Mathematical Sequences of Connected, Cumulative and Challenging Tasks in the Early Years

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This symposium reports on a project that focused on *Exploring the Use of Mathematical Sequences of Connected, Cumulative and Challenging Tasks (EMC^3*) with students in the early years (Foundation Level to Year 2). The project was funded by the Australian Research Council, Catholic Education Diocese of Parramatta and Melbourne Archdiocese Catholic Schools (LP180100600). Together with industry partners the EMC^3 project was designed to enhance the cognitive and affective experiences of students when learning mathematics by researching teaching approaches that utilise sequences of cognitively challenging tasks.

Paper 1: *Exploring the Potential of Sequences of Connected, Cumulative and Challenging Tasks in the Early Years* [Peter Sullivan, Melody McCormick]

This paper outlines the rationale for the teaching approach the EMC^3 project aimed at studying an approach to teaching and learning mathematics in the early years (students aged 5–9).

Paper 2: *Differentiating Mathematics Instruction through Sequences of Challenging Tasks in the Early Primary Years* [James Russo, Jane Hubbard]

This paper reports on post-program questionnaire data collected from 100 teachers who express their views about the effectiveness of various instructional approaches to support differentiation in mathematics.

Paper 3: *Changing Teacher Practices: A "Slow Burn" or Rapid with "Big Shifts."* [Sharyn Livy, Janette Bobis, Ellen Corovic, Maggie Feng]

This paper reports on interview data collected from five teacher educators who provided support to the teachers when trialing the EMC^3 resources. The focus of this presentation will be on the notable changes to teacher practices.

Paper 4: *The Nature of Leadership and Other Support that Facilitate Innovation and Improvement in Teacher Practice.* [Ann Downton, Janette Bobis]

The final paper reports on survey data collected from 70 teachers about the forms of support that assisted implementation of project resources—in-class support and facilitation of planning.

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Exploring the Potential of Sequences of Connected, Cumulative and Challenging Tasks in the Early Years

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This paper outlines the rationale for, and some elements of, a particular approach to teaching and learning mathematics in the early years. The researchers worked with two school systems to offer both centrally delivered and school-based teacher professional learning, which included the application of illustrative teaching resources. The project gathered a range of data from teachers and leaders on their dispositions and knowledge, as well as the opportunities and constraints they experienced, and the influence these variables had on planning, teaching and student learning outcomes.

The following outlines the rationale for, and some elements of, an Australian Research Council funded project aiming to study a particular approach to teaching and learning mathematics in the early years (students aged 5–9). This contribution provides background information relevant for the other presentations in the symposium. Fundamental to this approach to teaching was the use of sequences of connected, cumulative, and challenging tasks that focused on mathematical content and proficiencies represented in the *Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2020).

Even though it is common for teachers to develop understanding and foster mathematical fluency associated with particular concepts before problem solving and reasoning, termed teaching *for* problem solving (Schroeder & Lester, 1989), the project explored the potential of the reverse. That is, we considered the impact on student learning and engagement when teachers pose problems that allow for student reasoning to occur first, with the intention of building understanding leading to fluency subsequently; this is termed teaching *through* problem solving (Schroeder & Lester, 1989).

The project task design and pedagogical emphasis were informed by two characteristics articulated by the Organisation for Economic Co-operation and Development (OECD) (2019; 2021). The first, agency, relates to students having the ability and will to make active decisions to positively influence their own and others' learning. This implies that students see themselves as not only capable of thinking for themselves but also having the confidence and aspirations to learn. In order to exercise such agency and realise their potential, learners require time initially to struggle productively (Sinha & Kapur, 2021) with problems or tasks without being told what to do by the teacher/educator or other students. During this uninterrupted time students are able to choose their own strategy and form of representation. This pedagogical focus aligns with the teaching through problem solving approach. In terms of emphasising student agency, we encouraged the project teachers to plan experiences that were productively challenging for students. Sullivan et al. (2020) explained that:

> Challenge comes when students do not know how to solve the task and work on the task prior to teacher instruction. Other characteristics of such tasks are that they: build on what students already know; take time; are engaging for students in that they are interested in, and see value persisting with a task; focus on important aspects of mathematics (hopefully as identified or implied in relevant curriculum documents); are simply posed using a relatable narrative; foster connections within mathematics and across domains …. (pp. 32–33)

The second characteristic, inclusion, involves identifying learning experiences and associated pedagogies that maximise opportunities of all learners. As far as possible, in the

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approach we are exploring, all learners are given opportunities to think for themselves and, especially for students experiencing difficulty, are provided support to access the full curriculum. This is elaborated further in Russo and Hubbard (Paper 2) and includes learning experiences in which the activities and tasks are accessible, while still being productively challenging, and with explicit teacher attention to actions that address the needs of individual learners. There are three aspects of the recommended pedagogies that are intended to foster inclusion. First, teachers are encouraged to choose learning experiences that are not only readily accessible for all students but also have the potential for further exploration. Second, teachers prepare specific enabling prompts for students experiencing difficulty and extending prompts for students who complete the set work quickly (see Sullivan et al., 2006). Third, teachers use a particular lesson structure, as summarised below, consistently to provide students with confidence of the ways the lessons develop.

The specific aims of the project were to:

- explore the potential of sequences of connected, cumulative, and challenging tasks that build a trajectory of consolidated learning of mathematics;
- explore responses from teachers, leaders and students when this approach to teaching mathematics is enacted;
- make recommendations for resource developers, curriculum designers and providers of teacher professional learning.

An Instructional Model for Student-centred Structured Inquiry

The EMC^3 project described this approach to instruction as *Student-centred Structured Inquiry* (the key elements of this are elaborated in Sullivan et al., 2020). The approach is also described as cognitive activation. Caro et al. (2016) analysed results of PISA 2012 involving over 500,000 students and provided compelling evidence of the effectiveness of this perspective. Characteristics of cognitive activation include posing problems that require students to think for an extended time, to choose their own solution procedures, to learn from mistakes, to explain their solution strategies and to solve problems in different ways.

To communicate the various associated teacher actions, the project participants and researchers developed an instructional model with four phases: *Anticipate, Launch, Explore, and Summarise/review*. The language of the instructional model draws heavily on Smith and Stein (2011) who focus on orchestrating classroom discussions, an essential element of creating opportunities for fostering student agency and inclusion. The aim is to make it obvious to students they have a role to play in creating new knowledge.

Anticipate phase. This phase is central to all planning. It includes identifying the intended learning outcomes (what, why and how); developing helpful resources; predicting students' solutions, strategies and possible misconceptions; and considering pre-requisite and new language, as well as other aspects of planning.

Launch phase. This phase addresses language and representation associated with the intended learning experiences. It includes providing opportunities for students to develop fluency in the mathematical processes and procedures relevant to the experiences. It also involves posing tasks without informing students on how to solve the problem, an essential aspect of fostering agency.

Explore phase. In this phase teachers interact with students, encouraging persistence, posing prompts, and identifying interesting and perhaps unanticipated solutions, selecting some for later presentation.

Summarise/review phase. This phase involves the teachers selecting and sequencing student solutions to be shared. Engagement is promoted by supporting students while they present their solutions and encouraging active participation of others. A key element of this phase is the teacher synthesising the essential ideas that represent the learning intentions of the experience.

Importantly, the launch-explore-summarise/review process happens more than once for each learning experience, with the tasks for the subsequent cycles based on Variation Theory (Kullberg et al., 2013). The variations, as represented by this theory, are intended to draw the attention of students to key elements of concepts by varying some aspects while keeping other aspects invariant. In other words, task design involves creating new tasks from existing tasks by keeping some aspects the same but varying other aspects. The variant might be the context, with the concept(s) staying the same. Alternatively, the variant might be the sophistication of the concept (or even the concept itself), with the context staying the same. The explicit intention of the subsequent iterations of the model is to consolidate thinking activated by the initial experience (Dooley, 2012). This consolidation involves repeating the preceding three phases, noting that consolidation can be in a subsequent lesson.

An important feature of the instructional model is that, when consistently applied, it is argued to help students to moderate their anxiety by normalising uncertainty. Buckley and Sullivan (2021) argued that students who are anxious can manage the threat to their learning opportunity by specific behavioural strategies and through familiarity with this lesson structure.

Project Resources

The project team and participating teachers developed coherent and connected sequences, representing the content descriptions and proficiencies of the *Australian Curriculum: Mathematics* (ACARA, 2020). The sequences were intended to make the mathematical ideas central to the learning obvious to the students. Participating teachers were provided with illustrative resources to support the implementation of the pedagogical approach. An example of a low floor/high ceiling task, focusing on making and naming polygons, that is intended to be productively challenging for students aged 6–8 is as follows.

Making polygons out of trapeziums

Using some or all of four trapeziums (all the same), what polygons can you make?
Draw the new polygons on isometric dot paper and name them.
How are your new polygons the same? How are they different?

Students are provided with sets of trapeziums such as those in Pattern Blocks and isometric dot paper. The "floor" is when students make and draw one polygon. The "ceiling" is the possibility of making and drawing multiple different shapes (there are many). An example of an enabling prompt is "what shapes can you make with two trapeziums?" An example of an extending prompt is "draw a triangle made out of three trapeziums without using the materials". An example of a consolidating task is as follows:

Making polygons out of rhombuses

Using some or all of four rhombuses (all the same), what polygons can you make?
Draw the new polygons on isometric dot paper and name them.
How are your new polygons the same? How are they different?

Even though acknowledging individual students' thinking as paramount, both the mathematical focus and the pedagogical approach are intentional and go beyond unstructured inquiry or play (Bruner, 1961; Mayer, 2004). At the same time, the approach rejects the notion that the optimal way to teach mathematics is by explicitly telling students what to do, followed by practice. The teacher has an active role, but this happens after students have had the opportunity to engage in the mathematics and the contexts of the tasks. Likewise, students are exposed to illustrative worked examples, some of which can come from the students themselves.

By proposing carefully constructed and effectively trialled sequences supported by related professional learning, teachers can experience not only ways in which learning can be sequenced but also how sequences enhance learning opportunities for students. The goal of offering suggestions for teachers was to free up energy for them to engage with the complexity of converting tasks, lessons and sequences into learning experiences for their students. The aim was to support the development of manageable and sustainable teaching practices. Part of the professional learning for participating teachers was illustration of ways of adapting the contextual stories and including the level of challenge to suit their particular class and student context. Participating teachers took an active role in the adaptation of the tasks, lessons and sequences, not only improving on the initial designs but also gaining insight into the process of sequence creation.

The project partnered with two school systems that invited schools to participate. In each of three years, participating teachers were offered an initial day of professional learning on the goals and resources of the project, were supported in their schools by researchers and system educators and offered further professional learning. Resources were made available in both hard copy and electronically. There were two sets of participants for each partner over the three years (due to COVID challenges). Data were collected from teachers, school-based leaders and system educators through surveys in each year of the project. There were also interviews with teachers and educators, classroom observations, and assessment of student learning. The findings of the project are in the process of publication.

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