* It means to *fill up* or *put more*, *put more* means *to add* (+). On a chalkboard he/she wrote +, *stock, fill up, put more, add*. Next, he/she contextualized this vocabulary. When I go to the mercado or tienda or grocery store, I see the owner *stock* his shelf with cans of food. His shelf has five cans of food; he needs to stock five more. What does *stock* mean (writing *stock* on the chart board). Stock means to add (+), fill up, put more. How many cans did he stock? Five cans plus five more is ten. Now it is your turn." Student, "Stock . . . means to *fill up* or *to add* " Researcher, "Yes. Now can use it in an example?" Student, "I go to the store and see the owner stock candies. He stocks five candies and adds five more. This equals 10." Researcher, "Very good!"

If the student was unable to say, recognize, or contextualize the math word and use it in a math example, the researcher prompted the student with other contextualized examples until the word was understood. This was repeated 3 times with all the vocabulary that was covered that session.

Step 2: Comprehension strategies instruction. In each intervention session, the teacher modeled the entire process of reading a math word problem and applied strategies using a cue sheet developed by the researcher (see Figure 2). During this process, the student was asked about what he or she already knew about the word problem; he or she was also asked to identify the problem by determining the question and looking for vocabulary terms. If he or she was unable to do the task, the researcher provided scaffolding support. The researcher also gave the student feedback to identify the word that was introduced during the preteaching vocabulary session. Next, the researcher and student collaborated in finding the numbers to set-up the problem, to calculate and solve the problem. The student was directed to ask questions so the researcher could determine whether the child understood the problem and solved it correctly. After solving the problem, the student was asked to evaluate the answer. This step was an ongoing reciprocal process or what Campione and Brown (1987) called scaffolding by explicit cue through learning and transfer, in which the teacher enables the student to operationalize the newly learned information and use newfound evidence to solve the problem successfully and evaluate this solution.

Step 3: Dynamic testing (generality probes). Next, students were tested using probes (Figure 1). Students were given a set of problems with the math vocabulary and concepts reviewed during daily intervention. None of the participants required more than 15 min for this task. The researcher used prompts to assess word problem–solving accuracy. Students' responses to questions were scored, recorded, and given a 0 for incorrect and a 1 if correct. If the student could answer the probe questions and solve the problem correctly, the student was moved to the next word problem level in the following intervention session. If not, the student was given more practice at the current level until 100% problem-solving accuracy was achieved.

Word Problem Example: *Joseph must* <u>*stock*</u> *the grocery store shelves with 177 cans of chicken soup. He has already stocked 96 cans. How* <u>*many more*</u> *cans does Joseph need to stock?*

Examiner, "A word problem asks a question (point to the question): *How many more cans does Joseph need to stock?* Next, I will underline the important words in the question.

Examiner, "I know that the words *many more* means a number of cans are missing to make the total. What does the word *stock* mean? I do not understand this word *stock.* Let's see if we go to the *grocery store*, I see cans of *chicken soup* placed on the *shelves*. If I replace or substitute the word *stock* with *put on top* or *next* to each other does this make sense? How many more cans does Joseph need to put *on top* or *next* to each other? Yes, this makes sense. *Stock* can also mean to *add* (+) or *put more*.

Examiner, "*How many more cans does Joseph need to stock?* The word problem says *Joseph must stock the grocery store shelves with 177 cans of chicken soup. He has already stocked 96 cans.* I am going to circle these numbers, as these are the numbers I need to solve this problem. Okay, let's solve the problem. He needs to *stock* or *put on top* or *add* next to each other, writing 96 cans + _ cans = 177 cans. My answer is 81 cans; Joseph needs to stock 81 cans.

Examiner, "Okay, I need to check my answer. In the ones place 6 plus 1 equals 7, in the tens place 9 plus 8 equals 17, my answer is 177 cans. This is right. Joseph needed to stock 177 cans. Now it is your turn."

Figure 2. Dynamic strategic math cue sheet (abbreviated example).

Social Validity

Social validity was assessed using a three-question interview protocol. At the end of the study, the participants were interviewed to consider their opinions on strengths, weaknesses, and what they would improve on the DSM intervention. This interview was also done with their homeroom teacher.

Interobserver Agreement and Treatment Integrity

A bilingual trained classroom teacher and the first author alternated sessions in applying the DSM intervention. In ensuring that both the first author and teacher were conducting the DSM intervention appropriately, a treatment integrity checklist based on the sequence of probe statements for each intervention phase was developed. The checklist was completed at the beginning (two sessions), middle (two sessions), and end (two sessions) of the intervention phase. Instructional behaviors were observed that included deviations from the probe such as pacing, presentation, and scaffolding. Across these observation points, each independent observation indicated the consistent presence of intervention behaviors at 100%. The score was calculated based on a point-by-point method between the interventionist and observer. This method entails dividing the number of agreements between the probe responses by the number of disagreements and then multiplying by 100.

Results

Figure 3 shows word problem level achieved for each participant as a function of baseline, DSM intervention sessions, and generalization sessions. Visual analysis indicated increases in word problem solutions based on level of difficulty. Table 3 categorizes the number of word problems solved correctly and incorrectly for each session, and accuracy percentage score (APS) on word problem solving for each session is also listed. During each session, students were administered a set of four word problems. An asterisk (*) denotes start of the next level. Also shown are WJ NU III-ACH Test 10 pre–post test gains (Table 2). Finally, a chi-square test was computed on baseline and intervention categories. This analysis indicated a significant relationship, $\chi^2(1, N = 6) = 6.491$, $p = .01$, between intervention and word problem–solving growth.

Baseline Performance

Ely received a total of three baseline sessions, and his APSs were 25%, 25%, and 25%. Next, Liz was administered four baselines, and her APSs were 50%, 25%, 25%, and 25%. Rey was administered five baselines, and his APSs were 25%, 50%, 50%, 25%, and 50%. Also, George was administered six baselines, and his APSs were 50%, 50%, 75%, 75%, 50%, and 75%. Doris was administered seven baselines, and her APSs were 50%, 75%, 75%, 100%, 75%, 75%, and 75%. Finally, Beth was administered eight sessions, and each session APS was 50%, 50%, 75%, 50%, 75%, 75%, 75%, and 100%. Doris and Beth established a Level 2 word problem baseline. One reason may have been that both Doris and Beth had higher reading achievements level in English. Although the participants' APS on word problem solving was stable and had a flat trend during baseline, the low performance on more complex language and difficult word problems for all the participants indicated a need for intervention. Pretest scores on the WJ NU III-ACH Test 10 also indicated a need for further intervention.

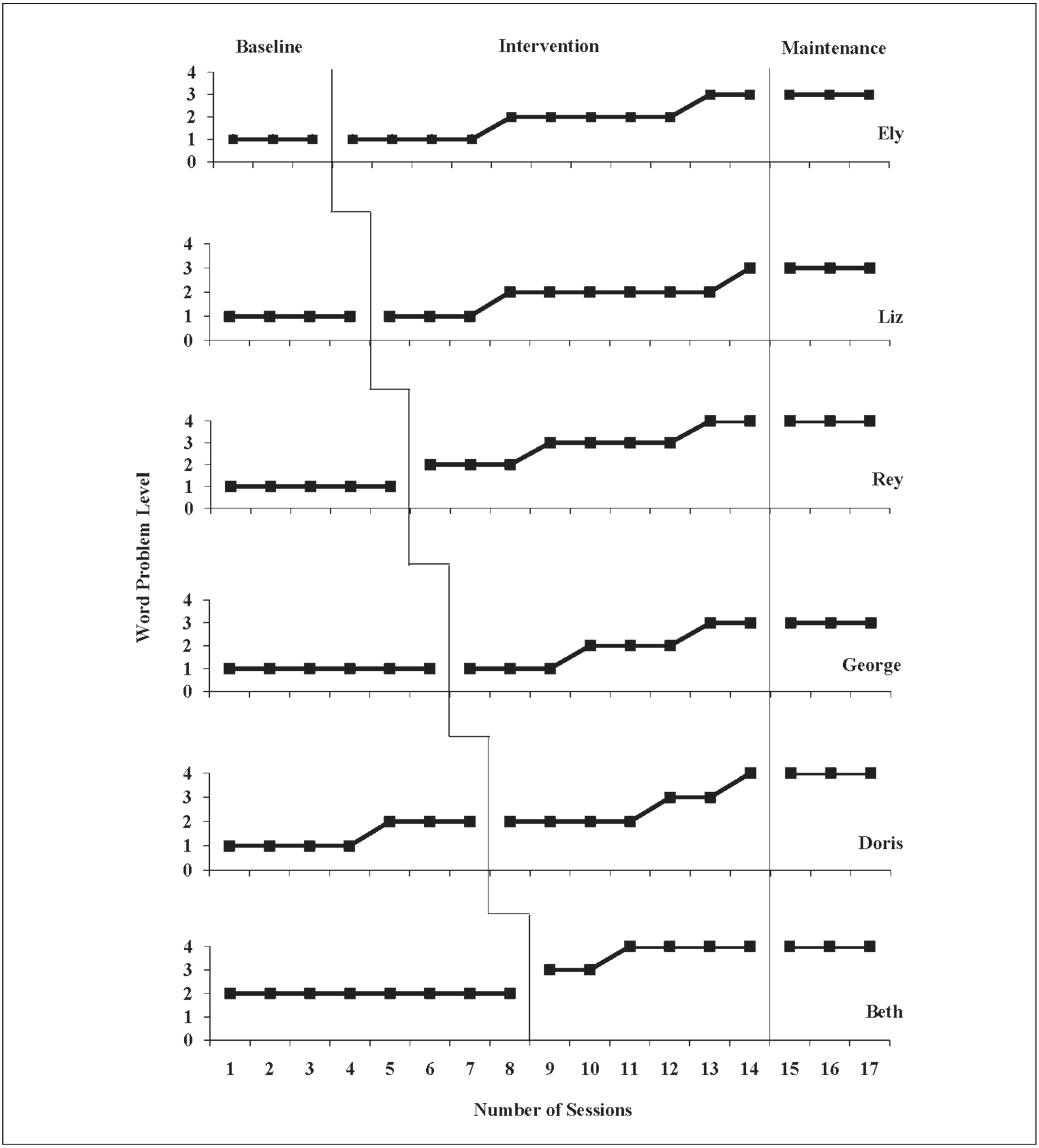

Figure 3. Word problem level achieved per session.

Intervention

As compared with baseline, the intervention condition produced an increase in both APS and level of word problem solved for all participants. After each intervention session, each participant was administered a problem-solving set of four problems similar to those used in baseline phase. Students' word problem–solving APSs during the maintenance phase are presented in Table 3, and word problem level achieved for maintenance phase are given in Figure 3.

Ely received 11 interventions sessions, his APSs on word problem sets were 75%, 50%, 75%, 100%, 50%, 50%, 75%,

Table 3. Word Problems Solved Correctly and Incorrectly for Each Student per Session.

	Ely							Liz						
	B		I		M		APS	B		I		M		APS
Session	IC	C	IC	C	IC	C	%	IC	C	IC	C	IC	C	%
1	3	1					25	2	2					50
2	3	1					25	3	1					25
3	3	1					25	3	1					25
4			1	3			75	3	1					25
5			2	2			50			1	3			75
6			1	3			75			1	3			75
7			0	4			100			0	4			100
8			*2	2			50			*2	2			50
9			2	2			50			3	1			25
10			1	3			75			2	2			50
11			1	3			75			3	1			25
12			0	4			100			3	1			25
13			*1	3			75			0	4			100
14			1	3			75			*1	3			75
15					1	3	75					1	3	75
16					0	4	100					1	3	75
17					0	4	100					0	4	100

	Rey							George						
	B		I		M		APS	B		I		M		APS
Session	IC	C	IC	C	IC	C	%	IC	C	IC	C	IC	C	%
1	3	1					25	2	2					50
2	2	2					50	2	2					50
3	2	2					50	1	3					75
4	1	3					25	1	3					75
5	2	2					50	2	2					50
6			1	3			75	1	3					75
7			1	3			75			*1	3			75
8			0	4			100			2	2			50
9			*3	1			25			0	4			100
10			2	2			50			*1	3			75
11			1	3			75			2	2			50
12			0	4			100			0	4			100
13			*2	2			50			*2	2			50
14			1	3			75			1	3			75
15					1	3	75					1	3	75
16					0	4	100					1	3	75
17					0	4	100					1	3	75

	Doris							Beth						
	B		I		M		APS	B		I		M		APS
Session	IC	C	IC	C	IC	C	%	IC	C	IC	C	IC	C	%
1	2	2					50	2	2					50
2	1	3					75	2	2					50
3	1	3					75	1	3					75
4	0	4					100	2	2					50
5	*1	3					75	1	3					75

(continued)

Table 3. (continued)

	Ely							Liz						
	B		I		M		APS	B		I		M		APS
Session	IC	C	IC	C	IC	C	%	IC	C	IC	C	IC	C	%
6	1	3					75	1	3					75
7	1	3					75	1	3					75
8			1	3			75	0	4					100
9			1	3			75			*1	3			75
10			1	3			75			0	4			100
11			0	4			100			*2	2			50
12			*1	3			75			1	3			75
13			0	4			100			1	3			75
14			*1	3			75			1	3			75
15					0	4	100					0	4	100
16					0	4	100					1	3	75
17					1	3	75					0	4	100

Note. B = baseline phase; I = intervention phase; M = maintenance phase; APS (%) = accuracy percentage score; IC = incorrect; C = correct; * = start of next level.

75%, 100%, 75%, and 75%, and he demonstrated a gradual increase in word problem level achieved (i.e., from Level 1 to word problem Level 3) from point of baseline. Liz received 10 intervention sessions, her APSs on word problem sets were 75%, 75%, 100%, 50%, 25%, 50%, 25%, 25%, 100%, and 75%, and showed an increase in not only problems solved but also word problem level achieved (i.e., from 25% word problem Level 1 to 75% word problem Level 3). Also, Rey received 9 intervention sessions, his APS showed a gradual increase in word problem level achieved. George received 8 interventions sessions and also showed a gradual increase from word problem Level 1 to word problem Level 3. Doris received 7 intervention sessions with APSs of 75%, 75%, 75%, 100%, 75%, 100%, and 75% and obtained a word problem Level 4. Finally, Beth received 6 intervention sessions, and her APSs were 75%, 100%, 50%, 75%, 75%, and 75%, and showed an immediate increase of word problem Level 2 to word problem Level 4 after two intervention sessions.

All students benefited from DSM intervention because they received instruction that was tailored to their oral language, vocabulary, and problem-solving needs from an interactive approach. First, students were directly and explicitly taught math vocabulary that connected to everyday vocabulary. Next, students were taught comprehension strategies that they would need to undertake in solving the problem. Finally, students were given time to practice their language skills by reading and interpreting problems through the use of probes. This not only helped the student(s) to practice their oral language skills but also helped them solve the problem(s).

Maintenance

Finally, to determine maintenance of learning, all students were individually administered a set of four math word problems per session for a duration of 3 weeks (one session per week) similar to those used during the preassessment phase. Students' word problem–solving APSs during the maintenance phase are presented in Table 3, and word problem levels achieved for maintenance phase are given in Figure 3. Students were administered the same version of the WJ NU III as a posttest. During this phase, all students sustained APS similar to the end of the intervention phase.

Ely's APSs on word problem sets were 75%, 100%, and 100%, and remained stable at Level 3. Liz's APS remained stable (e.g., between 75%, 75%, and 100%). Liz's APS on the word problem sets showed increase in not only problem solving but also word problem level achieved from the last point of the baseline phase (i.e., from 25% word problem Level 1 to 75% word problem Level 3). Rey showed an increase in word problem level achieved from the last point of baseline phase (from 100% word problem Level 1 to 75% word problem Level 3). Although his APS for word problems solved did not increase from baseline to maintenance phase, it did indicate that he was able to solve more difficult problems. Also, George's APS indicated an increase in word problem Level 1 during baseline to word problem Level 3 from maintenance (i.e., 50% to 75%). Doris showed an APS increase from baseline phase word problem Level 1 to 100% maintenance phase word problem Level 4. Finally, Beth's maintenance phase showed an increase from word problem Level 2 in baseline to Level 4 in maintenance.

In summary, during the maintenance phase, visual inspection of the data on students' word problem–solving accuracy and word problem level achieved indicates that they were able to maintain a high level of performance due to DSM intervention. Posttest scores on the WJ NU III-ACH Test 10 also indicate that students had shown improvement from DSM intervention.

Social Validity

Interview data indicate that all the participants were in agreement (100%) that DSM procedures were reasonable and effective. Several students commented around the theme "that I really liked how we talked about math." The homeroom teacher commented, "I really liked how the intervention was able to work with students' background knowledge and apply this to solving math problems. DSM is quite a straightforward approach that allows you to integrate reading comprehension strategies with math instruction. "The students recommended the use of more visual aids, and manipulative, that they could 'play with.'" The teacher would have liked a math journal for each student that would have helped them solidify their DSM understanding through writing.

Discussion

The present study indicates that DSM intervention improved ELL's ability to solve increasingly complex word problems during the intervention phase in comparison with the baseline phase, and this achievement was maintained during follow-up sessions 3 weeks after intervention. We infer that the intervention positively mediated math comprehension over time because it gave ELL performance-contingent feedback based on students' independent performance (known) and students' assisted performance (potential) that promoted their math development. Furthermore, DSM provided participants opportunities to learn and practice the academic language of mathematics by teaching them how to use their English skills and contextual experiences to solve word problems. Word problem–solving data indicated that participants could acquire word problem–solving proficiency with basic word problems once given appropriate language development instruction and allowed to practice. For example, once students were taught basic math vocabulary and terms and given reading practice with simple math problems, they calculated these problems quite easily. However, as word problems became more complex, solving these types of word problems became more challenging because of students' emerging English language acquisition and reading comprehension skills.

In addition, because daily math instruction focused on mastery of numbers, operations, and computation skills through addition, subtraction, multiplication, and division, these students had a good understanding of basic calculation principles. DSM was able to focus on the application of this understanding that not only modeled for them how to do numerical computations but also decide on the numbers to use, choose the numbers needed, do the necessary calculations, and then appropriately apply these skills across an array of increasing complex or challenging word problem–solving levels. For reasons like this, interventions like DSM may be able to provide feedback or practice prior to standardized testing, which can improve on ELLs' word problem–solving skills.

Finally, although the participants were reading below grade level prior to DSM implementation, all students showed gains because their learning obstacles revolved around the reading comprehension and vocabulary domains that ameliorated with added individualized practice. DSM improved on these constructs by (a) positively affecting students understanding of word problems, (b) instructing students to be aware of their comprehension through probes, (c) teaching students to understand word problem structure as they read, (d) guiding them to find the question in the word problem, (e) finding important vocabulary and numbers, (f) setting up the problem to solve, and (g) finally, checking their answer for accuracy. As an example, Rey, George, Doris, and Beth went from solving basic word problems during the baseline to solving highly complex word problems during the DSM and maintenance phases. As a result of DSM, all students were able to move from less to more complex math problems and maintained a high level of word problem–solving complexity and accuracy in the postintervention phase compared with the baseline phase.

Limitations of the Research

Although the results indicate that the DSM intervention improved students' mathematical word problem–solving skills, several limitations of the study suggest that caution must be exercised when interpreting the findings. First, this was a single-participant research design with a small sample size (six students) and, because of this, generalizing DSM effectiveness to another population may be limited. Moreover, few math studies have been conducted with ELLs and DA to compare similar results. Next, because students were provided individualized instruction, caution must be noted in applying these findings to small group or whole group classroom math instruction. Finally, the students received linguistically modified word problems as to control for linguistic complexity during intervention sessions. Although the intervention was based on research that promotes carefully designed instruction focusing on linguistic complexity, language modifications may have influenced students' performance, instead of the DA nature of instruction.

Implications for Practice

The findings from this study have implications for practice. First, with its emphasis on comprehension, DSM intervention helped at-risk students for math failure and/or learning disability not only acquire mathematical word problem–solving skills but also maintain these skills. Many times, ELLs may have strong basic mathematical skills in their native language; however, because of acquiring English as a second language, teachers may assume that their students may lack basic skills in English, focusing on teaching these skills first rather than focusing on instruction that enhances their word problem–solving comprehension. Consequently, math instruction may need to focus more on math comprehension strategies that give them the opportunities to develop the necessary language skills to solve word problems in English. Second, the effectiveness of DSM instruction suggests that ELLs are able to learn comprehension strategies in English that gives them structure in solving a word problem. Lack of attention, organization, and poor working memory capacity are the characteristics of learning/math disability, which implies comprehension difficulties (Swanson, Harris, & Graham, 2005). Finally, it is important for teachers to provide linguistically modified word problems that may improve students' cognitive efficiency and thus improve their word problem–solving skills.

Research Implications

The final rationale for DSM is that to date, there have been few quantitative studies of DA with ELLs. Granted that emerging strategy instruction research is indicating positive outcomes for students at risk for learning/math disabilities experiencing comprehension difficulties, these strategies may be too dependent on statistic assessments that view student achievement on a particular word problem as stable. Although scores based on static assessment predict performance in the intervention, these scores may not be predictive of what the student is capable of achieving. Because of this, a larger replication study needs to be conducted, examining linguistic modifications within DSM intervention procedure to see whether the same learning pattern holds.

Conclusion

The math literature indicates that students need to be provided with multiple strategies and approaches to further develop their mathematical thinking. Word problem performance improves as children gain a greater ability to (a) understand underlying arithmetic operations (e.g., Rasmussen & Bisanz, 2005; Zamarian, López-Rolón, & Delazer, 2007), (b) distinguish between types of word problems on a basis of mathematical operations (e.g., Fuchs & Fuchs, 2007; Rittle-Johnson, Siegler, & Alibali, 2001), and (c) choose an effective strategy (e.g., Geary, Hoard, Nugent, & Byrd-Craven, 2007). In addition, affecting these types of problem-solving processes can be difficult for ELLs because math problem–solving skills require working memory that involves the preservation of information while simultaneously processing the same or other information in a second language (Swanson, Kehler, & Jerman, 2009).

In summary, this study found that ELLs might have difficulty developing word problem–solving skills because of inadequate preparation in language development. DA may play a key role in mediating math comprehension skills, as it can determine whether substantive changes occur in child behavior if feedback is provided across an array of increasing complex or challenging tasks. Further research needs to be conducted in understanding how ELLs' appropriate problem-solving skills can be enhanced through dynamic analysis.

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