Teacher Actions to Progress Mathematical Reasoning of Five-year-old Students

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Opportunities for five students to access higher mathematics and critical thinking is often restricted until they have developed sufficient knowledge. This case study focuses on two reception class teachers and their teacher actions used to develop mathematical reasoning with their students. The findings illustrate the impact these teachers have on their students in the mathematics classroom by developing mathematical practices with five-year-old students that show that these students can critically reason and engage in mathematical practices.

Students in Aotearoa/New Zealand enter and attend school on or near their fifth birthday and are termed *New Entrant* students. They are assessed for mathematical knowledge within days of their entry to the schooling system. This should hold potential for them to then be given opportunities to learn mathematics from where they are on entry. However, Young-Loveridge (1998) contended that teachers then spend eighty percent of these students' first year teaching and practising existing skills. This means that a considerable proportion of New Zealand New Entrant students are provided with limited opportunities to succeed beyond the skills they enter school with as five-year-old students. Also, because of the repetitive form of instruction, they have limited opportunities to develop rich reasoning in mathematics (Hunter & Hunter, 2018). In contrast, if teachers scaffolded students by explicitly praising their engagement in mathematical practices and creating *talk rooms,* students would then be provided with opportunities to develop rich mathematical reasoning. However, we know that developing such classrooms is challenging, more so because there appears to be limited models for teachers to draw on to support their practice. The aim of this paper is to provide a possible model for New Entrant (reception) teachers, that could be representative of classrooms premised within such forms of inquiry and deep reasoning. The focus of this paper is to explore and examine two New Entrant teachers as they construct rich mathematical understandings with their five-year-old students. The question we ask is:

> *What actions do teachers take to provide all students with the opportunities to learn and communicate their mathematical reasoning in sense-making ways?*

Literature Review

Readiness is an important concept in mathematics teaching, but it can also be misinterpreted and used as a reason to delay allowing students opportunities to progress mathematically. Decades ago, Young-Loveridge (1987) warned that for many New Zealand students, mathematics was a subject that teachers used readiness mistakenly, to hold back on allowing their students to explore number concepts and key ideas until it has been achieved. This idea of readiness was developed from Piaget's (1952) view of cognitive development. Young-Loveridge explained how educators have taken from Piaget the idea that students are not ready to learn numerical mathematics until they have a base of cognitive abilities of conservation and classification. She says that there is no clear evidence to show that teachers should delay teaching rich mathematics until specific skills or knowledge have been acquired. MacGinitie (1969) also argued that to hold off exposing mathematical ideas or big ideas to New Entrant students was not an effective use of time. Nevertheless, this thinking has prevailed and in Aotearoa/New Zealand with a large amount of time placed on these young students

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gaining number knowledge to succeed in mathematics. This was exemplified in Aotearoa/New Zealand with mathematics teaching revolving around the New Zealand Numeracy Project (Ministry of Education, 2006). The Numeracy Project followed a linear trajectory of learning with students placed in ability groups. The initial focus for young children prescribed knowledge-based activities, which many teachers enacted in procedural and rote-type learning. A large focus of learning was around students *mentally* solving or using *imaging* to solve number problems. If unable to use a specific strategy students would remain at that level until the knowledge was learnt. This is despite many studies (e.g., Clements & Sarama, 2018; Steffe et al., 1976; Smith et al., 1981), which showed that students without what is considered necessary knowledge still benefit from deep mathematical understanding to the same degree as those who have the necessary knowledge.

In more recent times in Aotearoa/New Zealand, the focus of mathematics has been around students constructing reasoning embedded within a body of knowledge. As part of this, access to a range of materials and ways of representing reasoning was considered important for young children to construct rich forms of conceptual reasoning. Selling (2016) described how teachers should "attend to precision" (p. 547) as students use different representations. She states that it is important for teachers to name the representation being used as students show and explain their mathematical ideas. Mathematical practices are a set of skills or practices students use to become effective mathematicians. These practices include students justifying their ideas, making claims, the use of mathematical explanations, using representations and making generalisations – among others with and without materials (Selling, 2006). Selling also discussed the impact of teachers explicitly making connections across different representations by comparing solutions. Mueller and colleagues (2014) also showed how using materials supports students to clarify their ideas, think critically, and make connections across big mathematical ideas, and deepen their mathematical knowledge as connections and translations are made across representations. Hunter and Hunter (2017) added that for deepening students' mathematical knowledge they needed opportunities to develop mathematical practices and reasoned conceptual knowledge throughout all mathematical areas.

Mathematical practices are an important tool which affords students opportunities to develop deep conceptual understandings. Through mathematical practices Hunter and Hunter (2018) outlined how students can develop their mathematical identities and strengthen positive mathematical dispositions from as early as five years old. They explained that students need to be provided with the opportunities to productively engage in mathematical practices for the practices to become real and meaningful. Selling (2016) argued the importance of teachers making mathematical practices explicit to facilitate "equitable learning opportunities" (p. 511). She described how it promoted the students as doers of mathematics as their peers are exposed to the work or reasoning they are doing. Hunter (2009) also discussed the impact of teachers responding in the moment to students engaging in mathematical practices. In doing so, teachers supported the development of these mathematical practices as tools that supported mathematical learning.

Mathematical practices work in tandem with classrooms that are discourse heavy. Researchers (e.g., Chapin & O'Connor, 2007; Franke et al., 2009; Hunter & Hunter, 2018) described the importance of talk in the mathematics classroom. Mathematical talk provides opportunities for students to not only justify and explain their ideas but, through this process, clarify and make sense of their reasoning and that of their peers (Chapin & O'Connor, 2007). Chapin and O'Connor (2007) detailed a set of what they term talk-moves, which teachers can employ to position their students to sense-make in mathematics classrooms. Nevertheless, for students to participate in such learning environments a safe and secure classroom environment needs to be developed. Mutual respect and high expectations are crucial for students to contribute their reasoning and respond to their peers' questions (Hunter & Hunter, 2017). Many researchers (e.g., Boaler, 2008; Hunter, 2009; Hunter & Hunter, 2017) discussed the importance of teachers taking the time to teach and explicitly set up the norms expected in their classroom. Relating these norms and expectations to cultural experiences or values enhances the students' ability to connect to big mathematical ideas and experience success in real life mathematics (Bishop et al., 2009).

We began this paper by outlining the paucity of papers that provide teachers of very young children with models of talk-rooms to promote engagement in rich conceptual reasoning. The aim of this paper is to provide one possible exemplar of actions two teachers took to induct their young students into rich mathematical discourse environments.

Research Methods

This small-scale study was conducted at two New Zealand, primary schools with a diverse community of learners. These included students of Māori, Pāsifika, Indian, Korean, and New Zealand/ European ethnicities. The school was sited in a low-socioeconomic community. The teachers were experienced and of New Zealand/European ethnicity. Eight of Teacher A's students had attended school for two terms, two had attended school for six weeks, and three had attended school for less than two weeks. Teacher B's students had all attended school for less than ten weeks (n = 14).

Data consisted of video and audio recorded lessons observations wholly transcribed, and field notes. Analysis involved developing codes and notes and noticing the emerging common themes, which related to teacher actions to facilitate and progress mathematical reasoning amongst the new entrant students.

Findings and Discussion

Teachers structuring mathematics with ability grouping or streaming has been predominant in many New Zealand classrooms. This mode of grouping has traditionally begun almost as soon as students enter school as five-year-old students. In both classrooms described in this study, the teachers chose to structure their mathematical activity around the use of heterogenous grouping patterns, rather than taking a streamed approach. The classrooms also drew on pedagogical practices promoted within culturally sustaining (Paris, 2012), ambitious mathematics (Kazemi et al., 2009), which promoted engagement in inquiry and mathematical practices (Hunter, 2009). Lesson structures conformed to those outlined by Smith and Stein (2011). Problems were launched then students were given a short time to work on the problem with a buddy before returning to talk about their solutions in a larger group.

Providing Equal Opportunities for all New Entrant Students

From the data it is clear to see that both teachers considered that the benefits of having students in mixed ability groups outweighed possible deficits. Teacher A described and justified why she used multi-ability groups rather than streaming her students when she stated, "some of these students can add and write number sentences whilst some students can not recognise their numbers when counting to twenty – but they are all in the one group succeeding in the maths." Teacher B added to the discussion in describing how "there are some students in this group who struggle to form their numbers, so we provided materials, such as a hundreds boards to support them writing their numbers." Clearly, these teachers considered that it was their responsibility to ensure access to the mathematical reasoning through supportive means. This was evidenced in the substitution of the hundreds board rather than holding a child back because of the inability to record numbers.

Materials encourage students to solve mathematical tasks conceptually and a hands-on approach is needed for five-year-old students to develop mathematical representation. In the two classrooms the students were provided with many opportunities to use multiple resources to support them to develop their mathematical ideas. A range of materials was available in both classrooms and students were pressed to show their reasoning in whatever way they chose. For example, in the observed lesson Teacher A had a box of mixed materials the students could access at any stage of the task to support them with its solving and explaining. The teachers also supported the students' representations by modelling and highlighting representations and scaffolding them to connect to mathematical concepts. Their actions matched those promoted by both Mueller et al., (2014) and Selling (2016). These researchers demonstrated the power of teachers building on student representations, as either recorded or hands on explanations. In their research they showed how such actions supported the students to explain and clarify their reasoning. This was evident in this research.

Beyond having multiple forms of concrete and other representations the teachers also carefully launched the tasks to ensure all students had access to the context and the mathematics. This is essential with such young learners. For example, Teacher A used the repeat talk move multiple times as she developed what the task required them to do mathematically. This repetition provided students who had difficulties identifying specific numbers when reading the task the opportunity to understand the numbers they were working with. Making the repeat talk move public and explicit encouraged all students to be aware that the discussion was happening with the whole class, not just between the teacher and the individual student. Chapin and O'Connor (2007) described how teacher action of using repetition by different students highlights the importance of a contribution or essential point made.

Setting Expectations for Individual and Collective Engagement

In order for five-year-old students to develop effective mathematical practices teachers must provide opportunities for the students to engage and struggle with the mathematics. The tasks must be relevant and responsive to the cultural and social needs of the students. In doing so the students will see the mathematics as relevant and real to life (Bishop et al., 2009; Hunter & Hunter, 2017). Teacher A's task was written about a particular student's news item shared the day previously. The student reported it was her brother's birthday and they celebrated with homemade Indian naan. When launching the mathematical task, the teacher was able to bring this student into the conversation, and she shared her experience with her peers. The teacher then facilitated a discussion where the students compared their likes and dislike for naan. With the students able to relate easily to the context, the teacher was then able to form a challenging task around naan. As Teacher A described, "Fractions are huge concepts for New Entrant students to develop as they are so used to the bigger the number the bigger the value it holds, so we are flipping their thinking." This was shown when she generalised and made a connection to her thinking by asking, "If you were to have a piece of naan, would you want a half or a quarter?" One student said: "half because the piece is bigger." Listening carefully another student stated, "I agree, half a naan is bigger than a quarter of the naan." Illustrated in this exchange was the importance of the task and ensuring all students had access to the context. The students were clearly using the context to make sense of the mathematics. Although Smith and Stein (2011) previously described the importance of the launch and practical ways to implement it, their work was with much older students. In this classroom with young students, it remained both an important and effective tool, which served to progress the sense-making of them all and kept them engaged.

Establishing an expectation for engagement of all students includes maintaining the high cognitive demand in the task. Teacher B directly addressed the potential for students to opt out if they became daunted by the task. She stated, "Todays task is going to be a challenge, but I know you can do it—otherwise I wouldn't have written it." Through her explicitly voiced support the students responded positively. She maintained the pressure on them to persevere but at the same time she indicated to them that they were not alone in working with the mathematical task: "I can hear lots of people talking and collaborating with their thinking that means you are sharing your maths ideas and that is really important." Other researchers (e.g., Boaler, 2008; Hunter, 2009) have illustrated the powerful role such positive praise and support holds in maintaining students persevering in mathematics. Although both these research studies were in classrooms with older students, the benefits remained the same for these five-year-old students and their perseverance with the task at hand.

The egocentric behaviour of five-year-old students has the possibility to interfere with a teacher's drive for collective mathematical reasoning. Both teachers in this study attended to building norms, which included students' accountability, not only to themselves but to each other. For example, Teacher B emphasised the ways in which collective action enhanced the outcomes in saying: "We are going to learn and notice the maths together—let's use everyone's brains to succeed when solving this problem—just like when we use the parachute everyone needs to work together to make it succeed." These students had just come back into the classroom from using a parachute outside and the teacher directly supported the students to relate to this experience as shared success. This expectation that all students were to join in and problem solve together encouraged the five-year-old students to persevere with the challenging task at hand and show collective responsibility within each group. Teacher A also emphasised the role they needed to consider when working as a pair or a group and the importance of their contributions to the collective understandings when she stated, "When you come back together as a class to take it to the next step." On interview, Teacher A expanded her thinking about the role of the collective in gaining deeper and richer reasoning when she said,

> When the pairs of students solving the task together got as far as they could on their own and had as much success with their problem. Then as a whole group we take the problem as far as we can and have further success together as a whole.

Through this statement it is clear the teacher did not expect one person in the group to solve the problem and teach the others, the expectation was for all students to engage equally and contribute collectively. As Boaler (2008) showed, it is important for students to learn that working and reasoning together means that as a collective they are all more likely to reach higher mathematical understandings.

Deepening Students' Use of Mathematical Practices

Mathematical practices have received considerable attention in the past decade. Many researchers (e.g., Boaler, 2008; Hunter & Hunter, 2017, 2018; Selling 2016) have noted their importance in student engagement in successful problem solving within reasoned mathematical discourse. Most often studies related to mathematical practices have described their implementation with older students, but what makes this study different is that it took place successfully in classrooms with five-year-old students.

Expectations were explicitly placed on students making explanations that were mathematically based but within the contexts of the problems. For example, Teacher B modelled and supported her younger students to make an explanation. She used what Selling (2016) described as naming and highlighting to make the students aware when a mathematical practice was used. She was also frequently heard prompting and questioning the students to draw out the mathematical explanation. This is exemplified below:

> Student: This is a half.
>
> Teacher: What makes it a half?
>
> Student: There are two pieces and a line in the middle.

Teacher: A line in the middle?

Student: Yes, a line in the middle so both bits are the same size.

Teacher: So, you are saying: This is a half because there are two pieces, and each piece is the same size?

Student: Yes

Teacher: that is an excellent mathematical explanation that you have just made.

Clearly, two things happened in this exchange; the teacher drew out the information from the student so that a full mathematical explanation was constructed; at the same time the student retained authorship of the mathematical idea and explanation. Selling promotes naming such actions as central for all students to learn what makes an appropriate mathematical explanation. The mathematical status of the student was also raised in the teacher actions in supporting the student ownership of their mathematical reasoning (Boaler, 2008).

Likewise, when students made a complete mathematical explanation Teacher A commented audibly with statements like, “Wow, what a great mathematical explanation, as you have explained what it means to cut an item in half.” This teacher action highlighted publicly to the students the importance of what made this explanation a valid one. Selling (2016) explained the importance of such actions and the use of positive language when publicly naming mathematical practices such as “great” and “good” in developing a positive disposition in students and contributing to them constructing positive mathematical identities.

Without justification of mathematical explanations, generalising mathematical concepts often does not occur. In both these classroom lessons a natural focus was placed on requiring students to extend their mathematical arguments to providing reasons or justification. Teacher B consistently told her students to “Talk to each other about what you think and why.” Even with such young students yes or no answers were not accepted during lessons, with both teachers consistently prompting their students to explain why. As a result, the students in these lessons would make a mathematical claim and then almost naturally back it up with their justification. This can be heard in a student explanation, “We are folding this in half because then we will have two pieces, that means it is in half. If we are folding into quarters, we would have four pieces because quarters are something broken into four.” As Teacher A commented after the lesson, “The justification is really important. Even at five, they can do it. So, for this lesson, the keyword was ‘because’.” They know that when they give an answer they have to explain why, so ‘because’ and then their reasoning.” Analysis of student talk in the lessons showed a pattern in which the older students endeavoured to provide full and detailed mathematical explanations and justification. As a result, the younger children were being inducted into a mathematical environment that modelled reasoned mathematical discourse.

Teacher questioning had a key role in establishing a mathematical environment founded within reasoned discourse. Students were positioned to question the mathematics presented to them. The use of asking students to agree or disagree with a mathematical idea but expecting them to do so in a mathematical manner and using the word because was evident in both lessons. For example, when asked about their reasoning a student responded with, “I disagree because there are two dots on that side but there should be 3.” This statement was followed by the teacher then asking all the listeners to consider whether they agreed or disagreed with the statement. This encouraged the students to be active members of the lesson, and to not merely accept a rebuttal without considering all the mathematical reasoning being offered. Chapin and O’Connor (2007) described how such actions create a habit in students of reasoning and developing critical thinking. Such actions provide space for students to practise mathematical analysis and critical thinking.

Conclusion and Implications

We have illustrated in this article the importance of teacher actions for providing all students with opportunities to learn and communicate their mathematical reasoning in sense-making ways. Importantly, we showed the value of teachers believing in the potential of their New Entrant students to learn rich mathematics in sense-making ways on school entry and onwards. It was evident that these young students, when provided with opportunities to engage in mathematical practices, used solutions to develop productive mathematical discourse and conceptual understandings. These findings correspond with those described by previous researchers (e.g., Hunter & Hunter, 2018; Chapin & O'Connor, 2007; Selling 2016). However, in this research the students were of a considerably younger age than those in previous research.

Teacher actions are of critical importance in inducting students into such talk environments. As Hunter and Hunter (2017) described previously, learning to provide mathematical explanations and respond to questions take many students time and practice to learn. Within these two classrooms this was apparent but through careful scaffolding this occurred. The teachers used many actions described by Boaler (2009) and Selling (2016) to establish their safe supportive classrooms. Also evident was the model provided by older students in inducting their younger peers into an environment, which actively required interactive participation. Many New Entrant classrooms in Aotearoa/New Zealand hold a range of different aged children because they all enter school on their fifth birthday. Therefore, teachers need to consider how a constant flow of new students can be catered for. This research suggests that teachers should scaffold the older students their peers so that the mathematics community continues to develop.

Materials also hold an important place as scaffolding tools. It was clear that the two teachers recognised their importance in both supporting the students to learn to construct mathematical understandings and also to provide rich explanations and justification. This parallels what Mueller and colleagues (2014) illustrated in their research when they showed the importance of materials for students in reasoning critically and making connections across big mathematical ideas. Selling (2016) in her research also showed the importance of teachers making connections across making representations. Although she did this with senior students in this research it was shown as also valid and important to progress mathematical reasoning with significantly younger students.

Mathematical practices have received considerable attention in recent times but most often with older students. What was illustrated in this research was their place and importance in New Entrant classrooms. They appeared to provide these students with many opportunities to access mathematics with deep understandings. This was because they were in classrooms that promoted a talk-rich environment as promoted by other researchers including Chapin and O-Connor (2007), Franke and colleagues (2009) and Hunter and Hunter (2018, 2019). Teacher actions also matched those described previously by Selling (2016) and Hunter (2009). Like both these researchers showed, this research illustrates the importance of teachers responding in the moment to student engagement in the talk to support development of mathematical practices.

Although this research reports on only two teachers and their two lessons the results have potential to provide other teachers with a model of the types of action and talk they can promote in their classrooms with very young students. More research is needed to examine the outcomes of other classrooms on the mathematical achievement of young students.

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